Probabilistic logic and statistical inference

STATISTICAL THINKING IN PYTHON (PART 1)

Justin Bois

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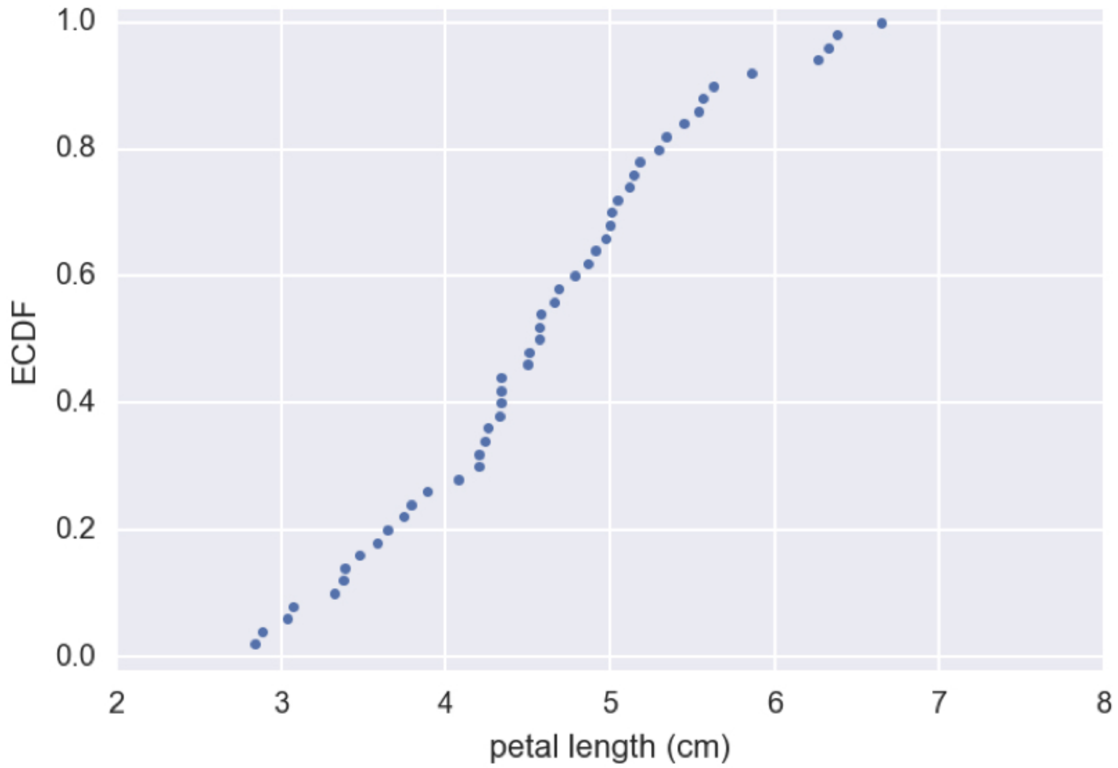
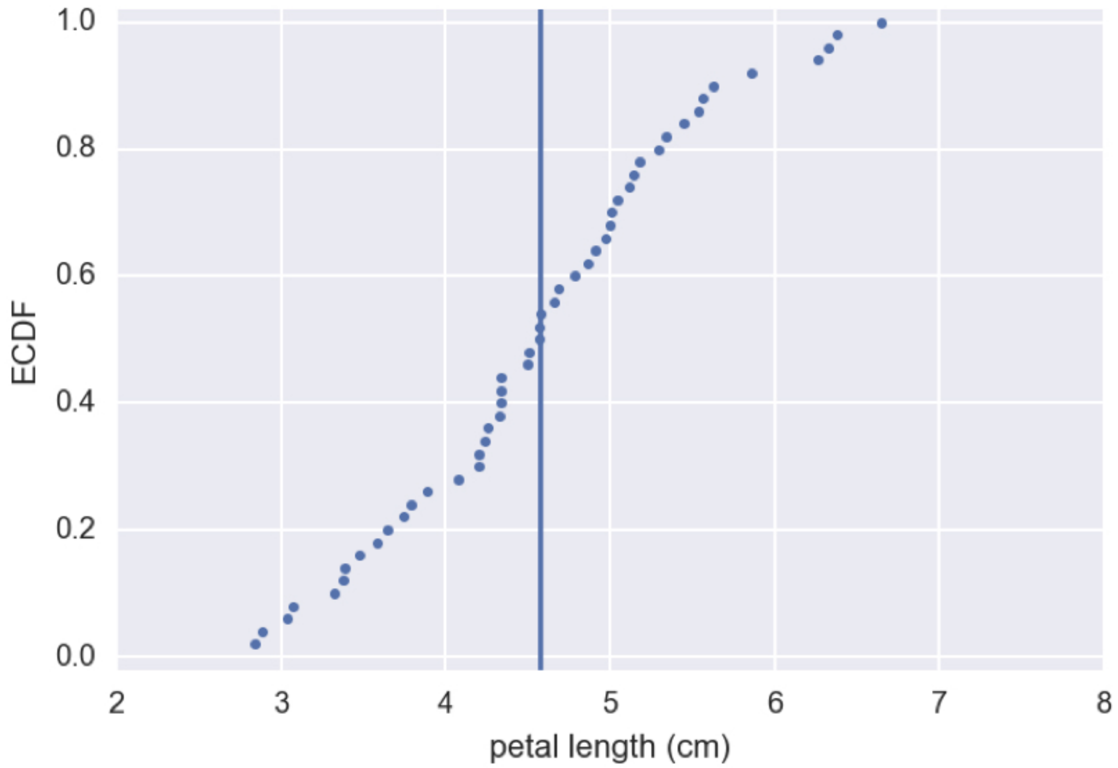
50 measurements of petal length



