ENGINEERING PHYSICS

Mohana Lakshmi, Ph.D.

Department of Science and Humanities

Electric Field : Any region where an electric charge experiences a force is called an electric field.

Magnetic Field : The region around a current carrying conductor or a permanent magnet where magnetic effects are experienced is called a magnetic field.

Both quantities are spatial 3D vectors that vary over time and may be denoted as

E[x, y, z, t] &

B[x, y, z, t]

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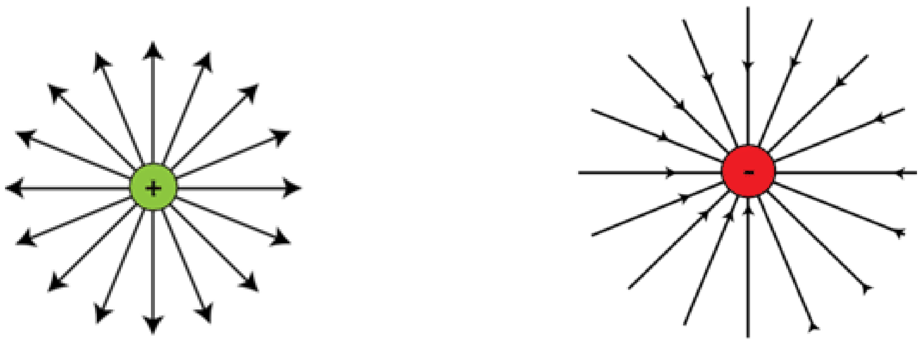
Concepts of Electric fields

Electric fields can be visualized through the electric flux lines

Electric field lines from positive and negative charges



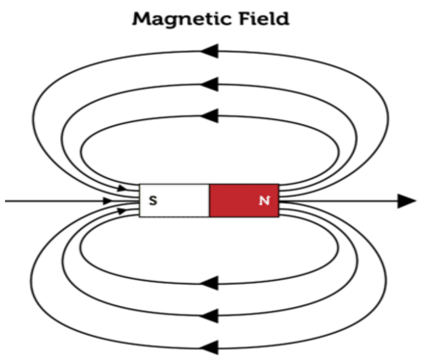
Images courtesy **Hyperphysics**, **Wikipedia**

Concepts of Magnetic fields

Magnetic dipoles

- *Magnetic mono poles do not exist*
- *Fields can be expressed in terms of the flux lines*
- *Flux lines are continuous from the north pole to the south pole*

Magnetic field lines of a magnetic dipole



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Del or Nabla operator - $\vec{\nabla}$

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The Nabla operator is a differential vector operator

- $\vec{\nabla} = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ ….. *Del operator*
- $\vec{\nabla}.\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \nabla^2$ ….. *Laplacian operator*
- $\vec{\nabla} \times (\vec{\nabla} \times A) = \vec{\nabla}(\vec{\nabla}.A) - \nabla^2 A$ ….. *Vector identity*

Operations with Del or Nabla operator - $\vec{\nabla}$

Operations with the Nabla operator (del operator)

- $\vec{\nabla}$ operates on a scalar to give a vector

  - Gradient of the scalar

- The dot product (.) of ∇ with a vector gives a scalar

  - Divergence of the vector

- The cross product (x) of ∇ with a vector gives a vector

  - Curl of the vector

Gradient of a scalar field

Gradient of a scalar $V(xyz)$

$$grad\, V = \nabla V = \hat{\imath}\frac{\partial V_x}{\partial x} + \hat{\jmath}\frac{\partial V_y}{\partial y} + \hat{k}\frac{\partial V_z}{\partial z}$$

The gradient of a scalar field gives a vector

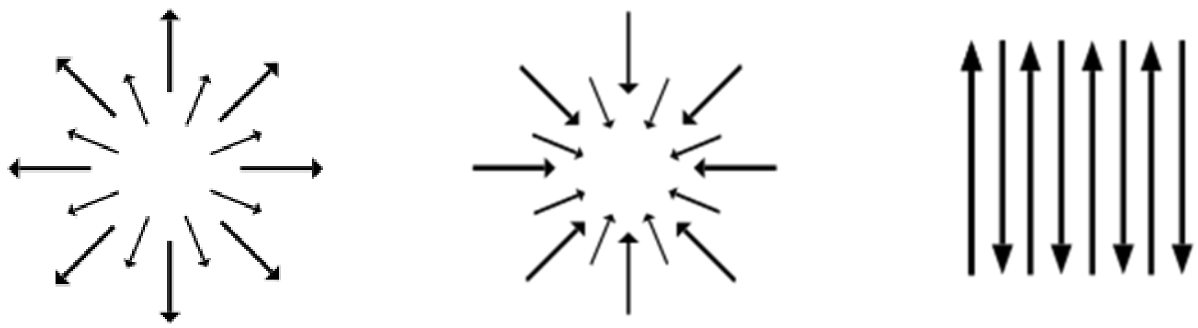
Gradient gives the rate of change of the property at any point and the direction gives the direction in which the change is maximum

Divergence of a vector field

Divergence of a vector $\vec{V} = \hat{\imath}V_x + \hat{\jmath}V_y + \hat{k}V_z$

$$Div\, V = \nabla . V = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

The divergence of a vector field gives a scalar



$\partial/\partial x(\mathbf{V}_x) > 0$
$\partial/\partial y(\mathbf{V}_y) > 0$
$\nabla\cdot(\mathbf{V}) > 0$

$\partial/\partial x(\mathbf{V}_x) < 0$
$\partial/\partial y(\mathbf{V}_y) < 0$
$\nabla\cdot(\mathbf{V}) < 0$

$\partial/\partial x(\mathbf{V}_x) = 0$
$\partial/\partial y(\mathbf{V}_y) = 0$
$\nabla\cdot(\mathbf{V}) = 0$

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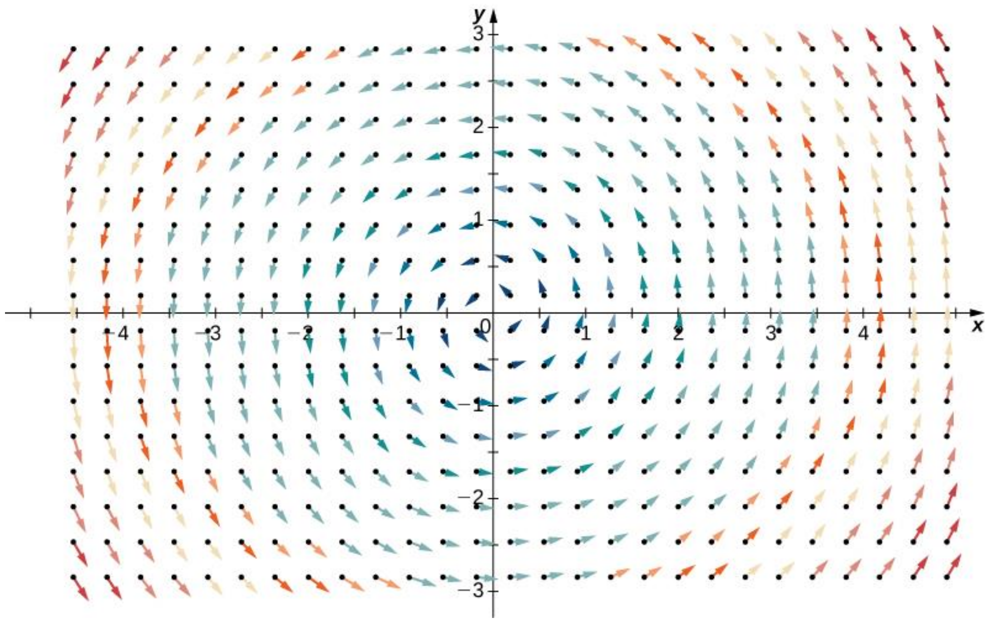
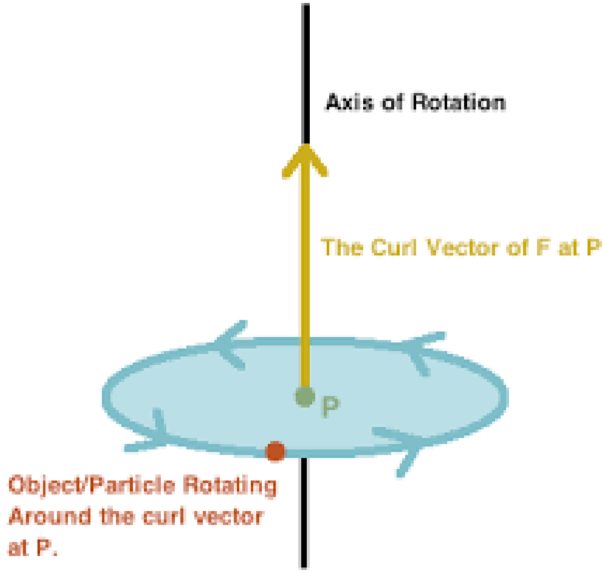
Curl of a vector field

$$curl\ A = \nabla \times A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

The curl of a vector is another vector





Relation b/w E & D and B & H

Two other vector fields are required when describing propagation of electromagnetism through matter.

The electric displacement 'D', (also electric flux density) and

The magnetic field intensity 'H', (also called the magnetizing force or the auxillary field)

$$D = \varepsilon E$$

$$B = \mu H$$

Where ε and μ are the electric permittivity and magnetic permeability of the material respectively.

Gauss' Law for electric fields

Gauss' Law states that the total electric flux through a closed surface is equal to **$1/\varepsilon_0$** times the net charge enclosed by the surface.

$$\oint \boldsymbol{E} \cdot \boldsymbol{ds} = \frac{q}{\varepsilon_0}$$

Or

$$\oint \boldsymbol{D} \cdot \boldsymbol{ds} = q$$

Gauss' Law for magnetic fields

According to Gauss theorem in magnetostatics, the net magnetic flux through an y closed surface is zero.

$$\phi_B = \oint \mathbf{B} \cdot \mathbf{ds} = 0$$

The number of lines emerging from any volume bounded by a closed surface is always equal to the number of lines entering the volume. Hence, the flux through any closed surface is equal to zero.

Faraday's Law of electromagnetic induction

According to this law, the negative time rate of change of magnetic flux linked with a circuit is equal to the emf induced in the circuit.

$$\text{Induced emf, } e = \frac{-d\phi_B}{dt}$$

The negative sign shows that the induced emf opposes the change in magnetic flux.

The work done in moving unit charge through the entire loop will be

$$W = \int dw = \oint E \cdot dl$$

$$\therefore \oint E \cdot dl = \frac{-d\phi_B}{dt}$$

Ampere’s Law

Ampere’s Law states that the line integral of magnetic flux density over any closed path is equal to the product of current enclosed by the path and the permeability of the medium.

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

Or

$$\oint \mathbf{H} \cdot d\mathbf{l} = I \qquad \because \mathbf{B} = \mu_0 \mathbf{H}$$

Ampere's Law

Also, the total current flowing through the surface area is given by

$$I = \int J.\,ds$$

Where, J is the current density.

Therefore, we can write

$$\oint B \cdot dl = \mu_0 \int J.\,ds$$

$$\text{Or} \quad \oint H \cdot dl = \int J.\,ds$$

This is known as Ampere's circuital law

Modified Ampere's Law

Ampere's law implies that a magnetic field can be produced only by flow of charges and this result was established based on experiments done on steady situations.

Maxwell showed that we run into difficulty when we apply Ampere's law for time-varying situations. He showed that we have to include another current called displacement current, which can also produce time-varying magnetic field.

Ampere-Maxwell's Equation

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

Four Basic Laws in Integral Form

1. Gauss' Law — $\varepsilon_0 \int E.dS = \sum q$
2. Gauss' Law for Magnetism — $\int B.dS = 0$
3. Faraday's Law — $\int E.dl = - d\phi_B/dt$
4. Ampere-Maxwell Law — $\int B.dl = \mu_0 [I + \varepsilon_0 d\phi_E/dt]$

Electric fields

Divergence of the electric field is given by the charge density divided by ε_o

$$\nabla . \vec{E} = \frac{\rho}{\varepsilon_o}$$

Magnetic Fields

Divergence of the magnetic field is uniformly zero

$$\nabla . \vec{B} = 0$$

This implies the absence of magnetic monopoles

Faraday's law of electromagnetic induction

The curl of the induced electric field in a closed loop is proportional to the rate of change of magnetic flux linked with the loop.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Ampere - Maxwell's law

Ampere - Maxwell circuital law

The curl of the magnetic field in a closed loop is equal to the sum of the current density and the displacement current due to the time varying electric field

$$\nabla \times \vec{B} = \mu_o \vec{J} + \mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t}$$

Maxwell's equations in Integral and differential forms

Integral form	Differential form
$\int_S \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$	$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
$\int_S \vec{B} \cdot d\vec{A} = 0$	$\nabla \cdot \vec{B} = 0$
$\oint_\Gamma \vec{E} \cdot d\vec{l} = -\frac{d\Phi(\vec{B})}{dt}$	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
$\oint_\Gamma \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \epsilon_0 \frac{\partial}{\partial t} \int_S \vec{E} \cdot d\vec{A} \right)$	$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{j} + \epsilon_0 \frac{\partial}{\partial t} \vec{E} \right)$

Maxwell's equations in free space

Free space implies – charges and currents do not exist

$$\vec{\nabla}.\vec{E} = 0 \qquad 1.$$

$$\vec{\nabla}.\vec{B} = 0 \qquad 2.$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad 3$$

$$\vec{\nabla} \times \vec{B} = +\mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t} \qquad 4.$$

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A general wave equation,

$$\nabla^2\vec{A} = \left(\frac{1}{v^2}\frac{\partial^2\vec{A}}{\partial t^2}\right),\ \text{with velocity} = v$$

Laplacian operator

$$\nabla^2 = \vec{\nabla}.\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Vector identity

$$\vec{\nabla} \times (\vec{\nabla} \times A) = \vec{\nabla}(\vec{\nabla}.A) - \nabla^2 A$$

Electric waves in free space

Taking the curl of Maxwell's equation 3

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

this reduces to, $\vec{\nabla}(\overrightarrow{\vec{\nabla}.\vec{E}}) - \nabla^2 \vec{E} = \left(-\frac{\partial \vec{\nabla} \times \vec{B}}{\partial t}\right)$

For free space, $\vec{\nabla}.\vec{E} = 0$ (Maxwell's equation 1),

Thus, $-\nabla^2 \vec{E} = \left(-\frac{\partial \vec{\nabla} \times \vec{B}}{\partial t}\right)$

Substituting for curl of B (Maxwell's equation 4)

$$\nabla^2 \vec{E} = \left(\mu_o \varepsilon_o \frac{\partial^2 \vec{E}}{\partial t^2}\right)$$

with $\mu_o \varepsilon_o = \frac{1}{c^2}$, wave equation for electric wave in free space,

$$\nabla^2 \vec{E} = \left(\frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}\right)$$

A wave equation for electric wave propagating in free space !

