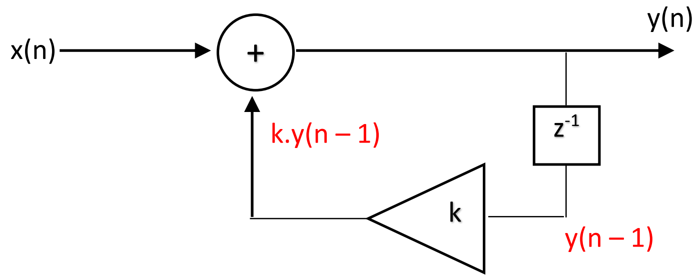
1)



=> $y(n) = x(n) + k.y(n-1)$

=> $Y(Z) = X(Z) + k.Z^{-1}Y(Z)$

=> $Y(Z) - kZ^{-1}Y(Z) = X(Z)$

=> $Y(Z)\ (1 - kZ^{-1}) = X(Z)$

=> $\frac{Y(Z)}{X(Z)} = H(Z) = \frac{1}{1 - kZ^{-1}}$

(nedensellik varsa YB çemberlerin dış bölgesi olmalıdır) => nedensel değilse => |z| < |k|

(kararlılık varsa YB birim çemberi içerir, başka bir deyişle z yerine 1 koyduğumuz zaman YB aralığını sağlamalıdır) => kararlı ise => |k| > 1 olmalıdır

=> |z| < |k| => $h(n) = -k^n u(-n-1)$

=> |k| > 1 => k = 2 dersek => $h(n) = -2^n u(-n-1)$

2)

$$\omega_0 = \frac{2\Pi}{T} = \frac{2\Pi}{2T_0} = \frac{\Pi}{T_0}$$

$$a_k = \frac{1}{T}\int x(t)e^{-jk\omega_0 t}dt = \frac{1}{2T_0}\int \left(\delta(t) - \delta(t - T_0)\right)e^{-jk\omega_0 t}dt =$$

$$\frac{1}{2T_0}\left(\int \delta(t)e^{-jk\omega_0 t}dt - \int \delta(t - T_0)e^{-jk\omega_0 t}dt\right) = \frac{1}{2T_0}\left(1 - e^{-jk\omega_0 T_0}\right) => \omega_0 = \frac{\Pi}{T_0} =>$$

$$a_k = \frac{1}{2T_0}\left(1 - e^{-jk\frac{\Pi}{T_0}T_0}\right) = \frac{1}{2T_0}\left(1 - e^{-jk\Pi}\right) => e^{-j\Pi} = cos\Pi - jsin\Pi = -1 =>$$

$$a_k = \frac{1}{2T_0}\left(1 - (-1)^k\right)$$

$$=> k\ çift => a_k = \frac{1}{2T_0}(1 - 1) = \frac{1}{2T_0}0 = 0$$

$$=> k\ tek => a_k = \frac{1}{2T_0}(1 - (-1)) = \frac{1}{2T_0}2 = \frac{1}{T_0}$$

3)

a)

$$x(t) = -\frac{1}{2}\delta(t+1) + \delta(t) - \frac{1}{2}\delta(t-1)$$

$$X(\omega) = \int x(t)e^{-j\omega t}dt = \int -\frac{1}{2}\delta(t+1)e^{-j\omega t}dt + \int \delta(t)e^{-j\omega t}dt + \int -\frac{1}{2}\delta(t-1)e^{-j\omega t}dt$$

$$X(\omega) = 1 - \frac{1}{2}\left(e^{-j\omega} + e^{j\omega}\right) = 1 - \frac{e^{j\omega} + e^{-j\omega}}{2} = 1 - \cos(\omega)$$

b)

$$T = T_1 => \omega_0 = \frac{2\Pi}{T} = \frac{2\Pi}{T_1}$$

$$a_k = \frac{1}{T}\int x(t)e^{-jk\omega_0 t}dt => \int x(t)e^{-j\omega t}dt = 1 - \cos(\omega) => \omega = k\omega_0 =>$$

$$\frac{1}{T}\int x(t)e^{-jk\omega_0 t}dt = \frac{1}{T_1}(1 - \cos(k\omega_0)) = \frac{1 - \cos(k\omega_0)}{T_1}$$

