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CSCI5521

Problem 1:

1.

$$||Xw - y||^2 = (Xw - y)^T * (Xw - y)$$

$$(Xw - y)^T * (Xw - y) = (X^T w^T - y^T) * (X w - y)$$

$$(X^T w^T - y^T) * (X w - y) = w^T X^T X w - y^T X w - w^T X^T y + y^T y$$

Setting the derivative of the function w to zero to minimize the objective function:

$$\frac{d}{dw}(w^T X^T X w - y^T X w - w^T X^T y + y^T y) = 2 X^T X w - 2 X^T y$$

$$2 X^T X w - 2 X^T y = 0$$

$$w = \frac{X^T y}{X^T X}$$

2.

$$||Xw - y||^2 + \lambda ||w||^2 = (Xw - y)^T * (Xw - y) + \lambda w^T w$$

$$(Xw - y)^T * (Xw - y) + \lambda w^T w = (X^T w^T - y^T) * (X w - y) + \lambda w^T w$$

$$(X^T w^T - y^T) * (X w - y) + \lambda w^T w = w^T X^T X w - y^T X w - w^T X^T y + y^T y + \lambda w^T w$$

Setting the derivative of the function w to zero to minimize the objective function:

$$\frac{d}{dw}(w^T X^T X w - y^T X w - w^T X^T y + y^T y + \lambda w^T w) = 2 X^T X w - 2 X^T y + 2\lambda w$$

$$2 X^T X w - 2 X^T y + 2\lambda w = 0$$

$$w = \frac{X^T y}{X^T X + \lambda}$$

Problem 2:

1.

$$\Pr(H,H,T,T,H) = p * p * (1-p) * (1-p) * p$$
$$\Pr(H,H,T,T,H) = p^3 * (1-p)^2$$

2.

a. $fair\ coin\ \boldsymbol{p} = \frac{1}{2}$

$$\Pr(fair\ coin \mid H,H,T,T,H) = \frac{1}{2}\ p^3 * (1-p)^2$$

$$\Pr(fair\ coin \mid H,H,T,T,H) = \frac{1}{2}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$$

$$\Pr(fair\ coin \mid H,H,T,T,H) = \left(\frac{1}{2^6}\right) = 0.0156$$

b. $biased\ coin\ \boldsymbol{p} = \frac{2}{3}$

$$\Pr(biased\ coin \mid H,H,T,T,H) = \frac{1}{2}\ p^3 * (1-p)^2$$

$$\Pr(biased\ coin \mid H,H,T,T,H) = \frac{1}{2}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2$$

$$\Pr(biased\ coin \mid H,H,T,T,H) = \left(\frac{2^2}{3^5}\right) = 0.0164$$

3.

$$p^3 * (1-p)^2 = p^3 * (1-p) * (1-p)$$
$$p^3 * (1-p) * (1-p) = p^5 - 2p^3 + p^3$$

$$\frac{d}{dw}\left(5p^4 - 82 + p^3\right) = 5p^5 - 8p^3 + 3p^2$$

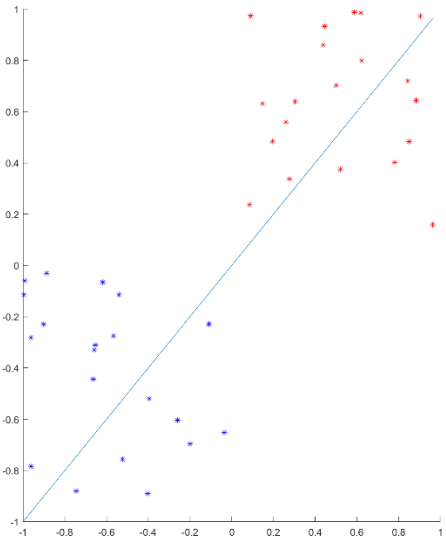
$$5p^4 - 8p^3 + 3p^2 = 0$$

Solve for p:

$$p = \frac{3}{5}$$

Problem 3:

1.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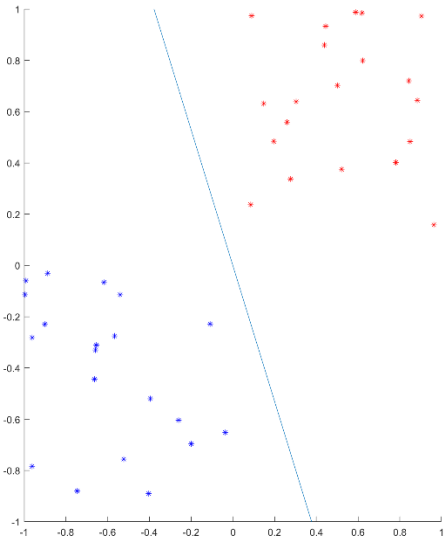
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