50 measurements of petal length



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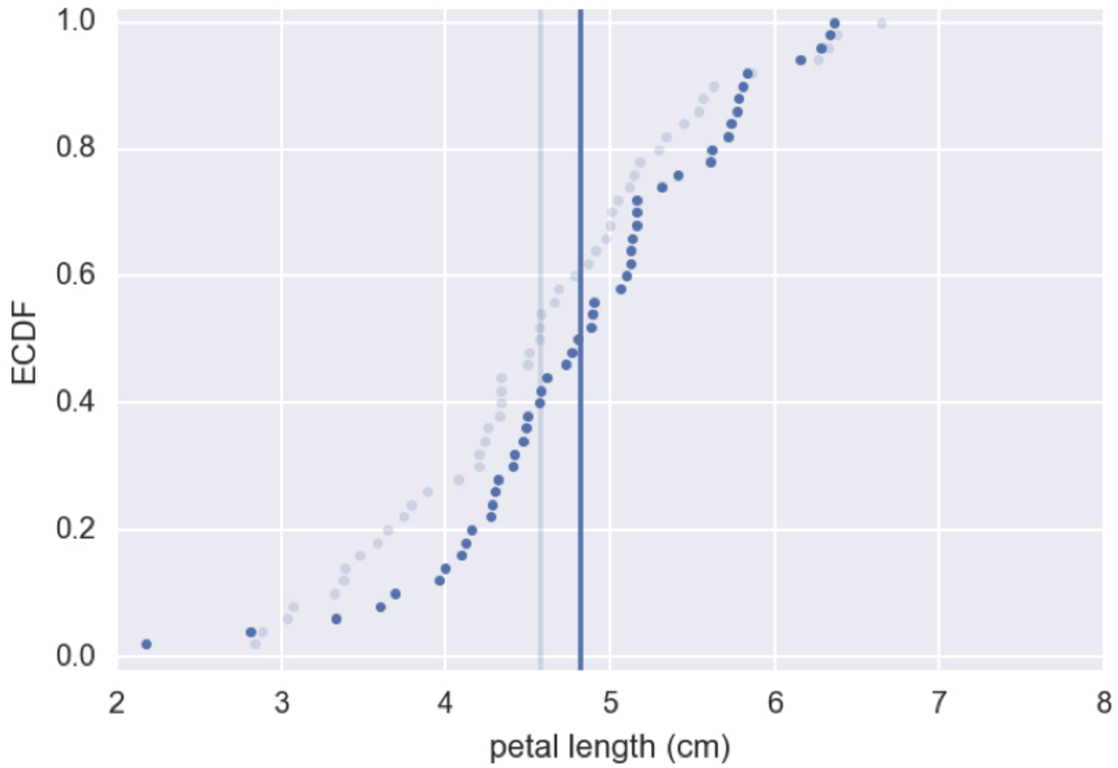
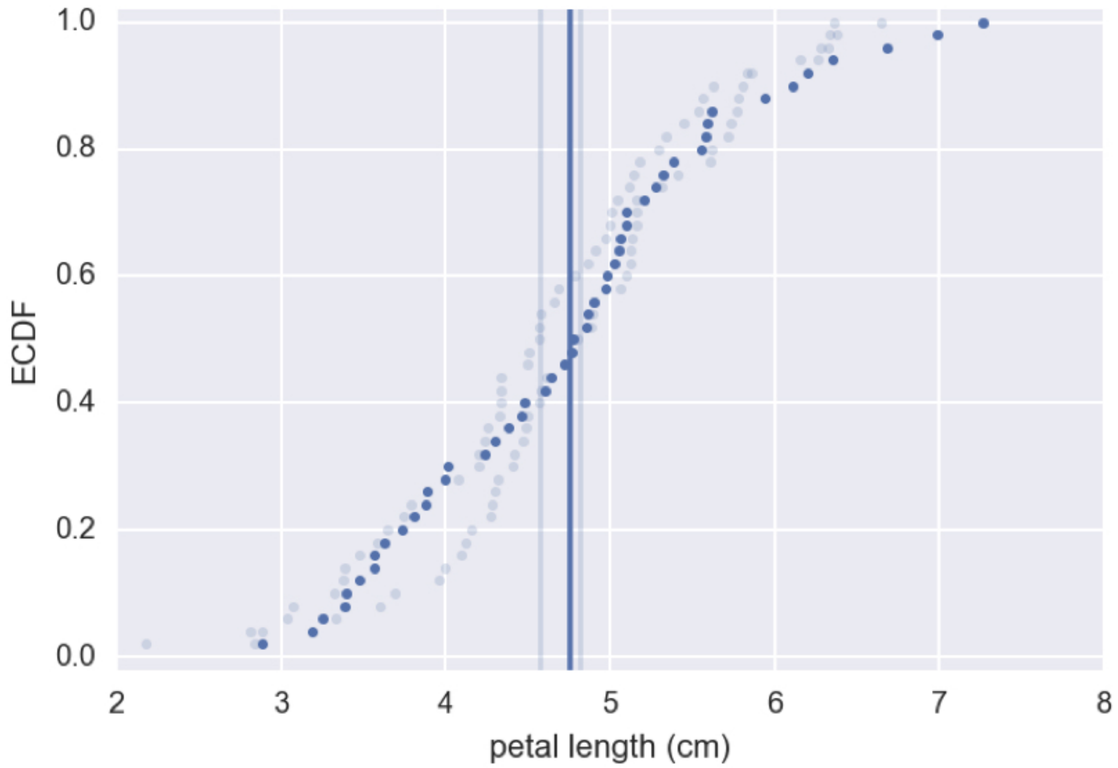
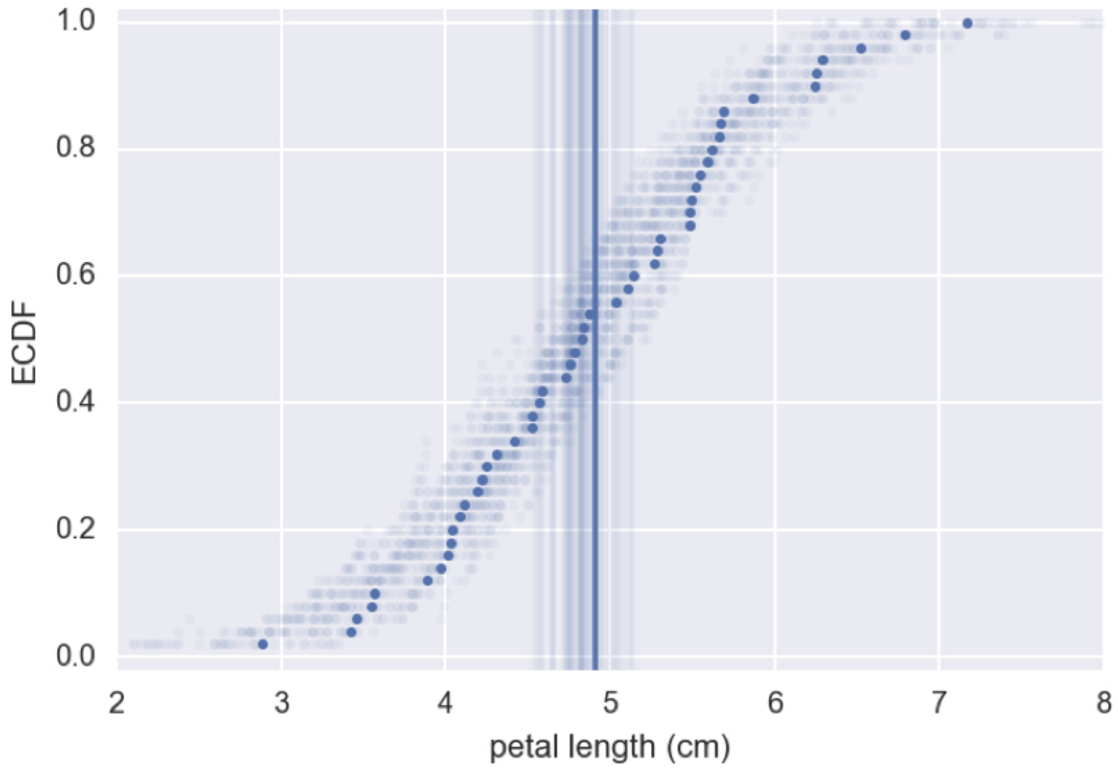