4)

$$x(t) = \frac{1}{2\Pi}\int X(\omega)e^{j\omega t}d\omega =>$$

$$\frac{1}{a^2 + t^2} = \frac{1}{2a} \cdot \frac{2a}{a^2 + t^2} = \frac{1}{2a}\left(\frac{1}{a + jt} + \frac{1}{a - jt}\right) = \frac{1}{2\Pi}\int X_2(\omega)e^{j\omega t}d\omega =>$$

$$\frac{2a}{a^2 + \omega^2} = \frac{1}{a + j\omega} + \frac{1}{a - j\omega} = \int_{-\infty}^{\infty} e^{-a|t|}e^{-j\omega t}dt =>$$

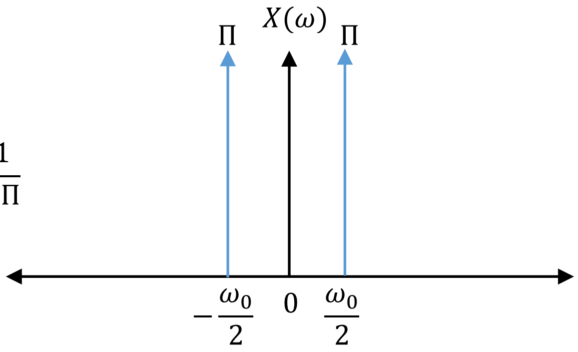
$$\frac{1}{2a}\left(\frac{1}{a + jt} + \frac{1}{a - jt}\right) = \frac{1}{2a}\left(\int_{-\infty}^{\infty} e^{-a|\omega|}e^{j\omega t}d\omega\right) = \frac{1}{2\Pi}\int X_2(\omega)e^{j\omega t}d\omega =>$$

$$X_2(\omega) = \frac{\Pi}{a}e^{-a|\omega|}$$

5)