Magnetic waves in free space

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Taking the curl of Maxwell's equation 4

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \times \left(\mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t} \right)$$

this reduces to, $\vec{\nabla}(\vec{\nabla}.\overrightarrow{B}) - \nabla^2 \vec{B} = \left(\mu_o \varepsilon_o \frac{\partial \vec{\nabla} \times \vec{E}}{\partial t} \right)$

[As per Vector identity $\vec{\nabla} \times (\vec{\nabla} \times A) = \vec{\nabla}(\vec{\nabla}.A) - \nabla^2 A$ **]**

For free space, $\vec{\nabla}.\vec{B} = 0$ **and** $\vec{\nabla} x \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ **(Maxwell's equation 3)**

Applying the above, $\nabla^2 \vec{B} = \left(\mu_o \varepsilon_o \frac{\partial^2 \vec{B}}{\partial t^2} \right)$

The general form of magnetic wave in free space at speed of light,

$$\nabla^2 \vec{B} = \left(\frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} \right), \text{ with } \mu_o \varepsilon_o = \frac{1}{c^2}$$

A magnetic wave propagating in free space with speed of light!

A wave equation for electric wave propagating in free space

$$\nabla^2\vec{E} = \left(\frac{1}{c^2}\,\frac{\partial^2\vec{E}}{\partial t^2}\right)$$

The general form of magnetic wave in free space at speed of light,

$$\nabla^2\vec{B} = \left(\frac{1}{c^2}\,\frac{\partial^2\vec{B}}{\partial t^2}\right), \text{ with } \mu_o\varepsilon_o = \frac{1}{c^2}$$

A general wave equation,

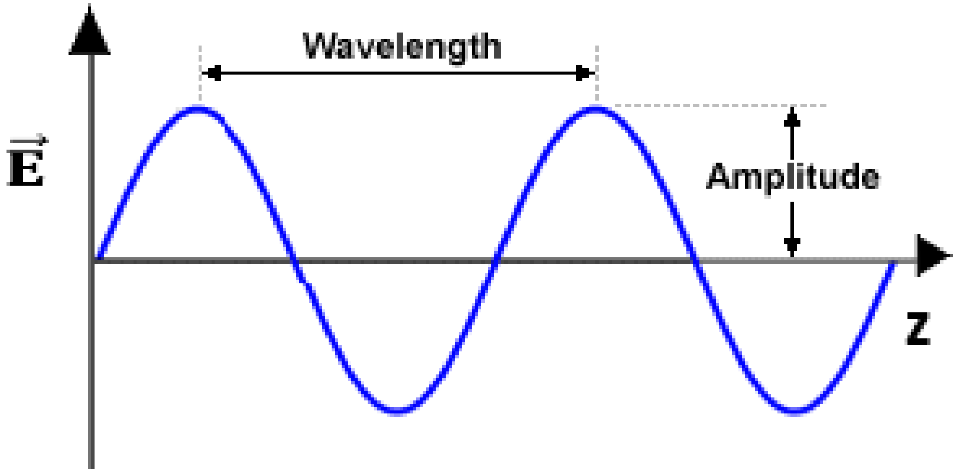
$$\nabla^2\vec{A} = \left(\frac{1}{v^2}\,\frac{\partial^2\vec{A}}{\partial t^2}\right), \text{ with velocity} = v$$

Electromagnetic waves in free space

Analysis: E and B are mutually perpendicular to each other

Consider a 1D electric wave E_x associated with EM radiation propagating in the Z direction as,

$$E_x = E_{ox}\,cos(\omega t + kz)\ or E_{ox}\,sin(\omega t + kz)$$



Plane E_x wave ($E_y = 0$) along z-direction

The electric field vector has only x component and other two components E_y and E_z are zero

The associated magnetic component of the EM wave is evaluated as,

Using Maxwell's third equation, $\vec{\nabla} x \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Evaluating curl of the electric field $\vec{\nabla} x \vec{E} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{bmatrix}$

$$= \hat{i} \times 0 + \hat{j} * \frac{\partial E_x}{\partial z} + \hat{k} * 0 = \hat{j} \frac{\partial}{\partial z}[E_{ox} \cos(\omega t + kz)]$$

$$= -\hat{j} * k * E_{ox} \sin(\omega t + kz)$$

Thus, $-\frac{\partial \vec{B}}{\partial t} = -\hat{j} * k * E_{ox} \sin(\omega t + kz)$

Integrating $-\frac{\partial \vec{B}}{\partial t}$ with respect to time gives magnetic component,

$$B = \hat{j} * \left(\frac{1}{\frac{\omega}{k}}\right) * E_{ox}\cos(\omega t + kz) = \hat{j}.E_x * \frac{1}{c}$$

($c = \frac{\omega}{k}$, is the velocity of the radiation)

Thus,

$$E_x = \hat{\imath}E_{ox}\cos(\omega t + kz)$$

$$B_y = \hat{j}\left(\frac{1}{\frac{\omega}{k}}\right) * E_{ox}\cos(\omega t + kz)$$

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Electromagnetic waves in free space-*Conclusion*

- Magnetic field is oriented in the Y-direction
- Perpendicular to the Electric field (X-direction).
- E and B are mutually perpendicular to each other.
- Magnetic field (B) of the EM wave is Y component
- In phase with the E field variations
- Phase velocity of the wave, $\mathbf{c} = \frac{\boldsymbol{\omega}}{\mathbf{k}}$
- Magnitude of B wave is $\frac{1}{c}$ times the magnitude of the E wave

$$|B_y| = \frac{|E_x|}{c}$$

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Electromagnetic waves in free space

- **EM waves have coupled E and B field components which are mutually perpendicular**
- **Both E and B are perpendicular to the direction of propagation**

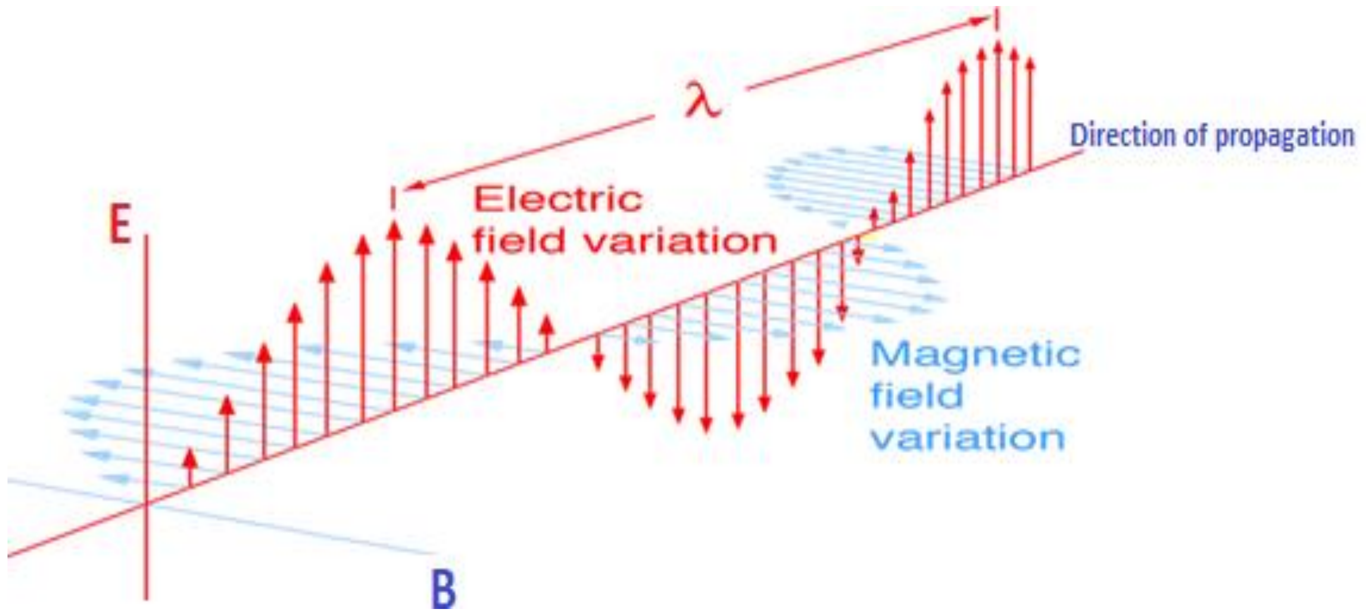


Image courtesy:Wikipedia

Energy in an electric field

The energy density of a wave is the radiant energy per unit volume.

The electromagnetic wave consists of electric and magnetic fields which independently can store energy.

The energy content of the electric component of the wave $= \frac{1}{2}\varepsilon_o E^2$

The energy per unit volume in an electric field is dependent only on the strength of the field !

Energy in a magnetic field

The energy content of the magnetic component of the wave

$$= \frac{1}{2}\frac{B^2}{\mu_o}$$

The energy per unit volume in a magnetic field is dependent only on the field strength!

Energy of EM waves

Energy content of the electric component

$$= \frac{1}{2}\varepsilon_o {E_x}^2 = \frac{1}{2}\varepsilon_o {E_{ox}}^2 \cos^2(\omega t + kz)$$

Energy content of the magnetic component $= \frac{1}{2}\frac{{B_y}^2}{\mu_o}$

Total energy content of the EM wave $= \frac{1}{2}\varepsilon_o {E_x}^2 + \frac{1}{2}\frac{{B_y}^2}{\mu_o}$

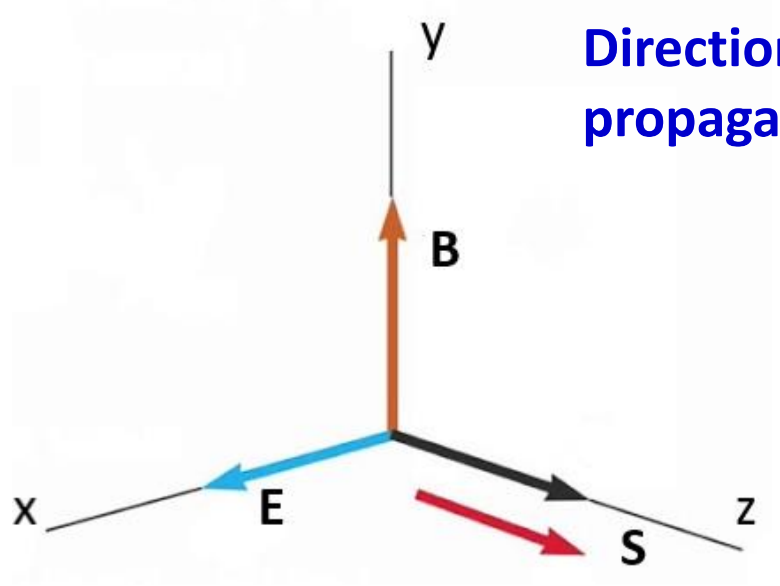
$$= \frac{1}{2}\varepsilon_o {E_x}^2 + \frac{1}{2}\frac{{E_x}^2}{c^2\mu_o} \quad \text{[Since, } B_y = E_x * \frac{1}{c}\text{]}$$

$= \varepsilon_o {E_x}^2$, **transported in the z-direction**

Poynting vector (Energy in an electromagnetic field)

Poynting vector ($\vec{S}$) describes the EM energy transported per unit time per unit volume

$$\vec{S} = \frac{1}{\mu_o} E \times B = c^2 \varepsilon_o E \times B$$



Direction of Poynting vector ($\vec{S}$) is the direction of propagation of EM waves

Average energy of EM waves

- **Average energy transported by an electromagnetic wave – energy transported in one cycle**

