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EE 5340

Bit Flip Code:

1. Preparation of the first qubit to be in superposition and apply CNOT-gate transformation to the second and their qubit controlled by the first qubit. The last 2 qubits are prepared in the |00> state:

```
qu <- intket(x=c(0,16), n=5, amplitudes=c(1,1))
qu <- controlled(gate=X(), n=5, cQubits=0, tQubit=1) %*% qu
qu <- controlled(gate=X(), n=5, cQubits=0, tQubit=2) %*% qu
```

The resulted quantum state:

$$\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{00000}> + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{11100}>$$

2. Creating bit-flip (X gate) error applied to one qubit at a time:

```
# bit-flip applied to the first qubit
qu <- tensor(X(), I(), I(), I(), I()) %*% qu

# bit-flip applied to the second qubit
qu <- tensor(I(), X(), I(), I(), I()) %*% qu

# bit-flip applied to the third qubit
qu <- tensor(I(), I(), X(), I(), I()) %*% qu
```

3. detecting the bit-flip error by measure the last 2 qubits which project error index:

```
qu <- controlled(gate=X(), n=5, cQubits=0, tQubit=3) %*% qu
qu <- controlled(gate=X(), n=5, cQubits=1, tQubit=3) %*% qu

qu <- controlled(gate=X(), n=5, cQubits=0, tQubit=4) %*% qu
qu <- controlled(gate=X(), n=5, cQubits=2, tQubit=4) %*% qu

qu <- measure(qu, 3, 4, l2r=TRUE)[[1]]
```

the resulted state for each of the bit-flip:

$$No\ error\ occurred = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{00000}> + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{11100}>$$

$$bit\ flip\ on\ 1^{st}\ qubit = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left|\mathbf{01111} > + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right|\mathbf{10011} >$$

$$bit\ flip\ on\ 2^{ed}\ qubit = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{01010} > + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{10110} >$$

$$bit\ flip\ on\ 3^{rd}\ qubit = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{00101} > + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}|\mathbf{11001} >$$

4. fix the detected error by applying CNOT-gate when necessary to the first 3 qubits:

```
qu <- controlled(gate=X(), n=5, cQubits=3, tQubit=1) %*% qu
qu <- controlled(gate=X(), n=5, cQubits=4, tQubit=2) %*% qu
```

If there is **no error** occurred, then qubits will preserve their original state:

$$\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left|\mathbf{00000} > + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right|\mathbf{11100} > \quad \rightarrow \quad \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left|\mathbf{00000} > + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right|\mathbf{11100} >$$

Fix the bit flip error occurred on the **first** qubit:

$$\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left|\mathbf{01111} > + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right|\mathbf{10011} > \quad \rightarrow \quad \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left|\mathbf{00011} > + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right|\mathbf{11111} >$$

Fix the bit flip error occurred on the **second** qubit:

$$\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left|\mathbf{01010} > + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right|\mathbf{10110} > \quad \rightarrow \quad \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left|\mathbf{00010} > + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right|\mathbf{11110} >$$

Fix the bit flip error occurred on the **Third** qubit:

$$\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left|\mathbf{00101} > + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right|\mathbf{11001} > \quad \rightarrow \quad \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\left|\mathbf{00001} > + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\right|\mathbf{11101} >$$

Density Matrices and Pauli Noise:

The following code produces the plot below:

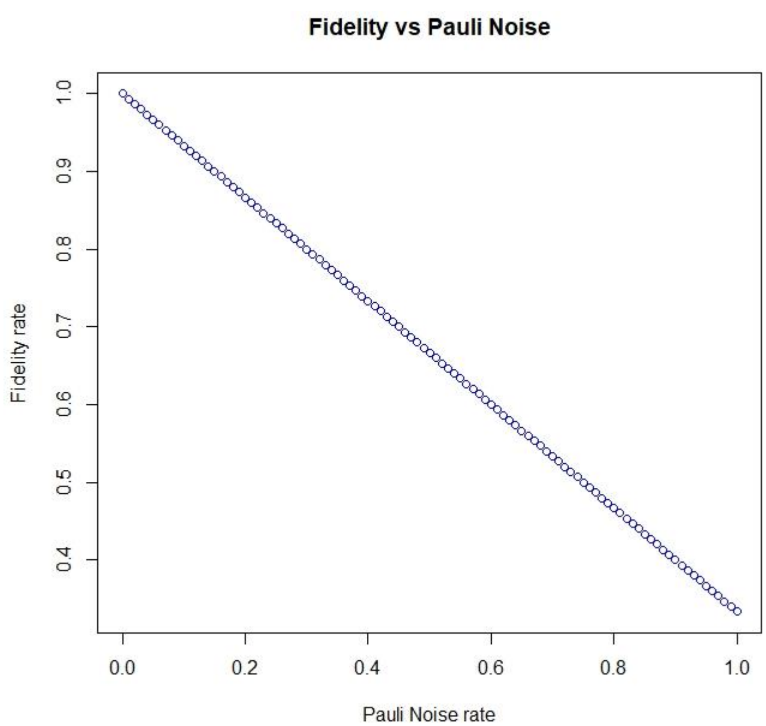
```
fidelity <- function(V,W){
    sum(diag( V %*% W ))
}

V <- convert_ket2DM(ket(1,0))

x_axis <- seq(0,1,length.out=101)

for (i in 1:length(x_axis)){
  W <- PauliNoise(p=V,e=x_axis[i])
  l[i] <- fidelity(V,W)
}
```

The produced plot:



The corresponding fidelity rates shown in the plot are following:

```
[1] 1.0000000 0.9933333 0.9866667 0.9800000 0.9733333 0.9666667
[7] 0.9600000 0.9533333 0.9466667 0.9400000 0.9333333 0.9266667
[13] 0.9200000 0.9133333 0.9066667 0.9000000 0.8933333 0.8866667
[19] 0.8800000 0.8733333 0.8666667 0.8600000 0.8533333 0.8466667
[25] 0.8400000 0.8333333 0.8266667 0.8200000 0.8133333 0.8066667
[31] 0.8000000 0.7933333 0.7866667 0.7800000 0.7733333 0.7666667
[37] 0.7600000 0.7533333 0.7466667 0.7400000 0.7333333 0.7266667
[43] 0.7200000 0.7133333 0.7066667 0.7000000 0.6933333 0.6866667
[49] 0.6800000 0.6733333 0.6666667 0.6600000 0.6533333 0.6466667
[55] 0.6400000 0.6333333 0.6266667 0.6200000 0.6133333 0.6066667
[61] 0.6000000 0.5933333 0.5866667 0.5800000 0.5733333 0.5666667
```

```
[67] 0.5600000 0.5533333 0.5466667 0.5400000 0.5333333 0.5266667
[73] 0.5200000 0.5133333 0.5066667 0.5000000 0.4933333 0.4866667
[79] 0.4800000 0.4733333 0.4666667 0.4600000 0.4533333 0.4466667
[85] 0.4400000 0.4333333 0.4266667 0.4200000 0.4133333 0.4066667
[91] 0.4000000 0.3933333 0.3866667 0.3800000 0.3733333 0.3666667
[97] 0.3600000 0.3533333 0.3466667 0.3400000 0.3333333
```

The plot indicates an inverse linear relationship between the Pauli noise injected in the system and the fidelity rate. The maximum amount of noise injected to the system (**ε = 1**) would have a **fidelity of 33.3%.**

We know that **ε** is the summation probability of the corresponding X-gate, Y-gate, and Z-gate errors:

$$\varepsilon = \varepsilon_X + \varepsilon_Y + \varepsilon_Z$$

Since the we set the fidelity to maximum $\varepsilon = 1$ and every gate error is linearly independent of each other and collectively exhaustive we get:

$$\varepsilon_X + \varepsilon_Y + \varepsilon_Z = 1$$

$$\varepsilon_X = \varepsilon_Y = \varepsilon_Z = \frac{1}{3}$$

Because **Z-gate** transformation has no effect whatsoever on **|0> state**, only X-gate and Y-gate influence the fidelity rate. Therefore, the fidelity rate of **|0> state** only dependent on ε_X and ε_Y and it can only reach a minimum of 33.3% when overall error rate (**ε = 1)** is set to maximum.