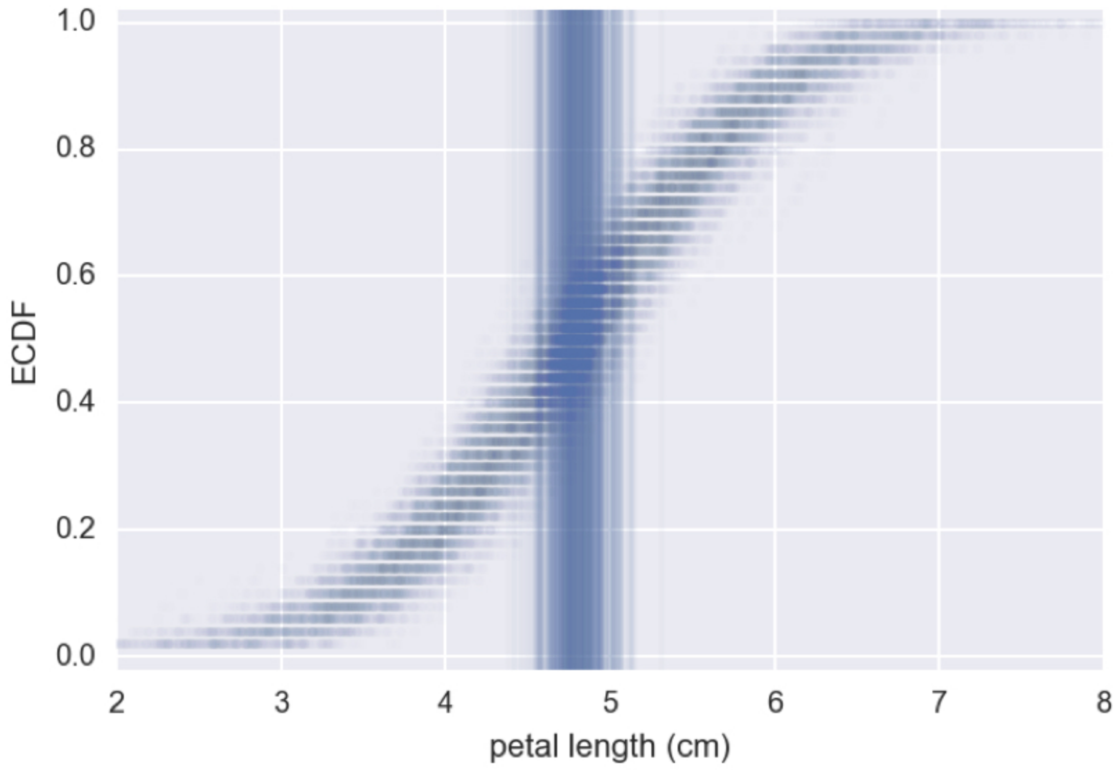
50 measurements of petal length



50 measurements of petal length



Repeats of 50 measurements of petal length



Let's practice!

STATISTICAL THINKING IN PYTHON (PART 1)

Random number generators and hacker statistics

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datacamp

Hacker statistics

- Uses simulated repeated measurements to compute probabilities.