- $$\langle Energy\rangle = \frac{c\varepsilon_o}{T}\int_0^T {E_x}^2\,\mathrm{dt}$$

$$= \frac{c\varepsilon_o}{T}\int_0^T {E_{ox}}^2 \sin^2(\omega t + kz)\,\mathrm{dt}$$

$$= \frac{1}{2}\varepsilon_o c {E_{ox}}^2$$

$$= \frac{1}{2}\mathrm{c}\frac{{B_{oy}}^2}{\mu_o}$$

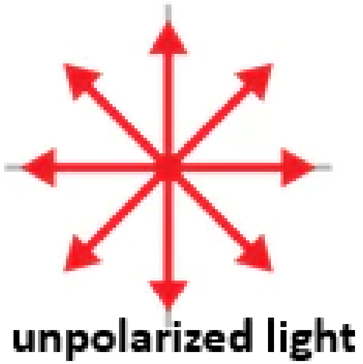
$$= \frac{1}{2}\frac{E_{ox}B_{oy}}{\mu_o}$$

Energy in an electromagnetic field

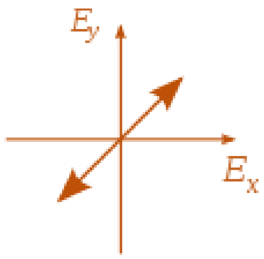
- **Energy transported is dependent on the amplitude of the electric and magnetic waves**
- **Energy is independent of the wavelength or frequency of the waves!!**

Polarisation of electromagnetic waves

- Natural light is generally unpolarized, all planes of propagation being equally probable
- Polarization of the electric wave



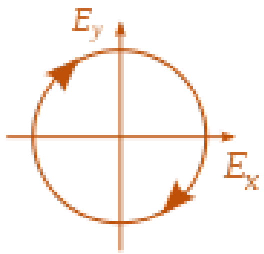
Plane polarized EM wave -
two waves in phase



Polarization of electromagnetic waves

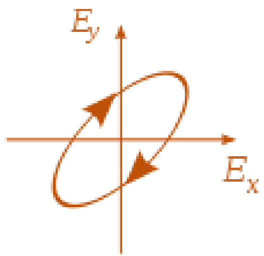
Circularly polarized EM wave –

two waves of equal amplitudes and out of phase by 90^o

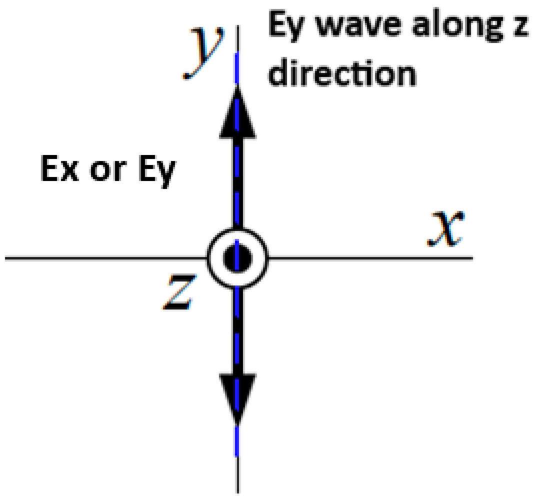


Elliptically polarized EM wave –

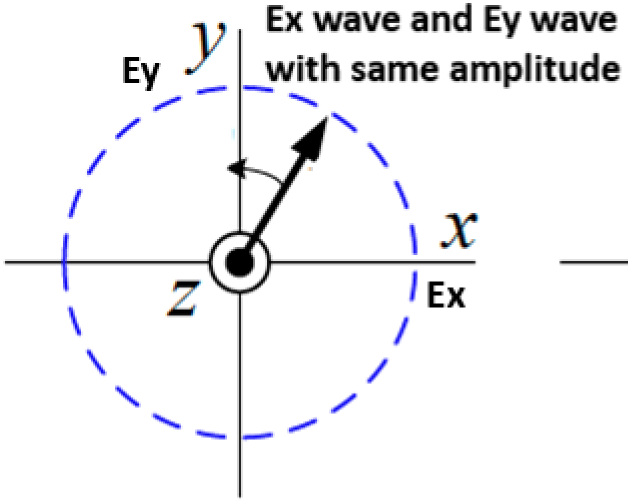
two waves of unequal amplitudes and out of phase <> 90^o



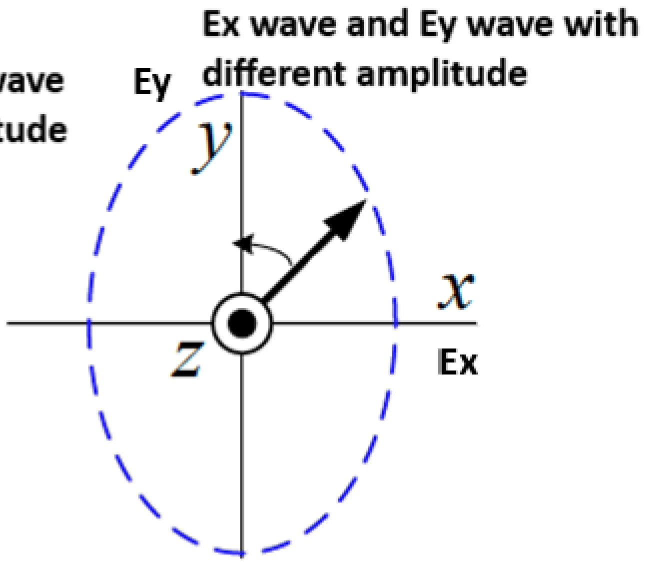
Polarization of electromagnetic waves



Linearly polarized



Circularly polarized



Elliptically polarized

Overview of failure of classical EM wave theory

- EM Radiation (e.g. Radio waves, microwaves, infrared, visible light, ultraviolet, x-rays and gamma radiation) - Described as mutually perpendicular sinusoidal electric and magnetic fields and perpendicular to the direction of propagation of the waves
- Classical wave theory - Assumed that energy content of the wave is proportional to the square of the amplitude of the waves (wavelength/frequency independence on energy!)
- Wave theory successfully explains the phenomena of reflection, refraction, interference, diffraction and polarization of light

Overview of failure of classical EM wave theory

Classical wave theory could not explain many observed phenomena

1. Black-Body Radiation Spectrum
2. Photo-electric Effect
3. Spectrum of Hydrogen Emissions (Atomic Spectra)
4. Compton Scattering

Resulted in the birth and rise of Quantum Mechanics!

Our focus: Black-Body Radiation and Compton Scattering

Classically the interaction of radiation with matter (by absorption and emission) gives the color of the material

Gustav Robert Kirchhoff found materials which absorb all incident rays

Such a material on heating would emit all wavelengths of radiation absorbed.

Black-body (not necessarily black!)

- absorbs all radiations falling on it
- Emits all wavelengths (frequencies) as it absorbed
- Emissivity is unity

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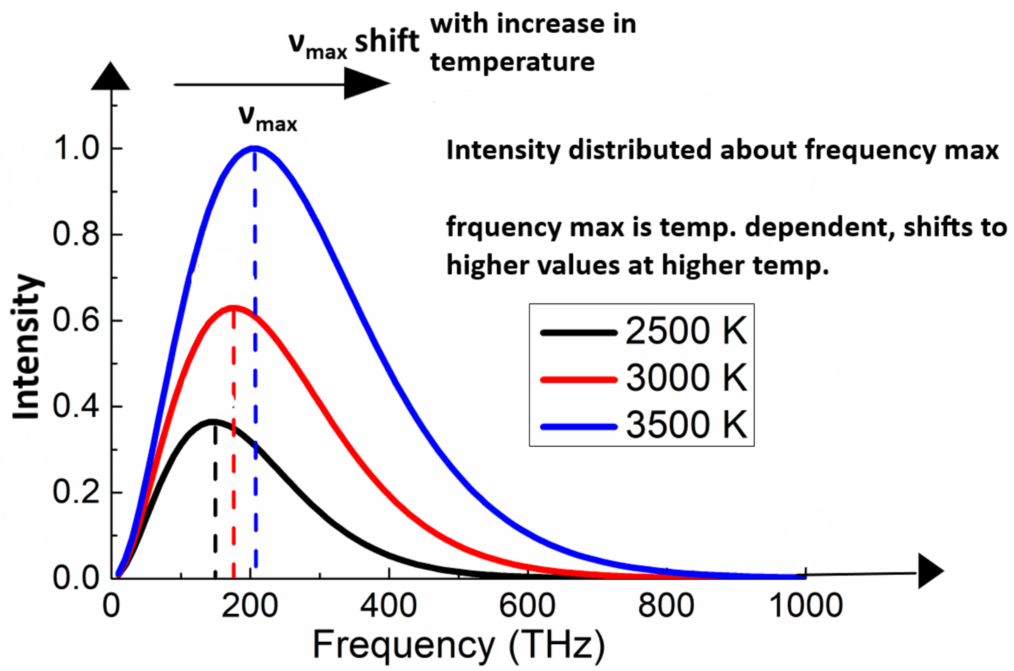
Black-body radiation spectrum

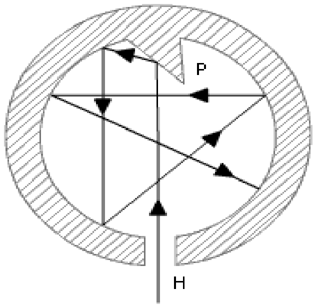
- Radiation depends only on the temperature of the object, and not on what it is made of **(material independent)**
- As the temperature increases, emits more energy **(Area under curve)**
- As the temperature of object increases, the peak wavelength becomes shorter **(higher frequency)**

(Blue stars are hotter than red stars!)



Black-body model (Cavity oscillators)

- Practically modeled as a cavity which does not allow any incident radiation to escape due to multiple reflections inside the cavity
- This cavity when heated, emit radiation of every possible frequency (rate of emission increases with temperature)



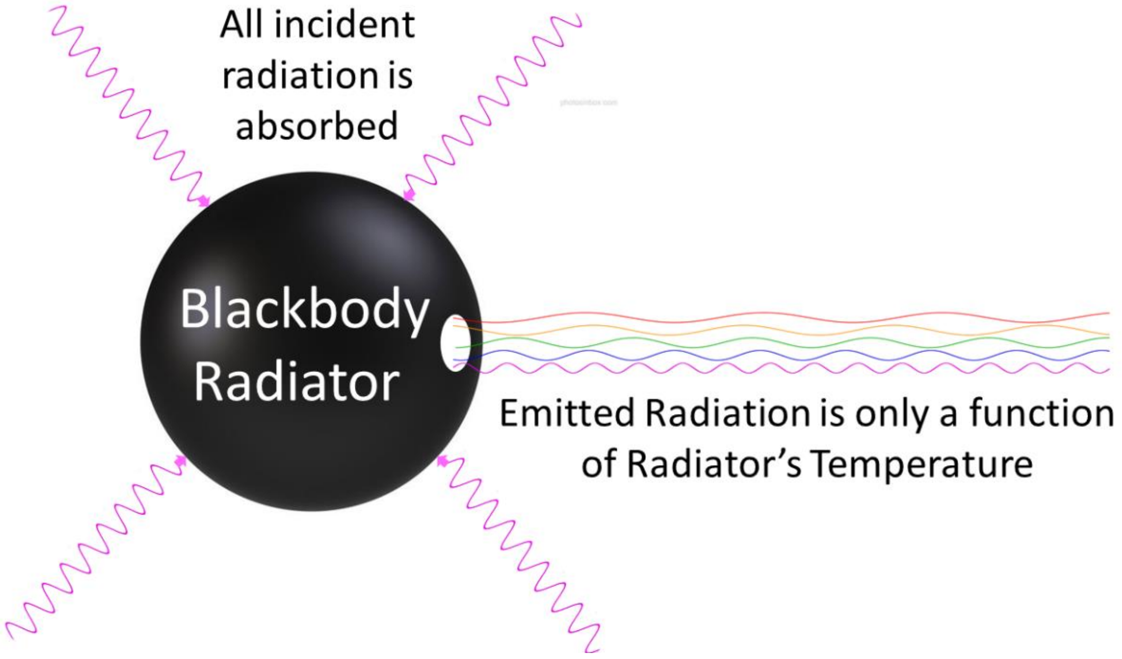
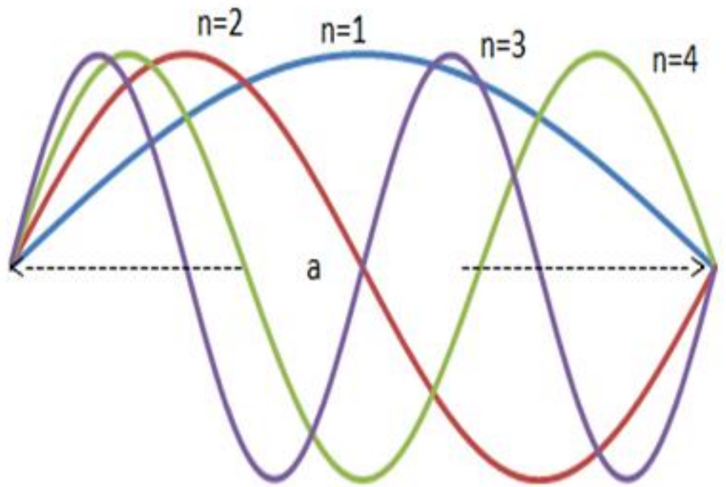


Image courtesy: https://chem.libretexts.org

Classical estimation of energy density

Analysis by Rayleigh-Jeans

- To understand the energy density of radiation assuming the black-body as a cavity oscillators (trapped oscillations of EM energy)
- Rayleigh and Jeans showed that the number of modes was proportional to the frequency squared
- The number of oscillators with frequencies between $\nu \text{ and } \nu + \delta\nu$ is calculated as $dN = \frac{8\pi}{c^3}\nu^2 d\nu$