$x_1(t) = \cos\left(\frac{\omega_0}{2}t\right)\cos(2\omega_0 t)$ =>

$X_1(\omega) = \frac{1}{2\Pi}$



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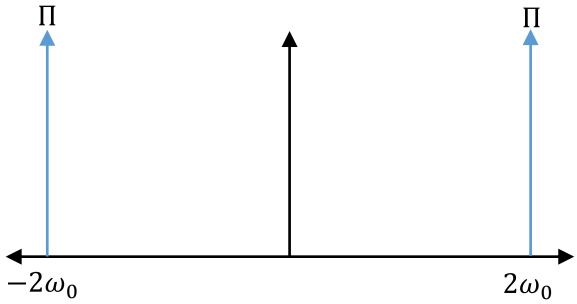
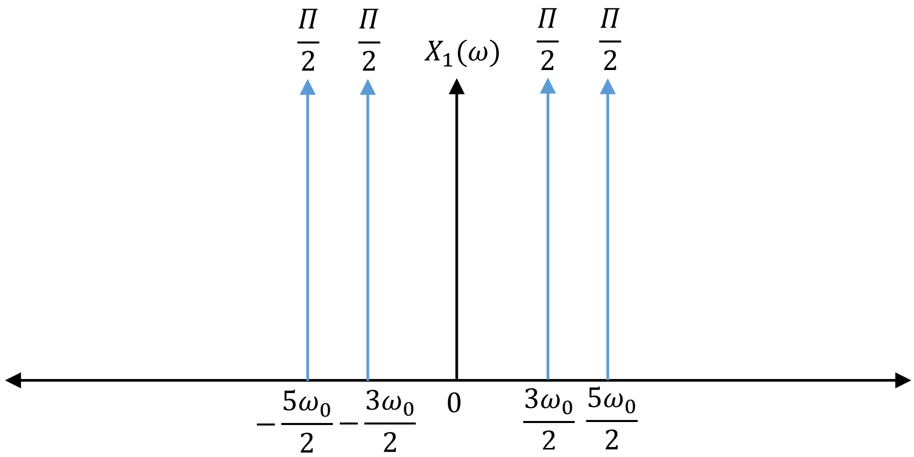
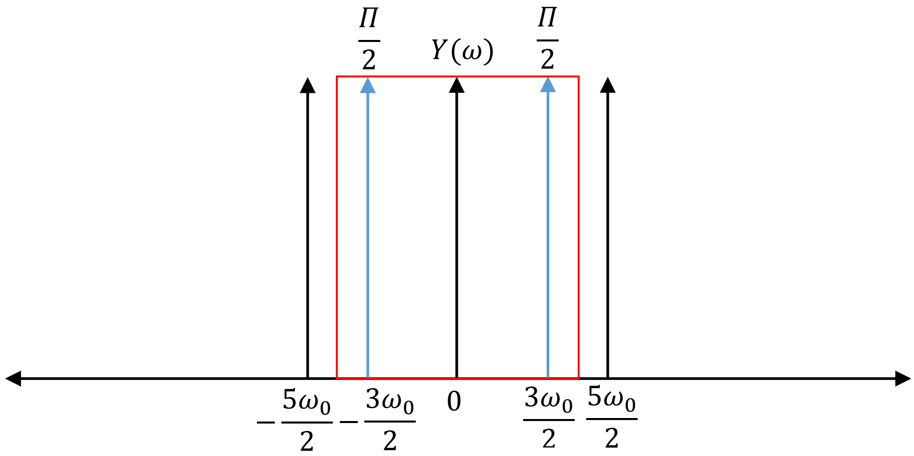