[1] Image: artist unknown

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The np.random module

- Suite of functions based on random number generation
- `np.random.random()` : draw a number between 0 and 1

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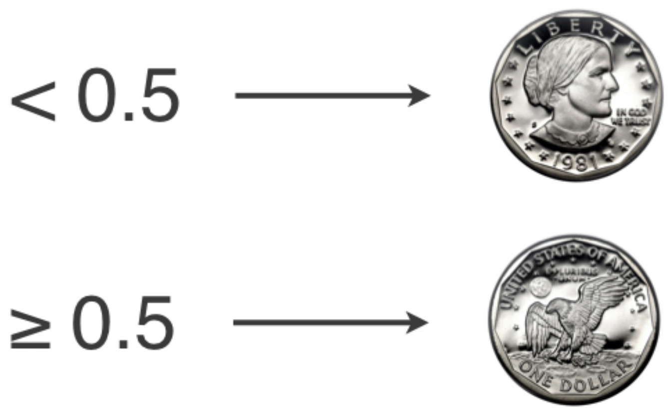

Bernoulli trial

- An experiment that has two options, "success" (True) and "failure" (False).

Random number seed

- Integer fed into random number generating algorithm
- Manually seed random number generator if you need reproducibility
- Specified using `np.random.seed()`

Simulating 4 coin flips

```
import numpy as np
np.random.seed(42)
random_numbers = np.random.random(size=4)
random_numbers
```

```
array([ 0.37454012,  0.95071431,  0.73199394,  0.59865848])
```

```
heads = random_numbers < 0.5
heads
```

```
array([ True, False, False, False], dtype=bool)
```

```
np.sum(heads)
```

```
1
```

Simulating 4 coin flips

```
n_all_heads = 0  # Initialize number of 4-heads trials
for _ in range(10000):
        heads = np.random.random(size=4) < 0.5
        n_heads = np.sum(heads)
        if n_heads == 4:
            n_all_heads += 1

n_all_heads / 10000
```

```
0.0621
```

Hacker stats probabilities

- Determine how to simulate data
- Simulate many many times
- Probability is approximately fraction of trials with the outcome of interest

Let's practice!

STATISTICAL THINKING IN PYTHON (PART 1)

Probability distributions and stories: The Binomial distribution

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Probability mass function (PMF)

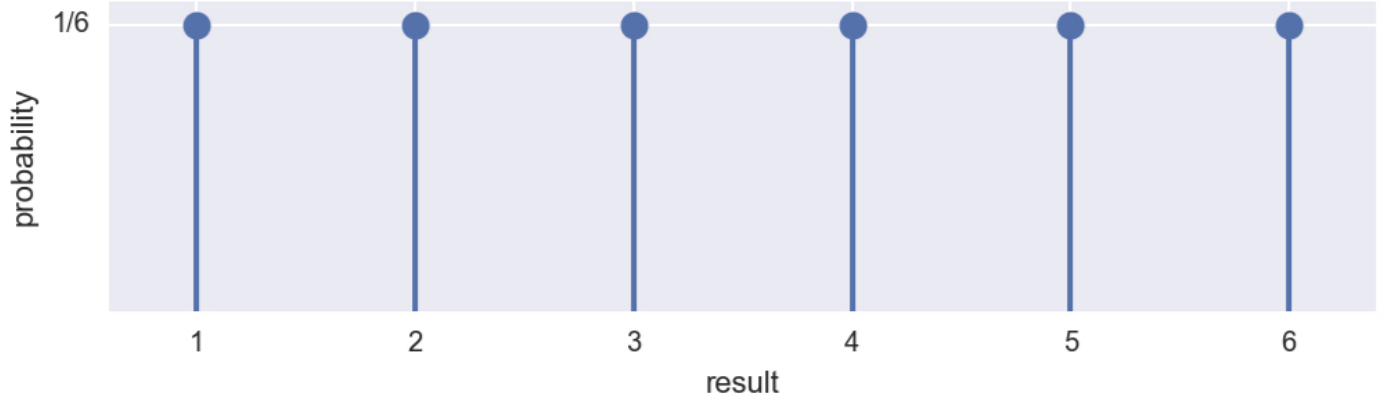
- The set of probabilities of discrete outcomes

Discrete Uniform PMF

Tabular

⚀	⚁	⚂	⚃	⚄	⚅
1/6	1/6	1/6	1/6	1/6	1/6

Graphical



Probability distribution

- A mathematical description of outcomes

Discrete Uniform distribution: the story

The outcome of rolling a single fair die is

- Discrete
- Uniformly distributed.