Blackbody radiation as cavity oscillations of EM energy

Classical estimation of energy density

The average energy of the oscillators is evaluated using **Maxwell-Boltzmann** distribution law as $\langle E \rangle = k_B T$

Thus, expression for the energy density (energy per unit volume) of radiations with frequencies between ν and $\nu + \delta\nu$ as

$$\rho(\nu)d\nu = \langle E \rangle dN = \frac{8\pi}{c^3}\nu^2 d\nu k_B T$$

This is the Rayleigh Jeans law which is in contradiction with the experimental observations

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Failure of Rayleigh-Jeans' law

Treating EM waves as classical oscillators failed to explain the experimental observations

(intensity of radiations were found to decrease with increase in frequency - termed as **ultra-violet catastrophe [high frequency region]**)

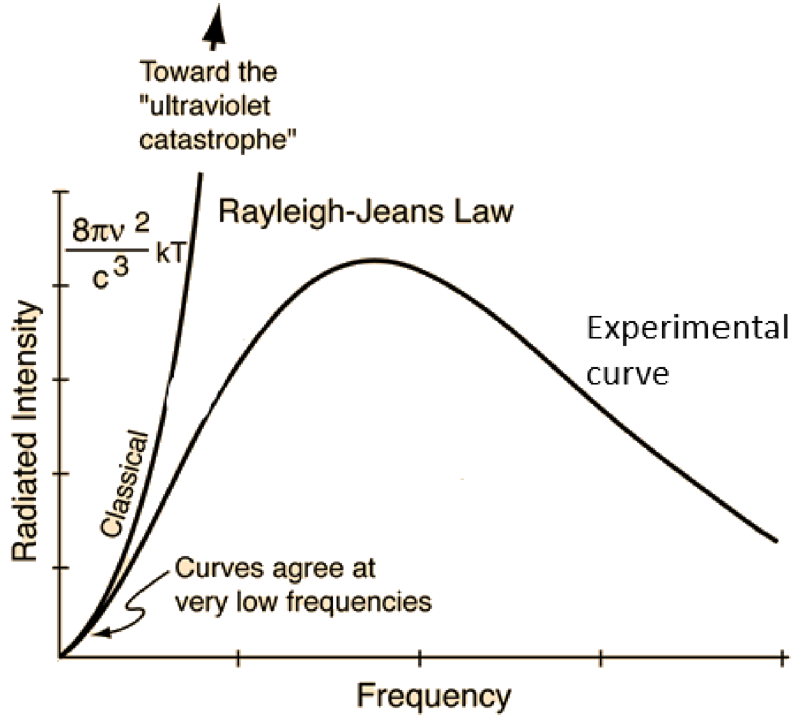


Image courtesy: hyperphysics

Max Planck’s analysis – Quantum theory of radiation

Solution to the failure of classical approach to understand the energy distribution in black body by Max Planck (quantum theory of radiation, 1900)

- This theory proposed that the energy of the harmonic oscillator (oscillator model of a black body) are restricted to multiples of a fundamental natural frequency $\boldsymbol{\nu}$times a constant $(\boldsymbol{h = 6.6x10^{-34} Js})$ ie.,$\boldsymbol{E = nh\nu}$
- Thus the radiations are from a collection of harmonic oscillators of different frequencies and the energy of the radiations has to be packets of $\boldsymbol{h\nu}$

Max Planck's analysis – Quantum theory of radiation

- The average energy of the oscillators were evaluated as, $\langle E\rangle = \frac{h\nu}{e^{h\nu/kT}-1}$
- Thus, the energy density of radiations

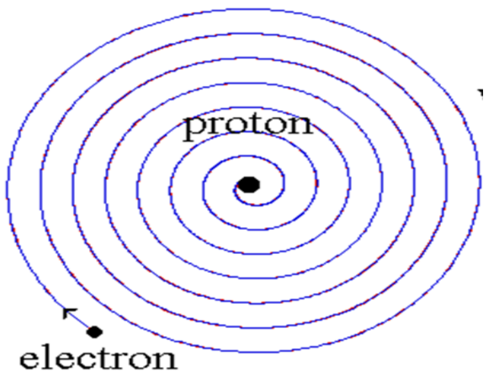
$$\rho(\nu)d\nu = Number\ of\ modes \times Average\ energy$$

$$\langle E\rangle dN = \frac{8\pi}{c^3}\nu^2 d\nu \frac{h\nu}{e^{h\nu/kT}-1} = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}\,d\nu$$

- Planck's expression gives excellent co-relation with experimental results
- *The foundation stone- for era of quantum physics!*

Failure of Classical Approach

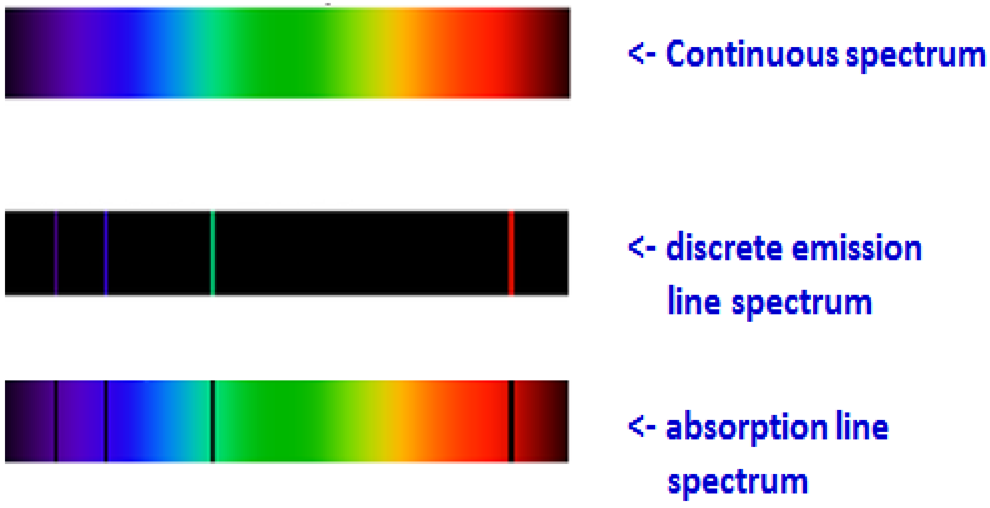
Classical physics tried to model the emission from atoms as that due to the orbiting electron, since an accelerated charge should emit electromagnetic radiation (light). However according to this model the electron should be continually losing energy and falls into the atom!



https://physics.weber.edu/carroll/honors/failures.htm

Atomic spectra

- Atomic absorption lines are observed in the solar spectrum, referred to as Fraunhofer lines
- Robert Bunsen and Gustav Kirchhoff discovered new elements by observing their emission spectra
- The existence of discrete line - Emission spectra
- Absence of discrete lines -Absorption spectra



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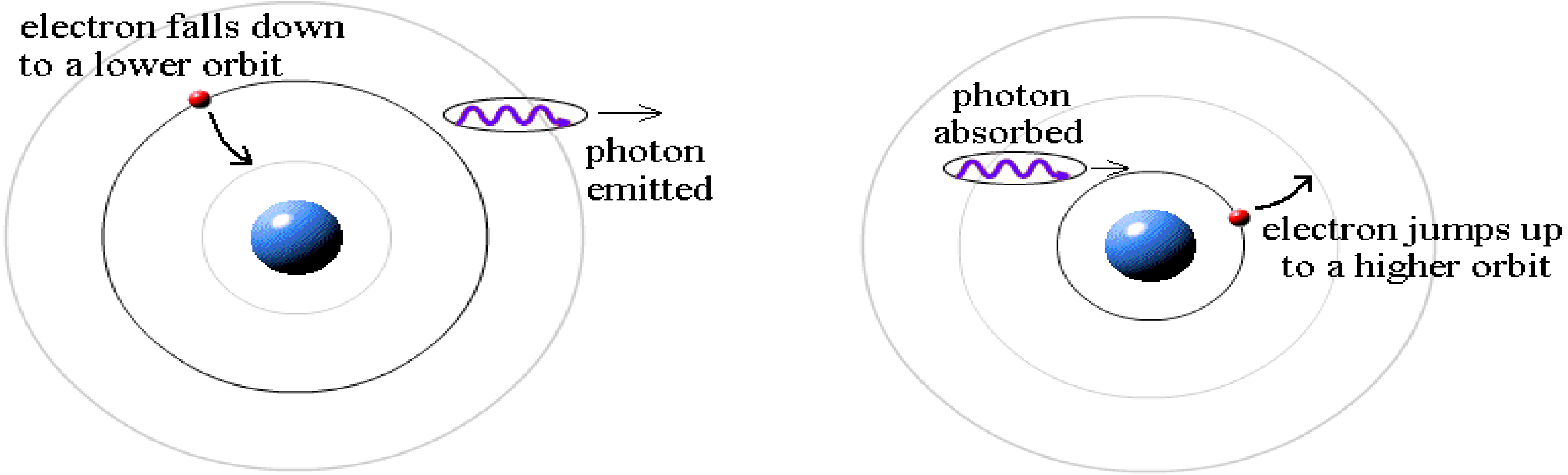
Atomic spectra analysis – Quantum explanation

- **Atoms of different elements have distinct spectra**
- **Atomic spectroscopy allows the identification of a sample's elemental composition**



The explanation of the line spectrum of atoms: in terms of transition between *quantized energy states of an atom*

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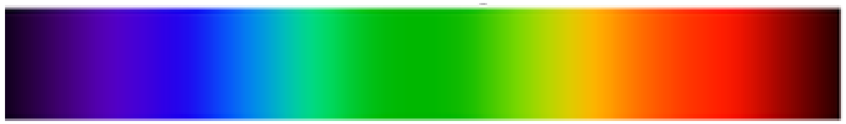
Atomic spectra

- *Continuous spectra from a source of white light*
- *Discrete emission lines*
- *Absence of lines from a continuous spectrum*



<- Continuous spectrum

<- discrete emission line spectrum

<- absorption line spectrum

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Photo electric effect

- Electron emission from metals under irradiation - Photo electric effect
- Instantaneous emission of electrons with kinetic energy dependent on wavelength of radiation
- Energy of photo electrons independent of intensity of radiation
- Failure of EM wave theory to explain observed results

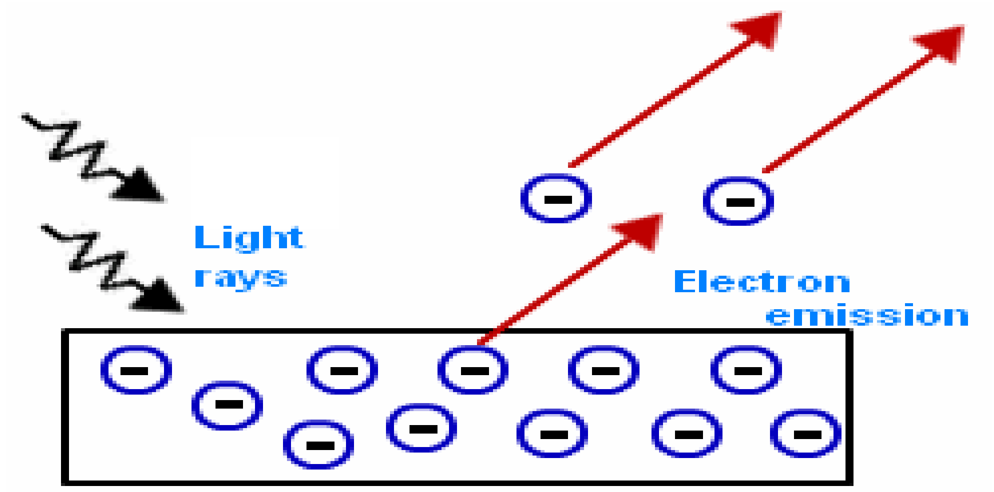
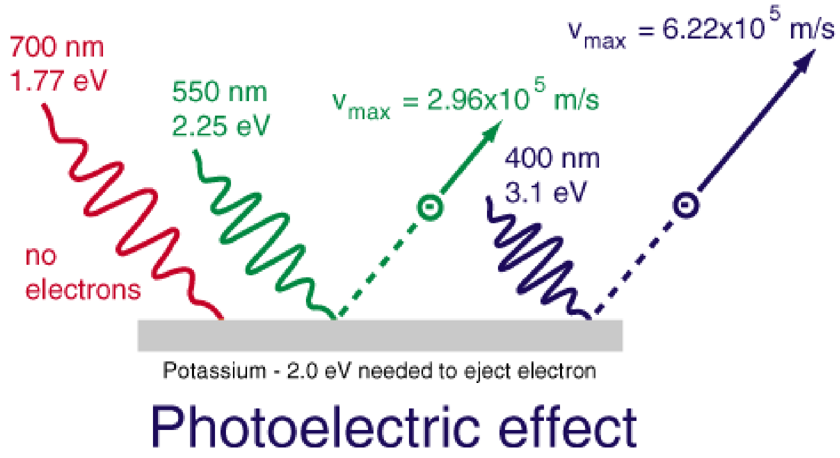


Photo Electric effect

- Einstein's concepts of photons
- Low energy electron – photon interaction
- Transfer of energy and momentum to the photo electron
- $h\nu = W + KE_e$
- Waves can have dual nature – depending on the nature of interaction with matter !



