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EE 5340

- We know that θ is between 0 and $\pi/2$. Therefore, we can use the following to find the angle:

$$\sin(\theta) = \frac{2\sqrt{1(16-1)}}{16}$$

$$\theta = \arcsin\left(\frac{2\sqrt{1(16-1)}}{16}\right) = \arcsin\left(\frac{2\sqrt{15}}{16}\right) = 0.5053\, rad$$

- The required Grover iteration required to reach a good solution us expressed as following:

$$R = \frac{\arccos\sqrt{\frac{M}{N}}}{\theta} = \frac{\arccos\sqrt{\frac{1}{16}}}{0.5053} = 2.986$$

$$\boldsymbol{R \approx 3}$$

- Meaning that the ideal number of Grover iteration is 3.

CODE:

- Starting by preparing the oracle function of Grover's algorithm

```
f <- function( x ){
  if(x == 2) 1
  else 0
}

g <- Uf(fun=f,n=4,m=1)
```

- Preparing the initial states of qubits to be in superposition before entering the Grover's algorithm

```
# n=4 || m=1
qu <-tensor(ket(1,1), ket(1,1), ket(1,1), ket(1,1), ket(1,-1))
```

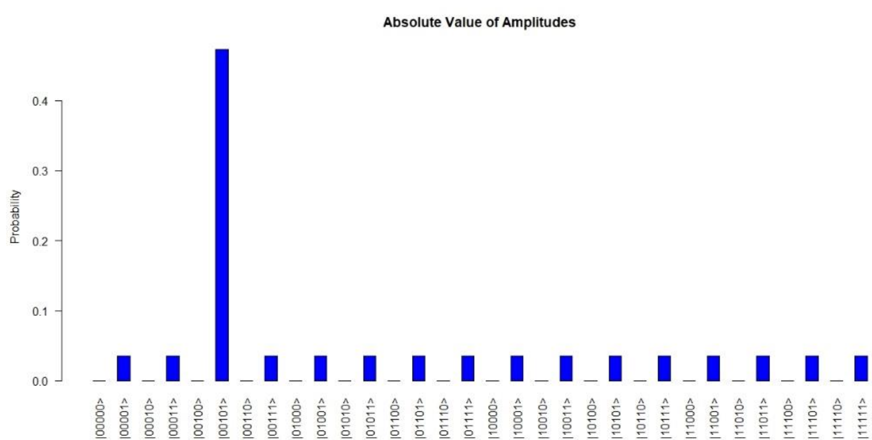
- Apply 6 iterations of the oracle and diffusion operator into the qubits:

```
# First iteration
qu <- g %*% qu
qu <- tensor(GroverDiffusion(4), I())%*% qu

# Second iteration
qu <- g %*% qu
qu <- tensor(GroverDiffusion(4), I())%*% qu

# Third iteration
qu <- g %*% qu
qu <- tensor(GroverDiffusion(4), I())%*% qu

# Fourth iteration
qu <- g %*% qu
qu <- tensor(GroverDiffusion(4), I())%*% qu

# Fifth iteration
qu <- g %*% qu
qu <- tensor(GroverDiffusion(4), I())%*% qu

# Sixth iteration
qu <- g %*% qu
qu <- tensor(GroverDiffusion(4), I())%*% qu

qu <- tensor(I(), I(), I(), I(), H()) %*% qu
```

The results of each iteration are noted below:

THE 1st ITERATOIN:

- The first iteration yells the following probabilities of all of N:

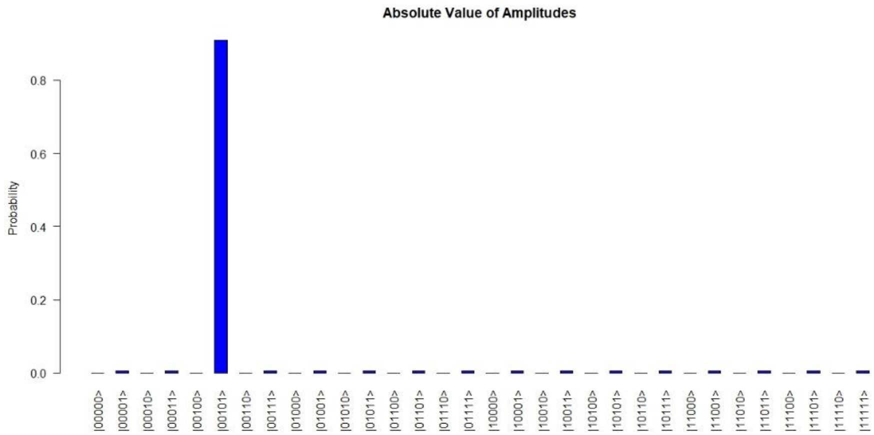


The probability of finding the solution index has increased drastically:

$$|sol >= \ |\mathbf{0010} > \left[\frac{|0 > -|1 >}{\sqrt{2}}\right] \ has\ an\ amplitude\ \mathbf{47.2}\%$$

All other non-solitons indexes have an amplitude of 3.51% each. Meaning, 52.65% probability of settling on non-solution.

THE 2^{ed} ITERATOIN:

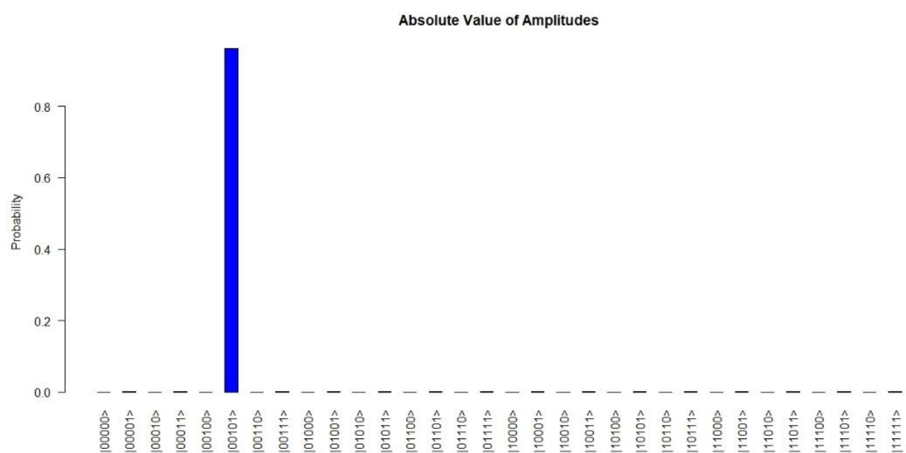


The probability of the solution index increased from 47.2% to 90.8% for the second iteration:

$$|sol >= \ |\mathbf{0010} > \left[\frac{|0 > -|1 >}{\sqrt{2}}\right] \ has\ an\ amplitude\ \mathbf{90.8}\%$$

Non-solution indexes amplitude has decreased to 0.61% each. 9.15% probability of settling on non-solution.

THE 3rd ITERATOIN:

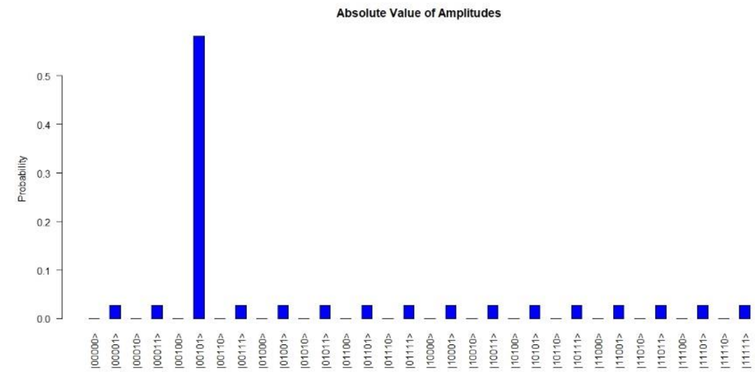


The probability of the solution index increased from 90.8% to 96.1% for the third iteration. Three interaction is the highest we can achieve based on the calculation we have done previously.

$$|sol >= \ |\mathbf{0010} > \left[\frac{|0 > -|1 >}{\sqrt{2}}\right] \ has\ an\ amplitude\ \mathbf{96.1}\%$$

Non-solution indexes amplitude has decreased to 0.25% each. 3.75% probability of settling on non-solution.