Binomial distribution: the story

- The number r of successes in n Bernoulli trials with probability p of success, is Binomially distributed
- The number r of heads in 4 coin flips with probability 0.5 of heads, is Binomially distributed

Sampling from the Binomial distribution

```
np.random.binomial(4, 0.5)
```

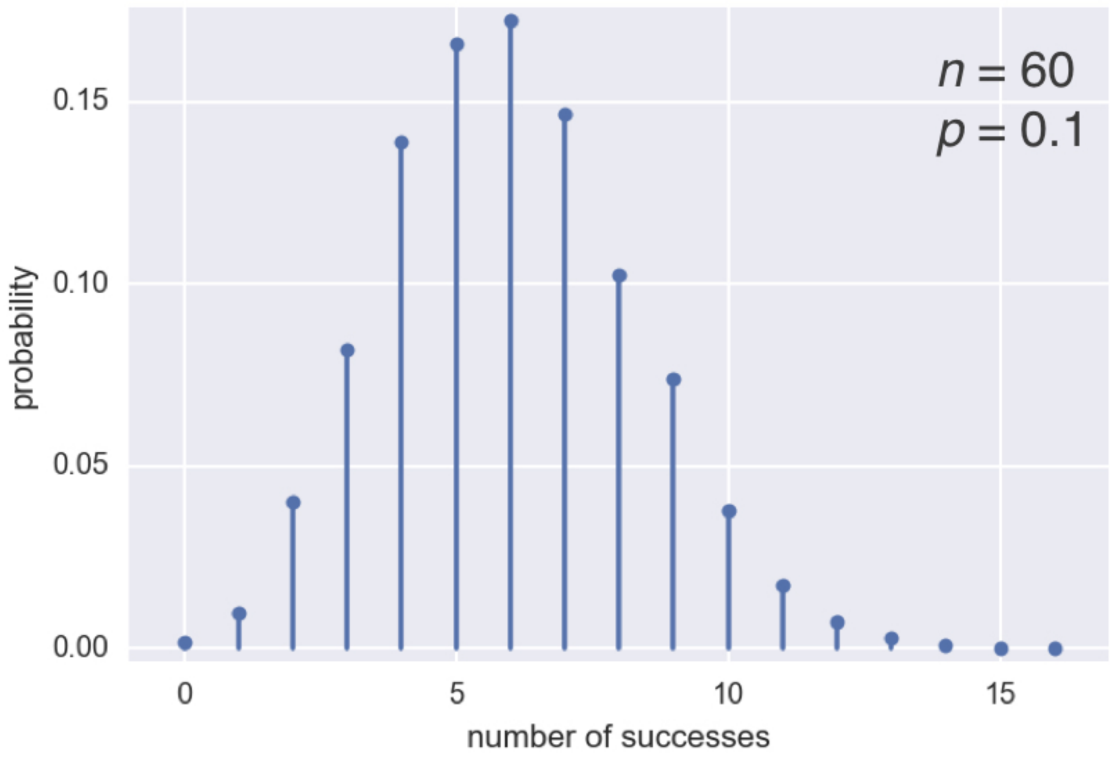
```
2
```

```
np.random.binomial(4, 0.5, size=10)
```

```
array([4, 3, 2, 1, 1, 0, 3, 2, 3, 0])
```