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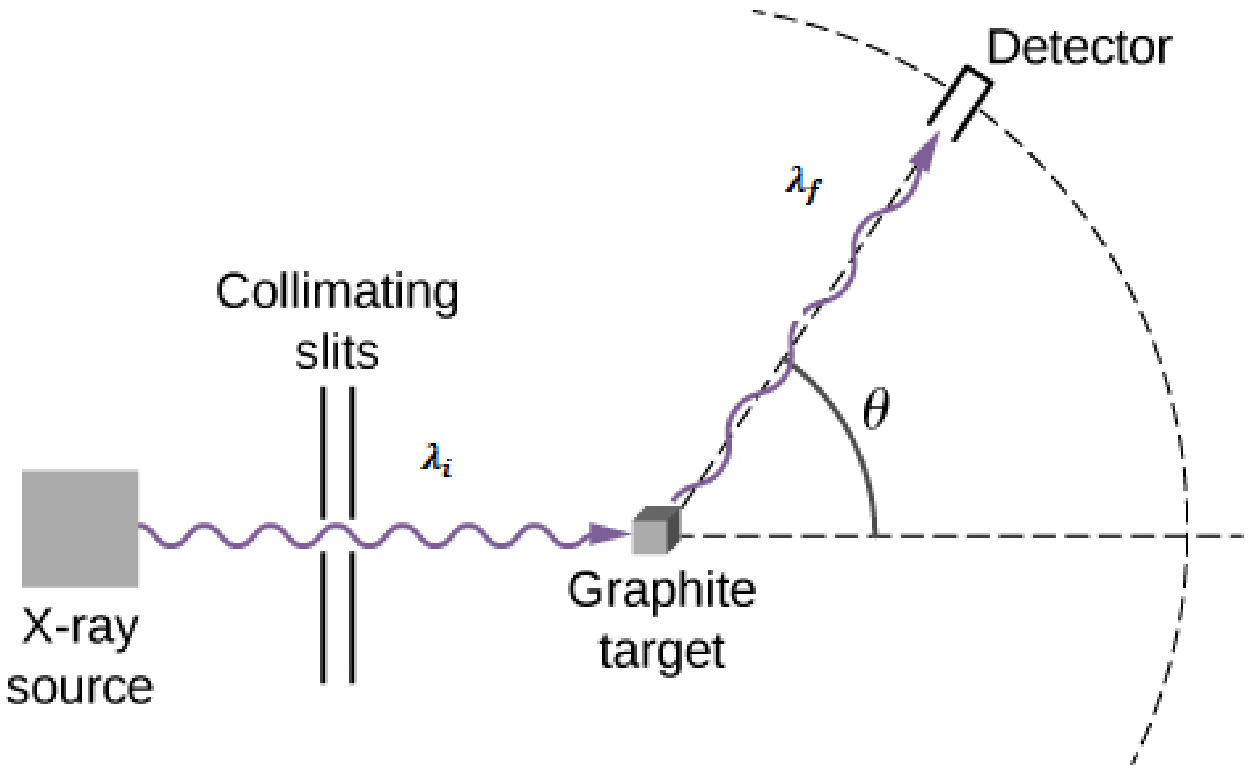
Scattering of X Rays by target materials-Compton effect

- X Ray scattering experiments with different targets
- Scattered X rays have a higher wavelength than the incident X rays



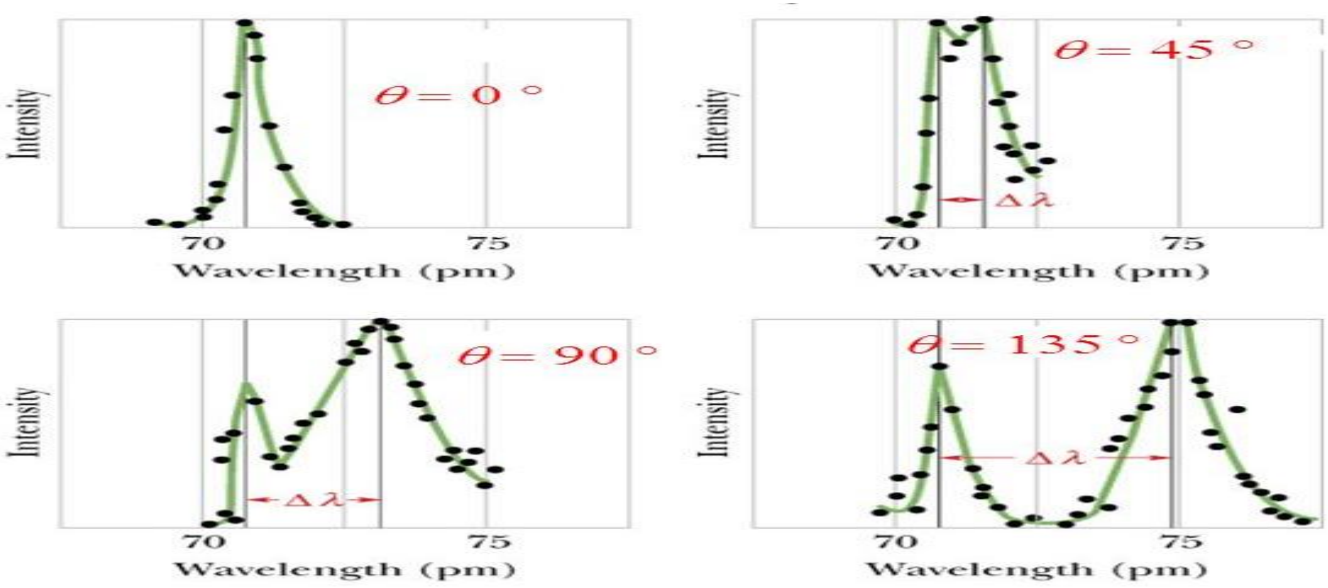
Compton shift

- The difference $(\lambda_f - \lambda_i)$ which indicates the enhancement in the wavelength is called the Compton shift.

➢ λ_i is the unmodified component

➢ λ_f is the modified component

- Compton shift increases with increasing scattering angle θ.



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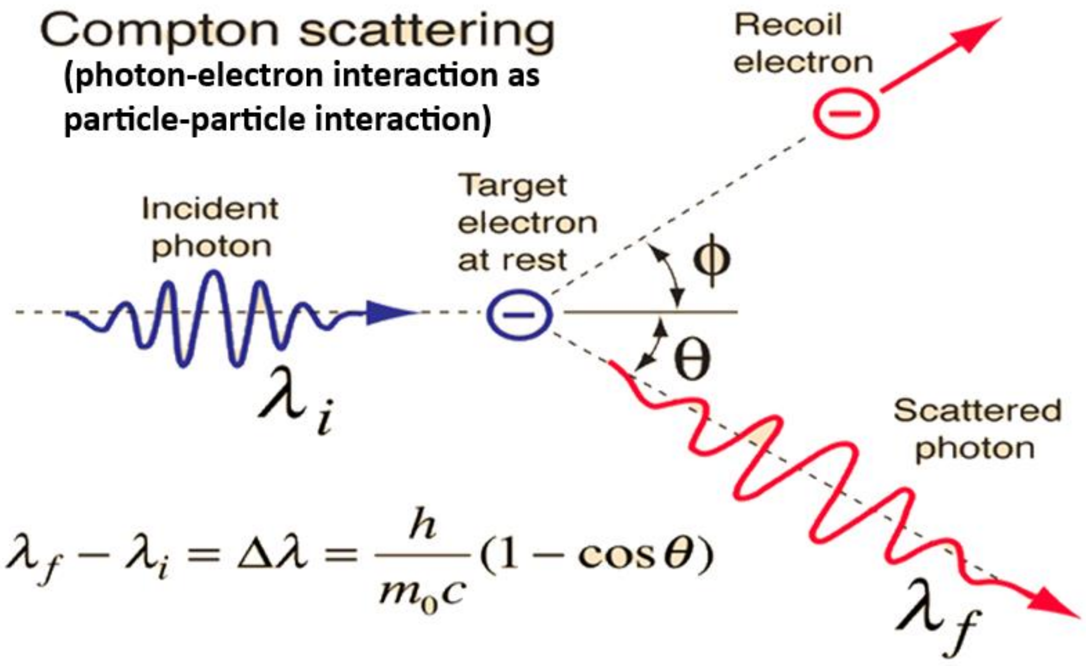
Experimental observations -Compton effect

- **Wavelength of scattered X rays is independent of the target material**
- **Wavelength of scattered X rays depend on the angle of scattering**
- **Scattering of EM waves with electrons do not explain the observed change in wavelength-*Classical explanation fails!***

Explanation of Compton effect



Relativistic concepts of momentum and energy

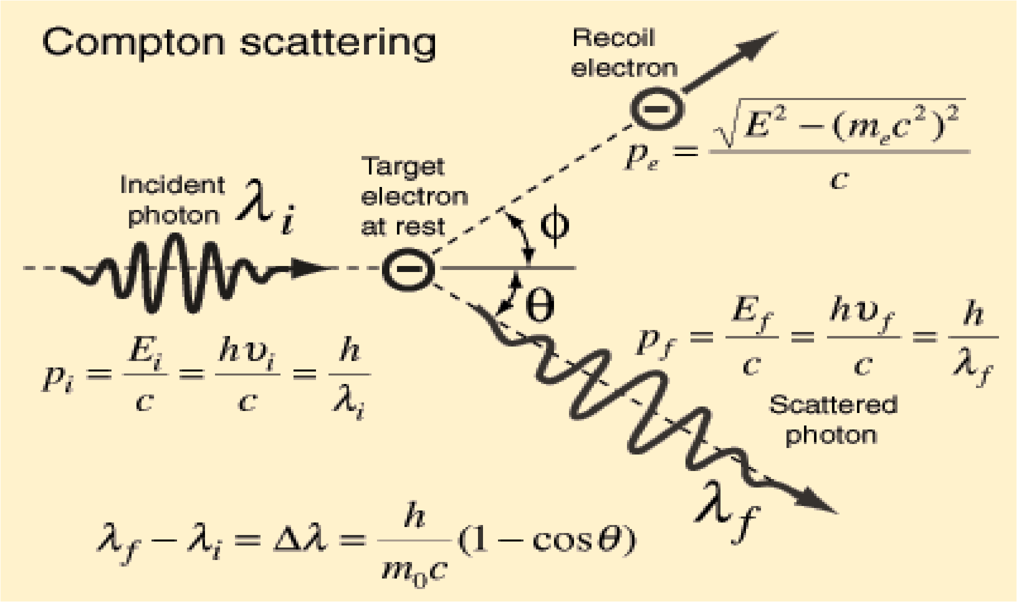
- *Rest mass energy of a particle given by*

$$E = m_o c^2.$$

- *the kinetic energy of a particle with momentum* p *is given by* pc

- *The total energy of the particle is given by*

$$E = \sqrt{p^2 c^2 + m_o{}^2 c^4}$$



Conservation of momentum in X ray scattering

- **Momentum conservation along the incident direction -**

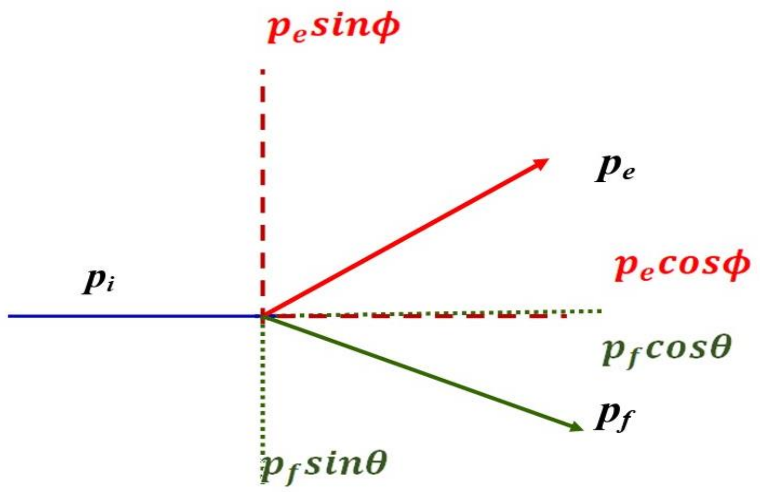
$$p_i + 0 = p_f cos\theta + p_e cos\phi.$$

- **Momentum conservation in a perpendicular direction -**

$$0 = p_f sin\theta - p_e sin\phi$$

- ***Conservation of momentum before and after collision***

$$p_e{}^2 = p_i{}^2 + p_f{}^2 - 2p_i\, p_f cos\theta \qquad \dots\ 1.$$



Conservation of energy

- *Conservation of energy before and after collision*

$$p_i c + m_o c^2 = p_f c + \sqrt{p_e{}^2 c^2 + m_o{}^2 c^4}$$

$$p_e{}^2 = p_i{}^2 + p_f{}^2 - 2p_i\, p_f + 2m_o c(p_i - p_f) \quad \text{--- 2}$$

- Comparing equations 1 & 2

$$-2p_i\, p_f + 2m_o c(p_i - p_f) = -2p_i\, p_f cos\theta \quad \text{---- 3.}$$

Compton Shift

- *With* $p_i = \frac{h}{\lambda_i}$ *and* $p_f = \frac{h}{\lambda_f}$ *equation 3. simplifies to*