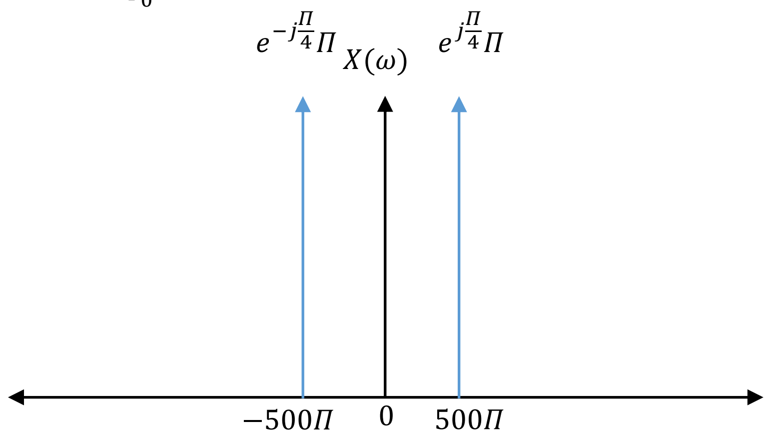


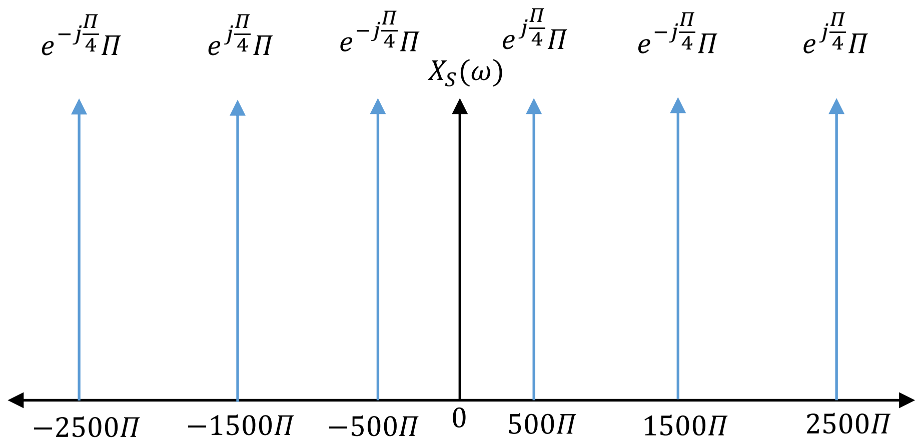


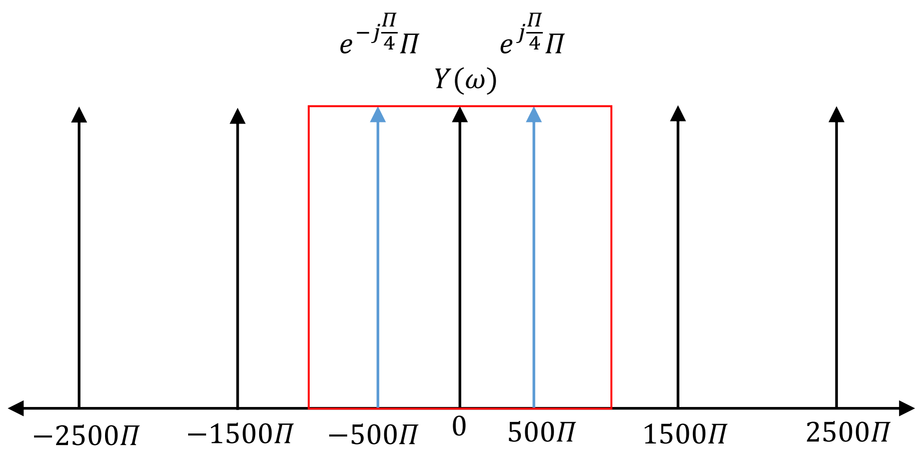
$y(t) = \frac{1}{2}\cos\left(\frac{3\omega_0}{2}t\right)$