The Binomial PMF

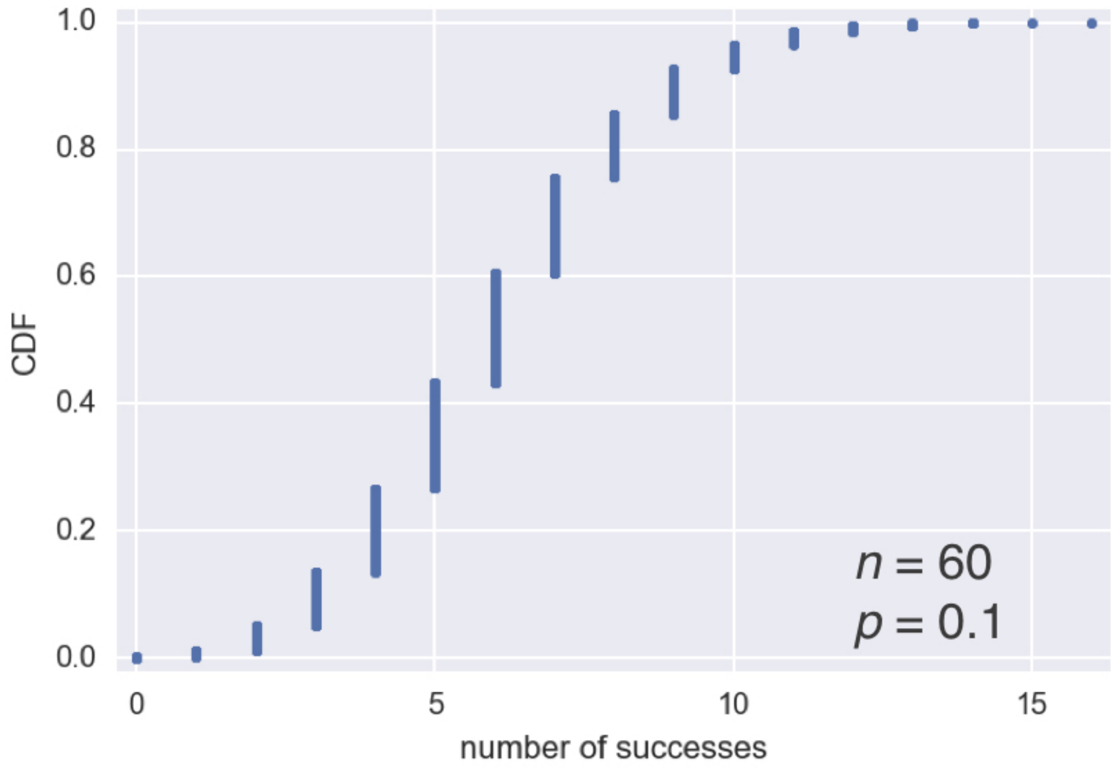
```
samples = np.random.binomial(60, 0.1, size=10000)
n = 60
p = 0.1
```



The Binomial CDF

```
import matplotlib.pyplot as plt
import seaborn as sns
sns.set()
x, y = ecdf(samples)
_ = plt.plot(x, y, marker='.', linestyle='none')
plt.margins(0.02)
_ = plt.xlabel('number of successes')
_ = plt.ylabel('CDF')
plt.show()
```

The Binomial CDF



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STATISTICAL THINKING IN PYTHON (PART 1)

Poisson processes and the Poisson distribution

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Poisson process

- The timing of the next event is completely independent of when the previous event happened