$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

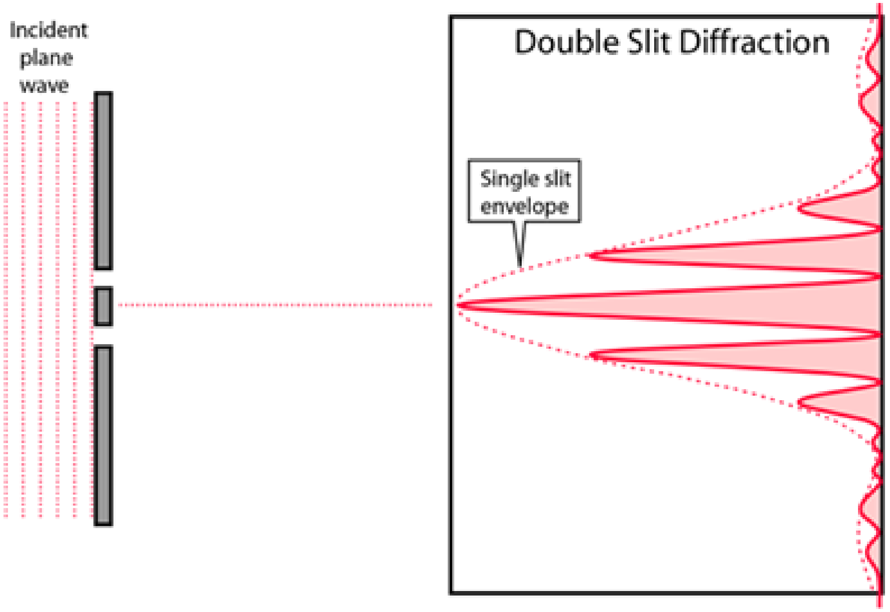
- $\Delta\lambda$ *is termed as the Compton shift*
- $\Delta\lambda$ *is independent of the incident wavelength*
- $\Delta\lambda$ *dependents only on the angle of scattering*
- $\frac{h}{m_e c} = \lambda_c$ is termed as the Compton wave length
- *For electrons* $\lambda_c = 2.42 \times 10^{-12}$ m

Compton effect

- *X rays – part of the EM wave spectrum*
- *Interaction of radiation with matter at sub-atomic matter requires radiation to be treated as particles - photons*
- *High energy photon – particle interaction explains the scattering phenomena*
- *Wave particle duality is a reality…*

Young's double slit experiment

- *Young's classic double slit experiment on interference and diffraction of radiations*
- *Characteristic wave experiment*



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de Broglie hypothesis

- *Based on the analysis of dual nature of radiation-*
- *Louis de Broglie hypothesis*
  - *Moving matter should exhibit wave characteristics*
  - *Wavelength of the associated waves* $\lambda = \frac{h}{p}$ *where* ***p*** *is the momentum of the particle*
- *Wavelengths of macro particles are extremely small to be measured*
- *Wavelengths of moving sub atomic particles can be in the measurable range* ($\sim 10^{-10} m$)

Dual nature of matter

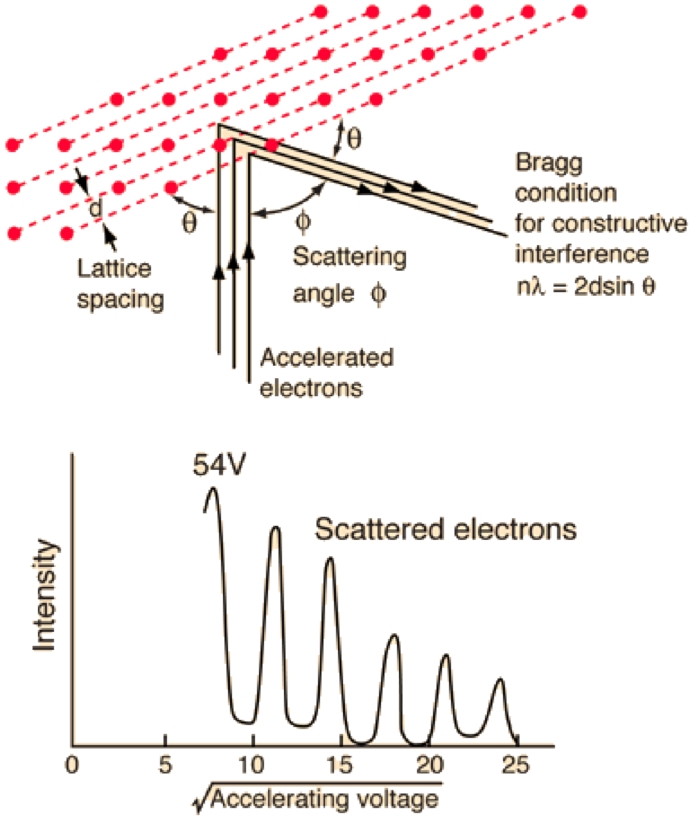
Experimental verification of de Broglie's hypothesis - Davisson and Germer's experiment (electron scattering by Ni crystals)

de Broglie wavelength $\boldsymbol{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}}$

Electron diffraction confirmed at particular settings (54 V, angle of scattering 50°)

Satisfied Bragg's law $\boldsymbol{\lambda = 2d \sin\theta}$, by 'electron waves'!

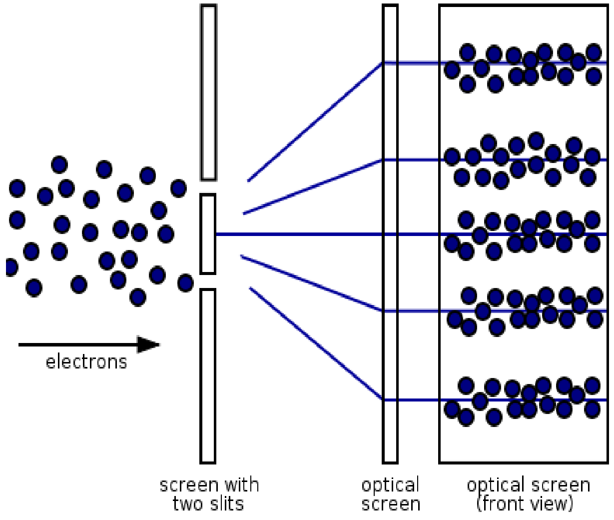
Conclusion: Dual nature of matter - *matter and matter waves*!

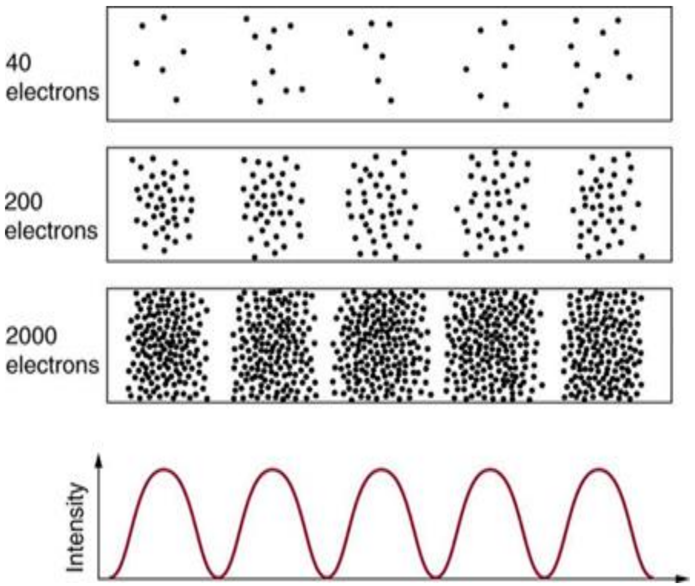


Davisson-Germer experiment

Double slit experiment with electrons

- ***Diffraction is characteristic wave phenomenon***
- ***Double slit experiment with a particle (single electrons or photons – one at a time) show wave nature – Particle diffraction!***





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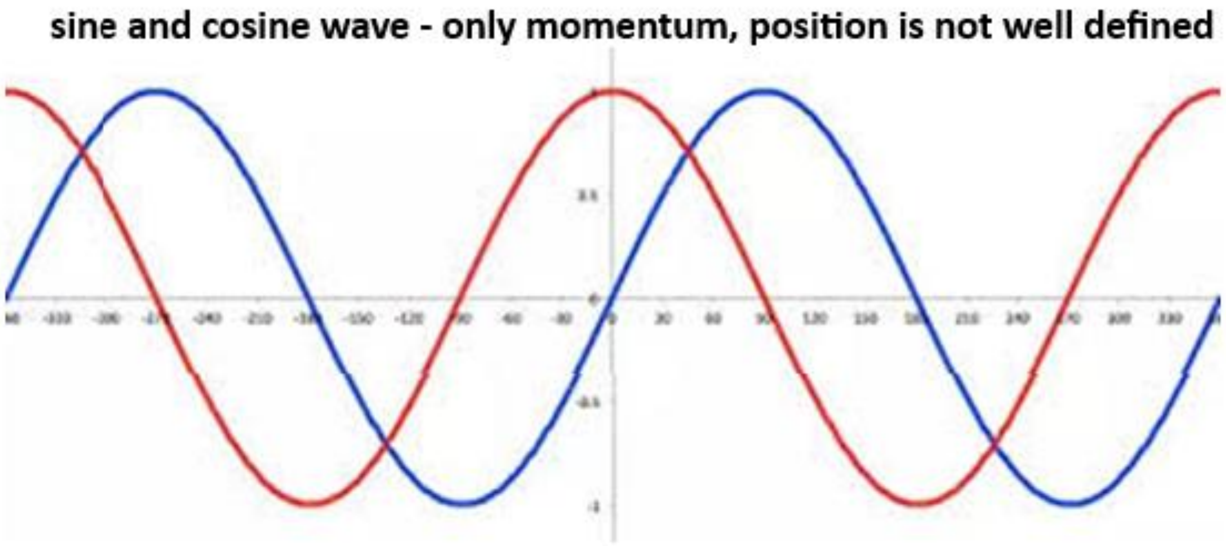
Concept of Matter waves

- ***Position and momentum are the two generalized parameters needed to describe the state of any system***
- ***Position and momentum are conjugate parameters***

$$v = \frac{dx}{dt}$$

Concept of matter waves

- **Need a mathematical concept to describe matter waves**
- **Any representative wave should be able to give information about the position and momentum of the system**
- **Simple sine or cosine waves fall short (Momentum can be inferred from wavelengths $p = h/\lambda$ but Position is not well defined)**



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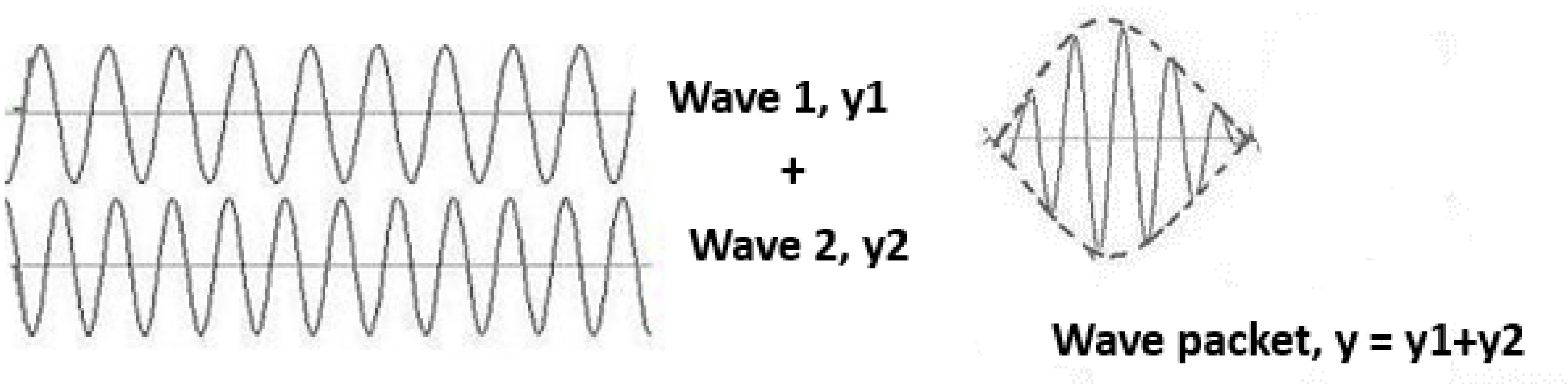
superposition of waves