6) $T_s = 1\ ms = \ 10^{-3}\ s\ \omega_s = \frac{2\Pi}{T_s} = 2000\Pi$

a) $\omega_0 = \frac{2\Pi}{T_0} = 500\Pi => T_0 \ = 0{,}004s$

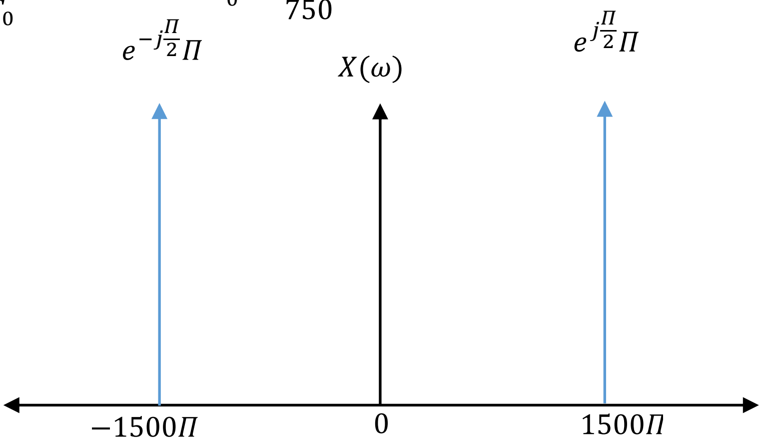


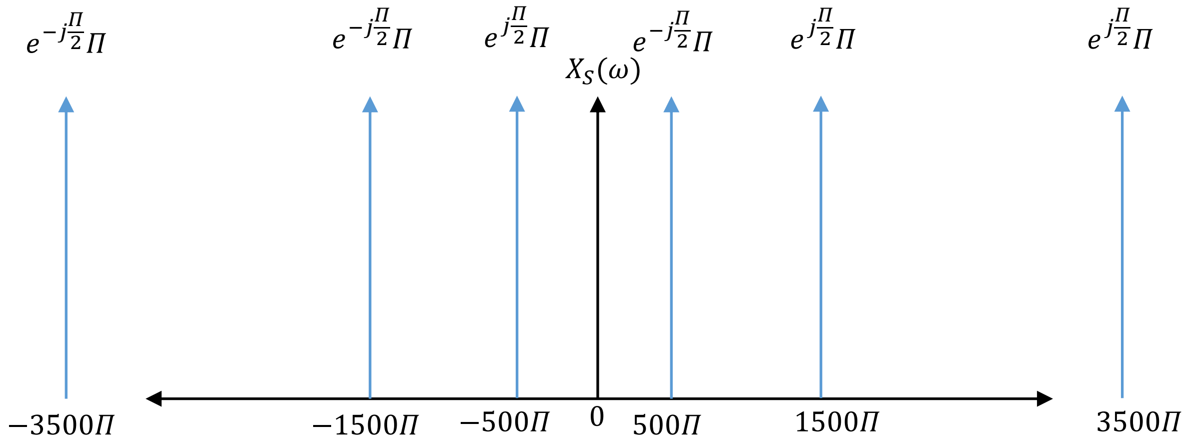


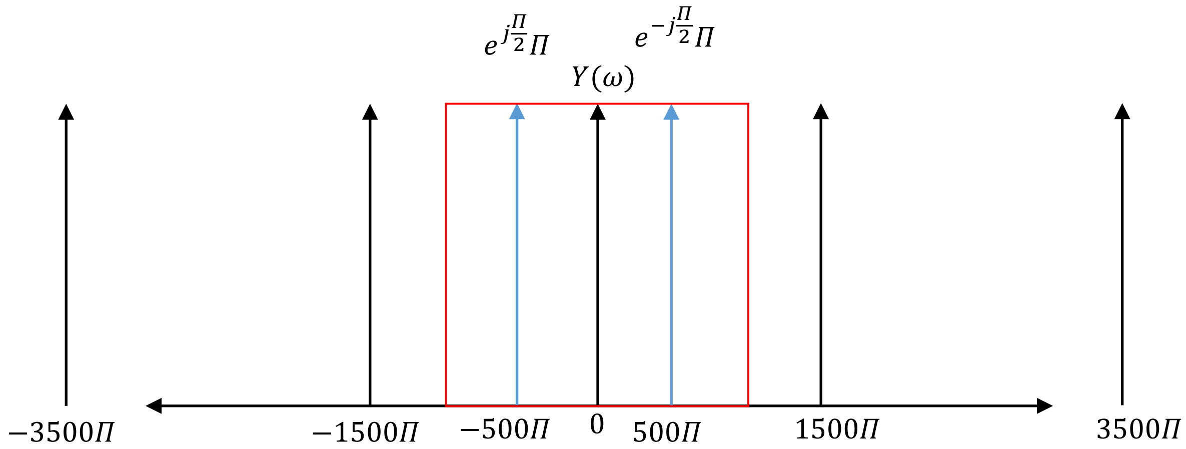


$y(t) = \cos(500\Pi t + \frac{\Pi}{4})$