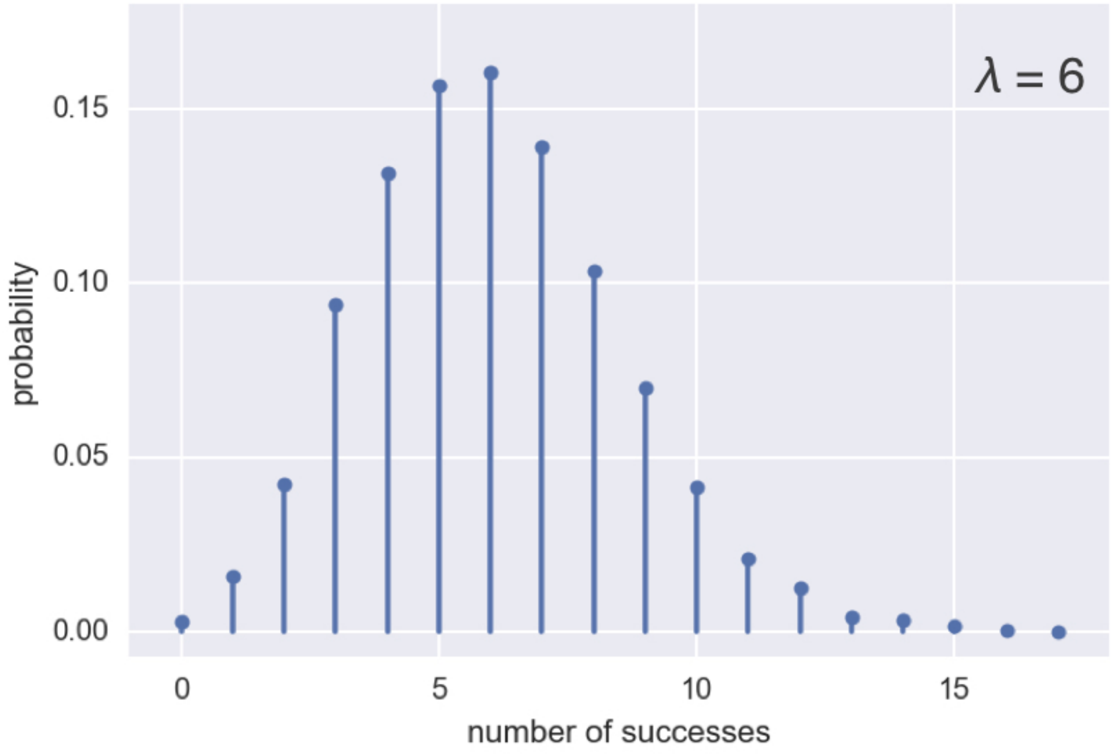
Examples of Poisson processes

- Natural births in a given hospital
- Hit on a website during a given hour
- Meteor strikes
- Molecular collisions in a gas
- Aviation incidents
- Buses in Poissonville

Poisson distribution

- The number r of arrivals of a Poisson process in a given time interval with average rate of ? arrivals per interval is Poisson distributed.
- The number r of hits on a website in one hour with an average hit rate of 6 hits per hour is Poisson distributed.

Poisson PMF



$\lambda = 6$
0.15
0.10
0.05
0.00
probability
0
5
10
15
number of successes

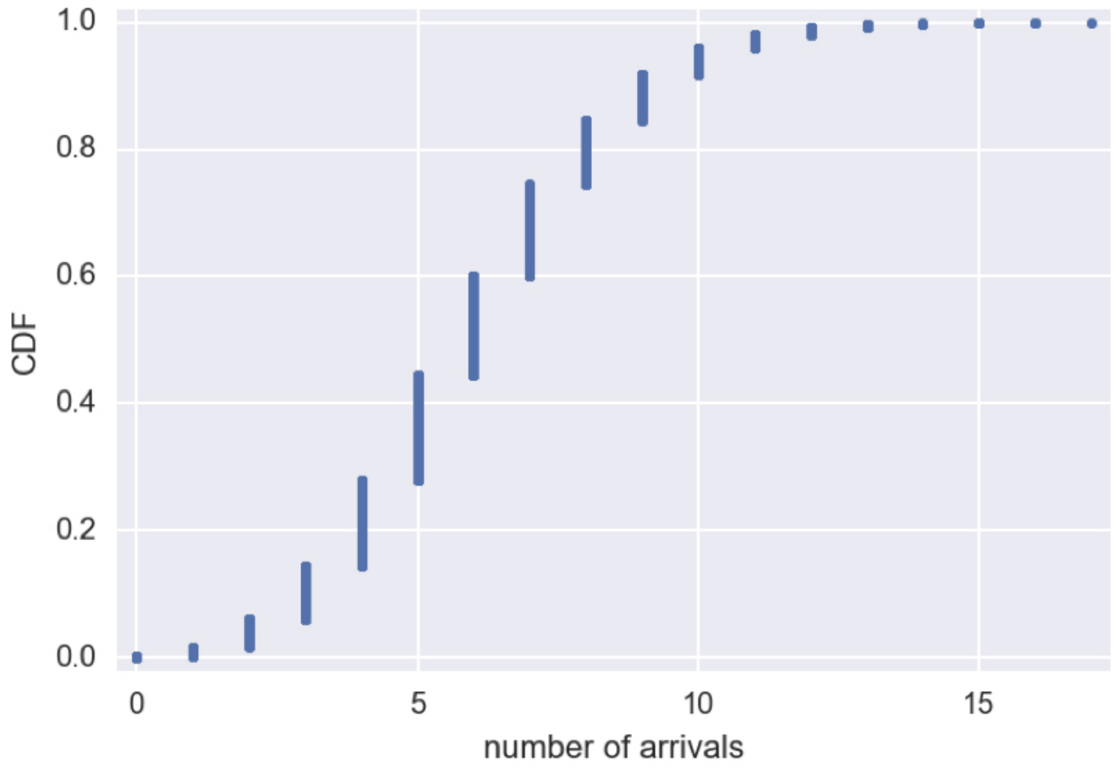
Poisson Distribution

- Limit of the Binomial distribution for low probability of success and large number of trials.
- That is, for rare events.

The Poisson CDF

```
samples = np.random.poisson(6, size=10000)
x, y = ecdf(samples)
_ = plt.plot(x, y, marker='.', linestyle='none')
plt.margins(0.02)
_ = plt.xlabel('number of successes')
_ = plt.ylabel('CDF')
plt.show()
```

The Poisson CDF



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