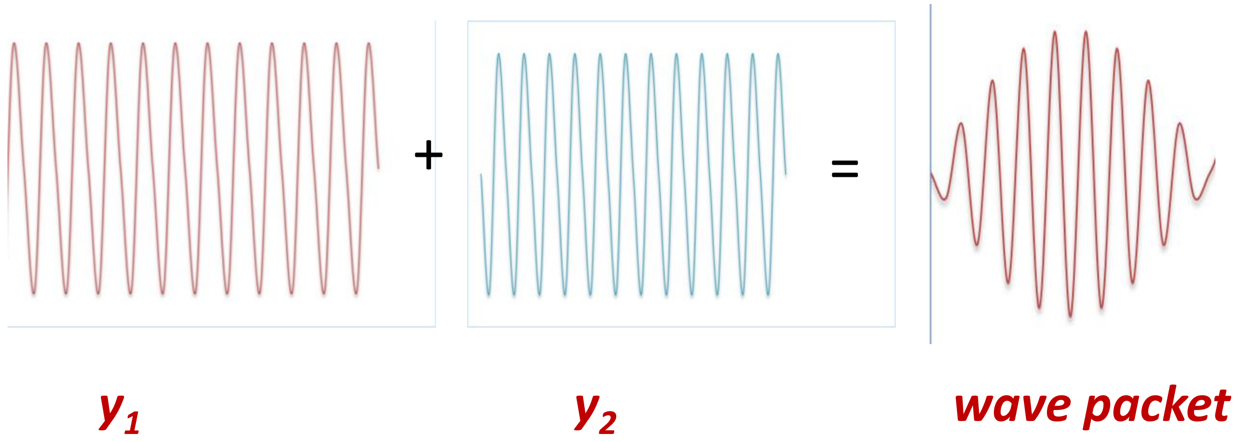
- Superposition of two waves - wave packets describe matter waves
- *Wave packets* describe matter waves - with a defined wavelength and an amplitude maximum
- Provide information about both position and momentum



superposition of two waves to a wave packet

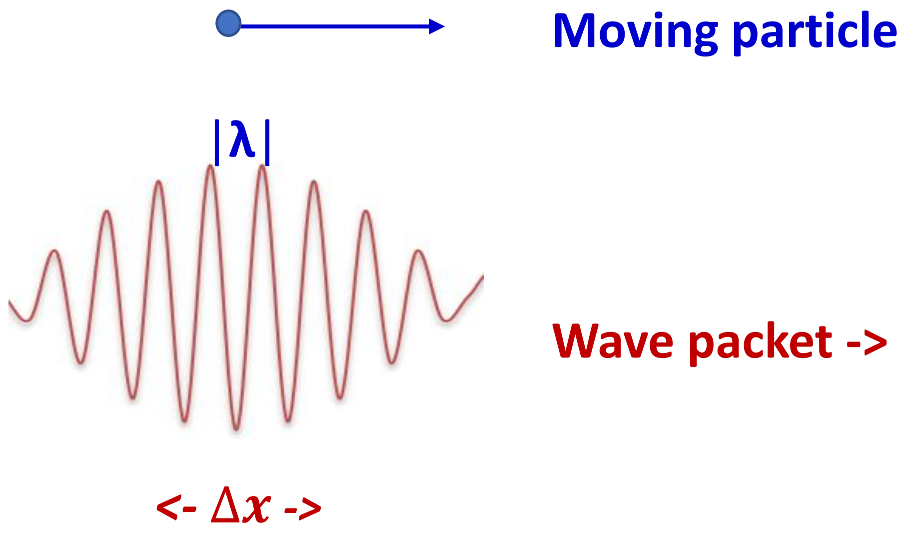
Wave Packets

- $y_1 = A\sin(\omega t + kx)$
- $y_2 = A\sin\{(\omega + \Delta\omega)t + (k + \Delta k)x\}$
- *Superposition*
- $y = y_1 + y_2 = 2A\sin(wt + kx).\cos\left(\frac{\Delta wt + \Delta kx}{2}\right)$



Wave Packets

- *From **k** we can infer **λ** - which defines momentum*
- *The spread around the central maximum can be the approximate position of the particle*



Phase and group velocities

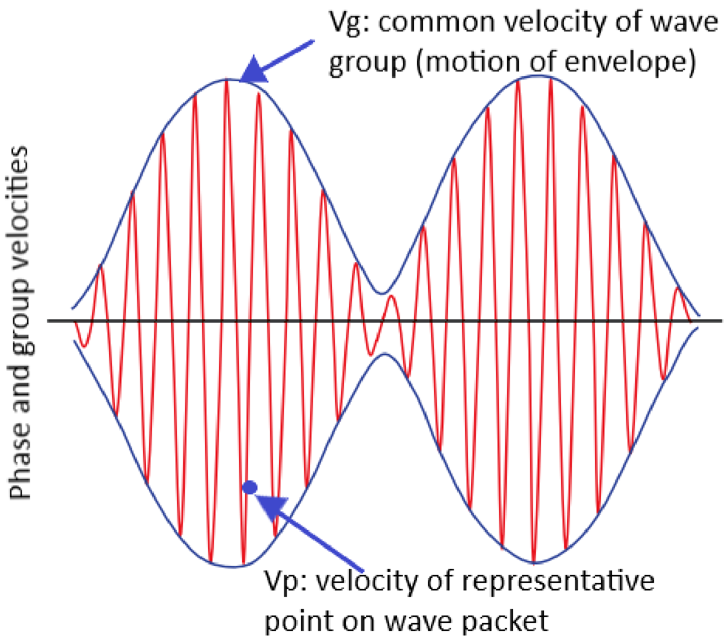
- $y = y_1 + y_2 = 2A sin(wt + kx) . \cos\left(\frac{\Delta wt + \Delta kx}{2}\right)$

- *The phase velocity of the wave packet is the velocity of a representative point on the wave packet*

$$v_{ph} = \frac{\omega}{k}$$

- *The group velocity of the wave packet is the velocity of common velocity of the superposed wave group*

$$v_g = \frac{d\omega}{dk}$$



Group and particle velocities

- ***The energy of the wave packet***

$$E = h\nu = \frac{h\omega}{2\pi} = \hbar\omega$$

The angular frequency $\omega = \frac{E}{\hbar}$ ***and hence*** $d\omega = \frac{dE}{\hbar}$

- ***The momentum of the particle***

$$p = \frac{h}{\lambda} = \frac{h\ .2\pi}{2\pi\ .\lambda} = \hbar k$$

The wave vector $k = \frac{p}{\hbar}$ ***and hence*** $dk = \frac{dp}{\hbar}$

- ***Group velocity***

$$v_g = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = \frac{p}{m} = v_{particle}$$

- ***Group velocity is reflecting the particle velocity***

Group and Phase velocity relation

- *Group velocity*

$$v_g = \frac{d}{dk}(\omega) = \frac{d}{dk}(v_{ph}k)$$

$$= v_{ph} + k\frac{dv_{ph}}{dk} = v_{ph} + k\frac{dv_{ph}}{d\lambda}\frac{d\lambda}{dk}$$

W.K.T., $k = \frac{2\pi}{\lambda}$ and $\frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2}$

hence $v_g = v_{ph} + \frac{2\pi}{\lambda}\left(\frac{-\lambda^2}{2\pi}\right)\frac{dv_p}{d\lambda}$

$$v_g = v_{ph} - \lambda\frac{dv_{ph}}{d\lambda}$$

- *Group velocity is dependent on the phase velocity and how the phase velocity changes with wavelength*

Group and Phase velocity relation

Group velocity = Phase velocity $v_g = v_{ph}$

$$-\lambda \frac{dv_{ph}}{d\lambda} = 0$$

- *Phase velocity does not change with wavelength*
- *The medium is non dispersive*
- *A dispersive medium is one in which*

  Group velocity <> Phase velocity

  - $v_g < v_{ph}$
  - $v_g > v_{ph}$

Group and Phase velocity relation

- $v_g < v_{ph}$ - *group velocity is half the phase velocity*
- $v_g = {}^{v_{ph}}/_{2}$

Group velocity of a wave packet is given by $v_g = v_p - \lambda \frac{dv_p}{d\lambda}$

- $\frac{dv_p}{v_{ph}} = \frac{1}{2}\frac{d\lambda}{\lambda}$ on integration yields
- $ln(v_{ph}) \propto ln\sqrt{\lambda}$ or $v_{ph} \propto \sqrt{\lambda}$
- This implies that the phase velocity is proportional to the square root of the wavelength

Group and Phase velocity relations

- $v_g > v_{ph}$ *- group velocity is twice the phase velocity*

- $v_g = 2v_{ph}$

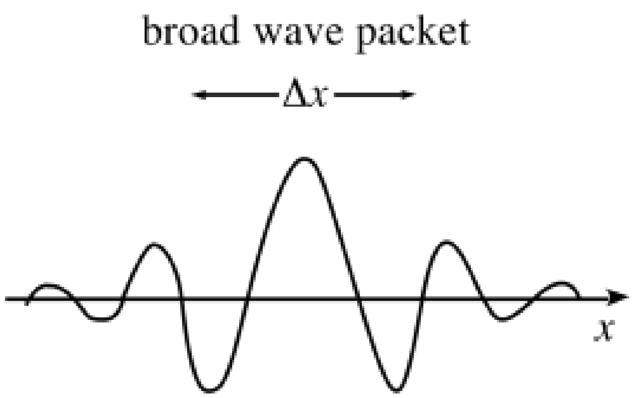
- $\frac{dv_p}{v_{ph}} = -\frac{d\lambda}{\lambda}$ on integration yields

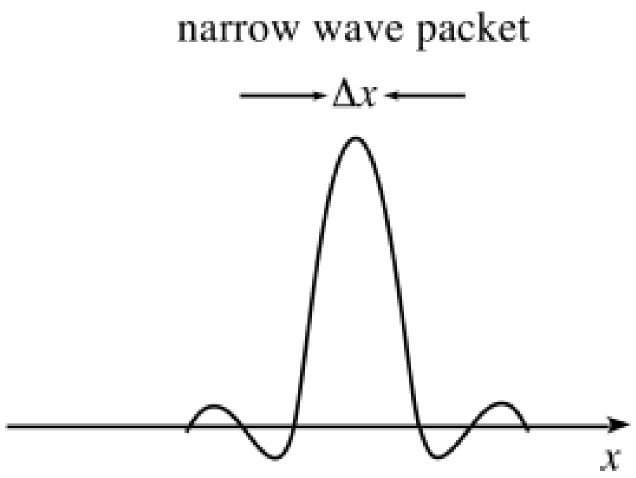
- $ln(v_{ph}) \propto ln\frac{1}{\lambda}$ or $v_{ph} \propto \lambda^{-1}$

- This implies that the phase velocity is inversely proportional to the wavelength

Analysis of Wave Packets

- *Wave packets are formed by the superposition of waves*
- *Wave packets have an inherent component of uncertainties*
- *A broad wave packet has a high uncertainty in position, but a high accuracy in momentum*
- *A narrow wave packet results in a high accuracy in position and a high uncertainty in the momentum*





Heisenberg's Uncertainty Principle

Position momentum uncertainty:

The position and momentum of a particle cannot be determined simultaneously with unlimited precision.

$$\Delta x . \Delta p \geq \frac{\hbar}{2}$$

Δx *- the uncertainty in the position*

Δp *- the uncertainty in the momentum*

Heisenberg's Uncertainty Principle

Energy time uncertainty:

The energy and life time of a particle in a state cannot be determined simultaneously with unlimited precision.

ΔE *- the uncertainty in the energy of the particle*

Δt *- the uncertainty in the life time of the particle in the state*

The product of the uncertainties $\Delta E . \Delta t \geq \frac{\hbar}{2}$

Uncertainty relation for circular motion:

The angular position and angular momentum of a particle in a circular motion cannot be determined simultaneously with unlimited precision.

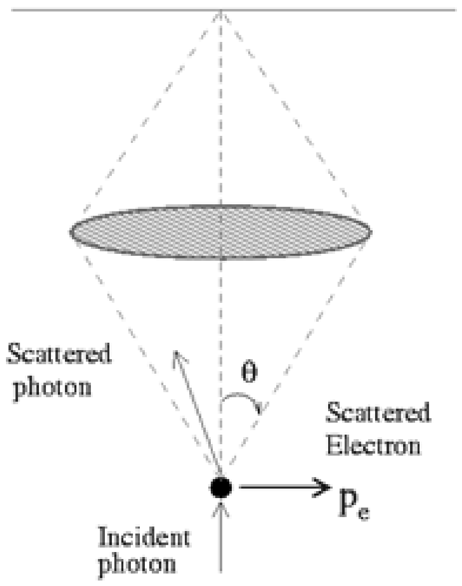
$\Delta\boldsymbol{\theta}$ *- the uncertainty in the angular position*

$\Delta\boldsymbol{L}$ *- the uncertainty in the angular momentum*

The product of the uncertainties $\Delta\boldsymbol{\theta}.\Delta\boldsymbol{L} \geq \frac{\hbar}{2}$

Gamma ray microscope a thought experiment

- *Experiment to "measure" the position of an electron*
- *Construct a "microscope" capable of imaging the electron*



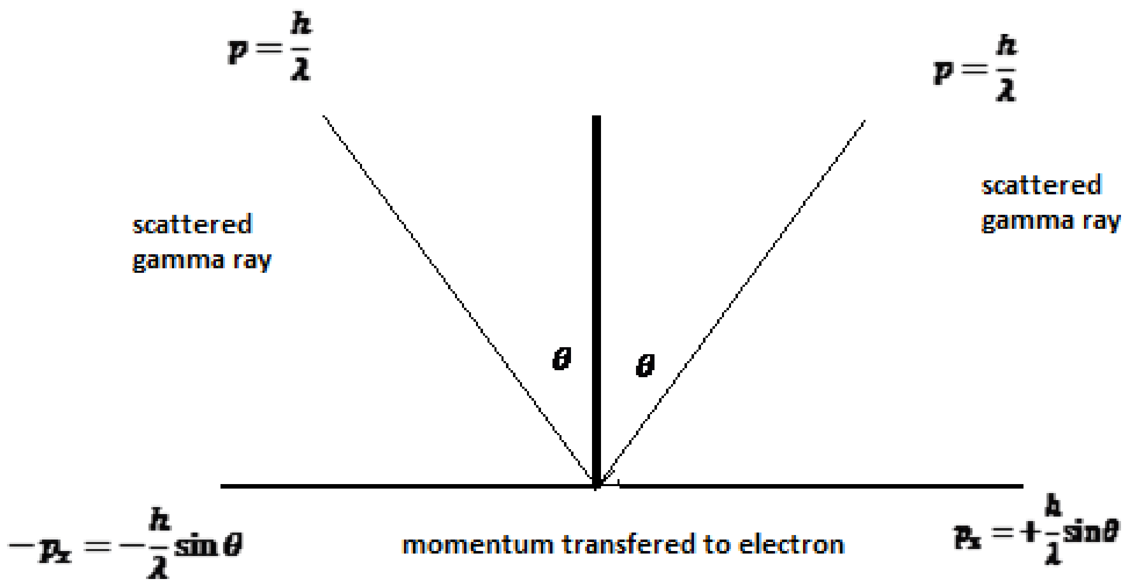
Gamma ray microscope a thought experiment