b) $\omega_0 = \frac{2\Pi}{T_0} = 1500\Pi => T_0 = \frac{1}{750}s$

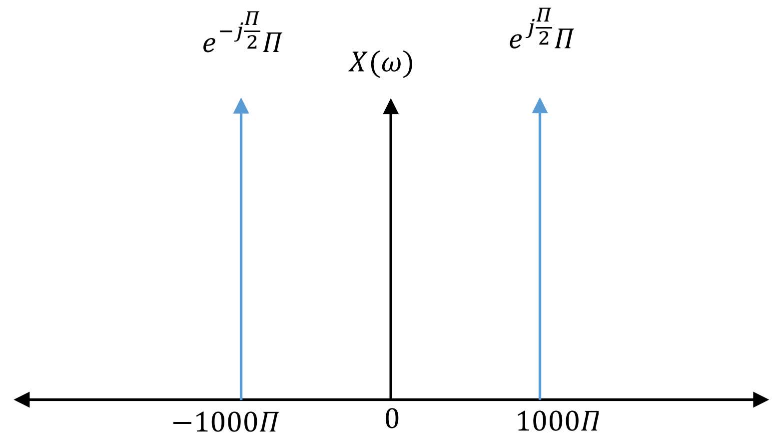


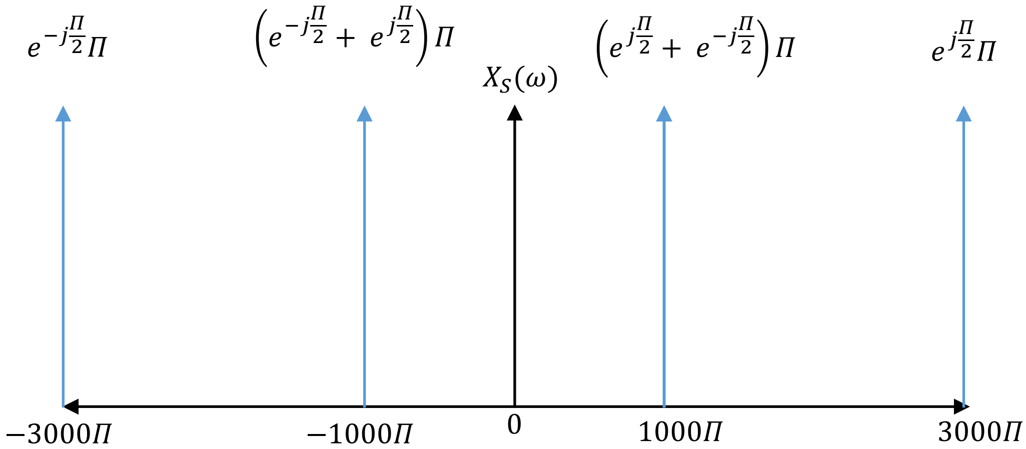


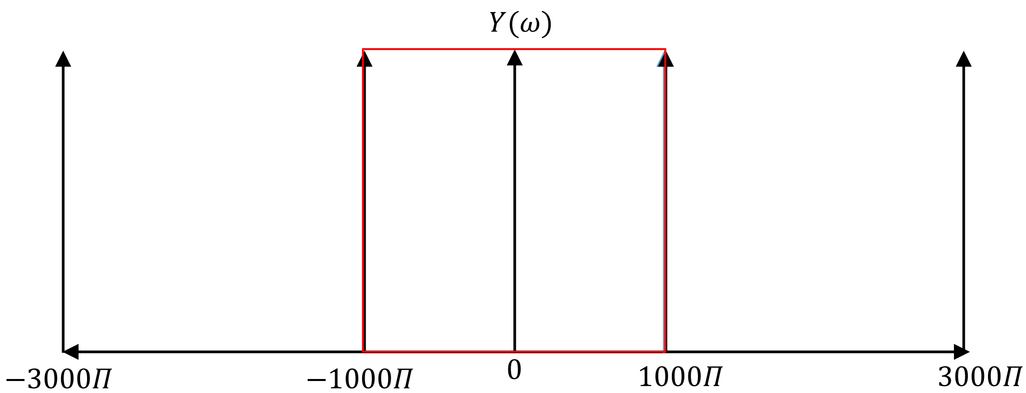


$y(t) = \cos(500\Pi t - \frac{\Pi}{2})$

c) $\omega_0 = \dfrac{2\Pi}{T_0} = 1000\Pi \Rightarrow T_0 = 0{,}001s$







$y(t) = 0$