THE 4th ITERATOIN:

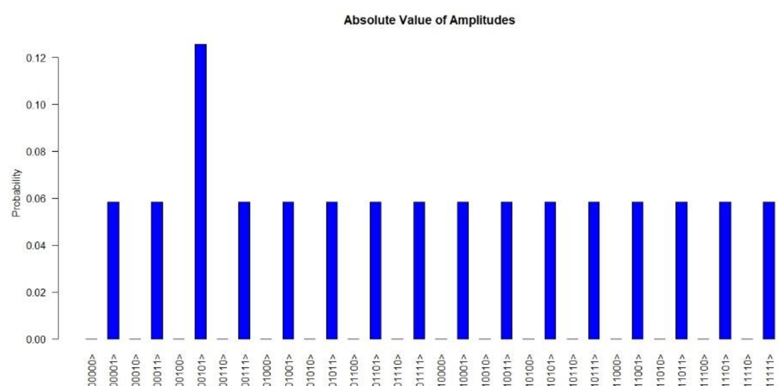


The probability of the solution index has majorly decreased from 96.1% to 58.1%. This is because we passed the range of angle from 0 to $\frac{\pi}{4}$ that are supposed to be bounded with.

$$|sol>= \ |\mathbf{0010}> \left[\frac{|0>-|1>}{\sqrt{2}}\right] \ has\ an\ amplitude\ \mathbf{58.1}\%$$

Each non-solution indexes have an amplitude probability of 4.23% each. 63.45% probability of settling on non-solution.

THE 5th ITERATOIN:

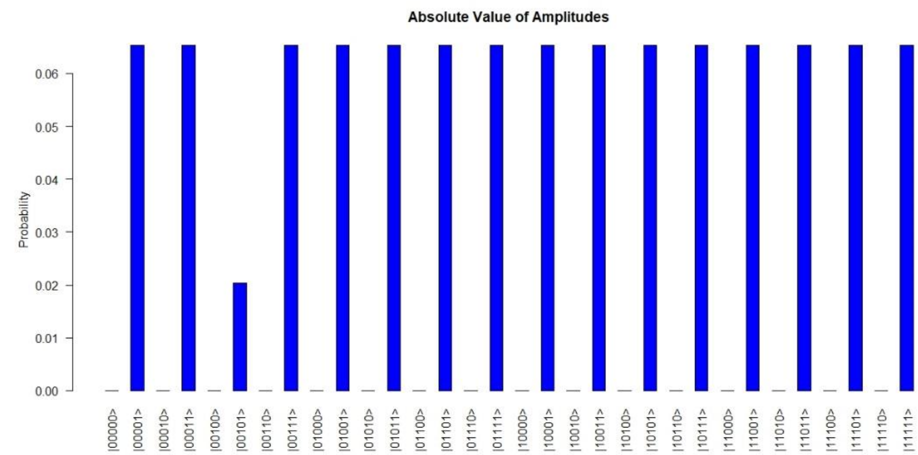


The probability of the solution index has decreased yet again from the last iteration to 12.5%. This is because we are getting further away from $|\beta>$ axis with additional iterations. Though, solution index has higher amplitude probability than non-solution still. $|\alpha>$ represents non-solution axis and $|\beta>$ represents solution axis. The goal is to be as close to $|\beta>$ axis as possible.

$$|sol>= \ |\mathbf{0010}> \left[\frac{|0>-|1>}{\sqrt{2}}\right] \ has\ an\ amplitude\ \mathbf{12.5}\%$$

non-solution indexes have an amplitude probability of 5.83% each. 87.45% probability of settling on non-solution.

THE 6th ITERATOIN:



The probability of the solution index still in declined definition from 12.5% of the last nitration to 2.03%. his time amplitude probability of the solution is less than other non-solutions which say that it's 97.9% not finding the solution. This is because it has reached close to $|\alpha>$ axis on the other side where it started. Additionally, it's even closer to $|\alpha>$ axis now that before starting because the probability now is a lot less than starting what we started.

$$|sol>= \ |\mathbf{0010}> \left[\frac{|0> -|1>}{\sqrt{2}}\right] \ has\ an\ amplitude\ \mathbf{2.03}\%$$

non-solution indexes have an amplitude probability of 6.53%.