- *Wavelength of radiation to be comparable to size of electron*
- *Gamma rays wavelengths* $\approx 10^{-12}\ m$
- *Resolution of the microscope =>* $\Delta x \approx \frac{\lambda}{sin\theta}$

Uncertainty in the position of the electron

Gamma ray microscope a thought experiment

- *High energy gamma rays impart momentum to the electrons – Compton effect with gamma rays*
- *Electron momentum along x direction* $\pm\frac{h}{\lambda}sin\theta$
- *Minimum electron momentum uncertainty* $\approx \Delta p = 2\frac{h}{\lambda}sin\theta$



Gamma ray microscope a thought experiment

- *Product of the uncertainties*
- $\Delta x \times \Delta p = 2\frac{hsin\theta}{\lambda} \cdot \frac{\lambda}{sin\theta} = 2h$
- *Greater than* $\frac{h}{4\pi}$ *!*
- *Confirms that the position and momentum cannot be simultaneously determined accurately.*

Non-existence of electron inside the nucleus

- *In a radioactive β decay an electron is emitted from the nucleus with energies of the order of 8 MeV*
- *Estimate the energy of the electron from the uncertainty principle*
- *Electron if present in the nucleus has to be confined to the diameter of the nucleus*
- *The uncertainty in the position is* $\Delta x \approx 10^{-14} m$

Non-existence of electron inside the nucleus

- *Uncertainty in the momentum of electron confined to the nucleus*
- $\Delta p = \frac{h}{4\pi\Delta x} = \frac{h}{4\pi \times 10^{-14}} = 5.28 \times 10^{-21}\ kg\ ms^{-1}$
- *Minimum momentum of the electron cannot be lesser than the uncertainty in momentum* $p \approx \Delta p$
- *A simple non relativistic estimation for the energy*
- $E = \frac{p^2}{2m} = \frac{\Delta p^2}{2m} = \frac{1}{2m}\left(\frac{\hbar}{2.\Delta x}\right)^2 \approx 96\ MeV$
- *Relativistic calculations yield energy >20 MeV*
- *Hence electrons cannot be part of the nucleus !*

Wave functions

- *In case of matter waves, the quantity that varies periodically is called "Wave function"*
- Ψ *is a function of position and time -* $\Psi(x, y, z, t)$
- *The wave function contains all the possible information about the system.*
- *The state of a system in motion can be represented by a wave function -* Ψ
- *The wave function in accounts for wave like properties of a particle.*

Characteristics of wave functions

- *A wave function in 1 dimension*

  $\psi = Ae^{i(kx-\omega t)}$ *where A can be real or imaginary*

- *The well-behaved wave function has to be*
  - *Finite, Continuous and Single valued (FCS)*

Characteristics of wave functions

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- *The derivatives of* Ψ *with respect to the vari*

$$\psi = Ae^{i(kx-\omega t)}$$

$$\frac{\partial\psi}{\partial x} = Ae^{i(kx-\omega t)}.\, ik = ik.\psi$$

$$\frac{\partial\psi}{\partial t} = Ae^{i(kx-\omega t)}.\,(-i\omega) = -i\omega.\psi$$

inherit the properties of Ψ *and hence has to be*

- *Finite, Continuous and single valued (FCS)*

Physical significance of the Wave functions

- ***By itself the wave function has no physical significance***
- ***In general - Ψ is usually a complex quantity***
- ***It is related to probability of finding a particle at a given place at a given time.***
- ***As inferred from the concept of a wave packet Ψ is a probability amplitude***

Normalization of wave functions

- *Given the probability amplitude $\psi = Ae^{i(kx-\omega t)}$ which can be imaginary in the general case*

- *ψ^* is the complex conjugate of the wave function*

$$\psi^* = A^* e^{-i(kx-\omega t)}$$

- *Where A^* is the complex conjugate of A*

- *The product $\psi^*\psi$ is $|\psi|^2$ - a real number*

- *The square of the probability amplitude is the probability density*

- *Which gives the probability of finding the particle in unit length of space*

Normalization of wave functions

- *The integral* $\int \psi^* \psi \, dx = 1$

  should give the total probability in the range where the function is defined

- ψ *is a localized function ->* $\psi \to 0 \; as \; x \to \pm\infty$

- $\int_{-\infty}^{+\infty} \psi^* \psi \, dx = 1$

- *The given wave function must be normalizable*

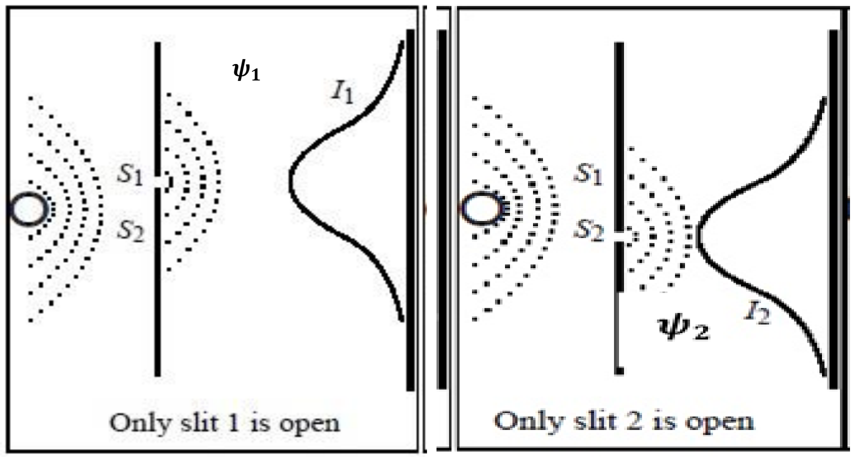
- *The process of normalization gets the right form of A*

Wave function as a state function

- *The wave function satisfying all the conditions is a state function*
- $\psi = Ae^{i(kx-\omega t)}$
- $k = \frac{\hbar}{p}$ and $\omega = \frac{\hbar}{E}$
- *The wave function* $\psi = Ae^{\frac{i}{\hbar}(px-Et)}$
- *The wave function can provide information about the state of the system*

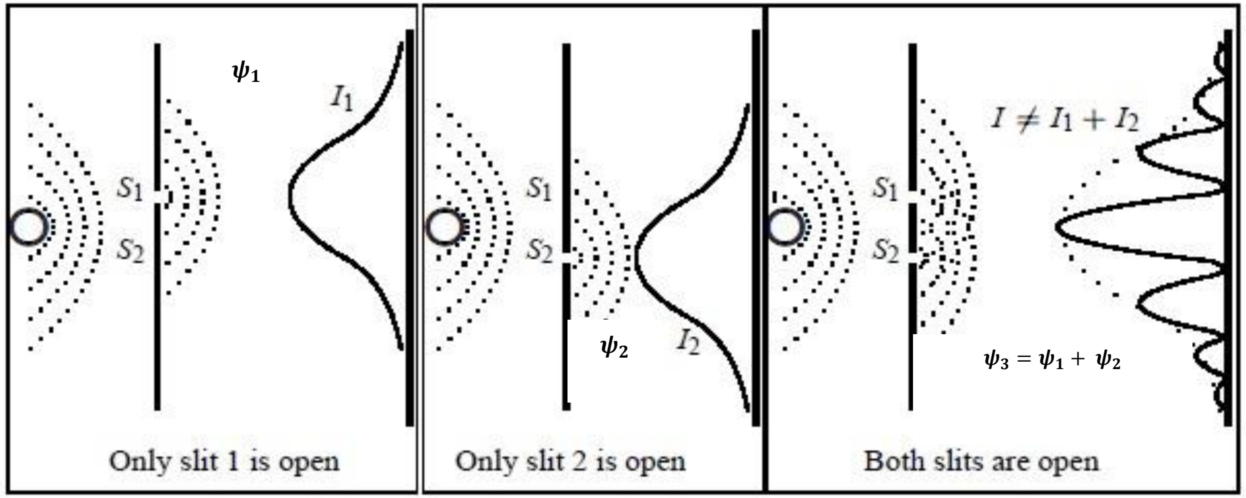
Double slit experiment revisited

- ψ_1 *is the wave function for photons from slit 1*
- $I_1 = |\psi_1|^2$ *is probability the photon reaches the screen*
- ψ_2 *is the wave function for photons from slit 2*
- $I_2 = |\psi_2|^2$ *is probability the photon reaches the screen*



Double slit experiment revisited

- $\psi_3 = \psi_1 + \psi_2$ *is the superposed wave function for photons from both slits*
- $I_3 = |\psi_3|^2$ *is the combined probability of photons reaching the screen* $I_3 \neq I_1 + I_2$
- $|\psi_3|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1{}^*\psi_2 + \psi_1\psi_2{}^* \neq |\psi_1|^2 + |\psi_2|^2$



Linear superposition of wave functions

- *The number of photons emerging from the slits can be different*
- *The superposed wave function* $\psi_3 = m.\psi_1 + n.\psi_2$
- *m and n are arbitrary constants*
- *This is the principle of linear superposition of wave functions*

Observables

- ***All experimentally measurable parameters of a physical system are observables***
  - ***Position***
  - ***momentum***
  - ***Energy of a state***
  - ***life time of electrons***
  - ***Spin of a system***

Operators

- ***Wave functions contain information about the quantum system***
- ***Mathematical operators can be used to extract information about the physical state in terms of the observables***
- ***In general, a physical parameter A of a system has an operator $\hat{A}$.***

Operators

- *A normalized wave function contain information about the quantum system* $\psi = Ae^{\frac{i}{\hbar}(px-Et)}$ *- eigen function*
- *A mathematical operator* $\hat{A}$ *operating on the wavefunction can result in the eigen value* A *of the observable*
- *The eigen value equation* $\hat{A}\psi = A\psi$

Operators

Momentum operator –

$$\psi = Ae^{\frac{i}{\hbar}(px-Et)}$$

- ***The partial derivative of ψ with respect to position yields***

$$\frac{\partial\psi}{\partial x} = \left(\frac{ip}{\hbar}\right)\psi$$

$$\left\{-i\hbar\frac{\partial}{\partial x}\right\}\psi = p\,\psi$$

- ***The momentum operator*** $\hat{p} = \left\{-i\hbar\frac{\partial}{\partial x}\right\}$

Operating on the eigen function yields the momentum eigen value

Operators

Kinetic energy operator –

$$\psi = Ae^{\frac{i}{\hbar}(px-Et)} \text{ and } \frac{\partial\psi}{\partial x} = (\frac{ip}{\hbar})\psi$$

- *The second derivative of ψ with respect to position yields*

$$\frac{\partial^2\psi}{\partial x^2} = \left(\frac{ip}{\hbar}\right)^2 \psi$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = \frac{p^2}{2m}\psi = KE\psi$$

The kinetic energy operator $\widehat{KE} = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right\}$

operating on the eigen function yields the eigen value of the kinetic energy of quantum system

Operators

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Total energy operator –

$$\psi = Ae^{\frac{i}{\hbar}(px-Et)}$$

- *The derivative of* ψ *with respect to time yields*

$$\frac{\partial\psi}{\partial t} = \left(-\frac{iE}{\hbar}\right)\psi$$

$$\left\{i\hbar\frac{\partial}{\partial t}\right\}\psi = E\psi$$

The total energy operator $\hat{E} = \left\{i\hbar\frac{\partial}{\partial t}\right\}$

Operating on the eigen function yields the eigen value of the total energy of quantum system

This is also called as the Hamiltonian operator $\hat{H}$

Operators

Position operator -

- *The position operator has to be discussed in the momentum space*

- *The position operator* $\hat{x}$ *operating on* ψ

$$\hat{x}\psi = x\psi$$

- *yields the eigen value of position of the quantum system*

Operators

Potential energy operator -

- *Potential energy operator is not explicitly described*
- *The eigen value of the potential energy can be inferred as the difference of the total energy and the kinetic energy*
- *The eigen value equation for the potential energy is*

$$\hat{V}\psi = V\psi$$

Expectation values of observables

Quantum mechanics predicts only the most probable values of the observables of a physical system

- the expectation values $\equiv$ the average of repeated measurements on the system.

The eigen value equation for momentum

$$\hat{p}\psi = p\psi$$

The operation $\psi^*\hat{p}\psi = \psi^* p\psi = p\psi^*\psi$

$\psi^*\psi$ is the probability density

$p\psi^*\psi$ should be the probability of the eigen value

Expectation values of observables

$$\psi^*\hat{p}\psi = \psi^* p\psi$$

The spread in the wave packet can yield a range of $p\psi^*\psi$

Integrated over the range of x for the extend of the wave packet

$$\int \psi^*\hat{p}\psi\, dx = \int \psi^* p\psi\, dx = \langle p \rangle \int \psi^*\psi\, dx$$

$\langle p \rangle$ is the most probable value of the momentum.

Thus the expectation value of the momentum is written as

$$\langle p \rangle = \frac{\int \psi^*\hat{p}\psi\, dx}{\int \psi^*\psi\, dx}$$

Expectation values of observables

In general an operator $\widehat{A}$ *of the observable* A

gives the expectation value of the observable

$$\langle A \rangle = \frac{\int \psi^* \widehat{A} \psi \, dx}{\int \psi^* \psi \, dx}$$

THANK YOU

Mohana Lakshmi, Ph.D.

Assistant Professor, Department of Science and Humanities

mohanalakshmi@pes.edu

+91 998623409