Dynamic Modeling VI

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10:35 AM

$$k_d = \frac{-(2a+b)I_{Tot} - C_F}{H(s)} \qquad k_p = \frac{(2ab+a^2)I_{Tot}}{H(s)}$$

$$k_I = \frac{-a^2 b\, I_{Tot}}{H(s)}$$

$$H(s) = \frac{p}{s^3 + qs^2 + us + p}$$

$$k_d = \frac{-(2a+b)I_{Tot} - C_F}{\frac{s^3+qs^2+us+p}{p}} = \frac{-(2a+b)I_{Tot}\cdot p - C_F \cdot p}{s^3+qs^2+us+p}$$

$$\frac{k_d s^5 + (k_d q + k_p)s^4 + (k_d u + k_p q + k_I)s^3 + (k_d p + k_p u + k_I q)s^2 + (k_p p + k_I u)s + k_I p}{I_{Tot} s^6 + (I_{Tot} q + C_F)s^5 + (I_{Tot} u + C_F q)s^4 + (I_{Tot} p + C_F u)s^3 + (C_F p + k_d p)s^2 + k_p p s + k_I p}$$

$$(s-a)^2(s-c)^2(s-b)^2$$

$$= (s^2 - 2as - a^2)(s^2 - 2cs - c^2)(s^2 - 2bs - b^2)$$

$$= (s^4 - 2cs^3 - c^2s^2 - 2as^3 + 4acs^2 + 2ac^2s - a^2s^2 + 2a^2cs + a^2c^2)(s^2 - 2bs - b^2)$$

$$= [(s^4 - 2(c+a)s^3 + (4ac + a^2 - c^2 - a^2)s^2 + 2(ac^2 + a^2c)s + a^2c^2](s^2 - 2bs - b^2)$$

$$= [(s^4 - 2(c+a)s^3 + (4ac - c^2)s^2 + 2(ac^2 + a^2c)s + a^2c^2](s^2 - 2bs - b^2)$$

$$\Rightarrow [(s^4 - 2(c+a)s^3 + (4ac - c^2)s^2 + 2(ac^2 + a^2c)s + a^2c^2] \cdot s^2$$

$$* \quad = s^6 - 2(c+a)s^5 + (4ac - c^2)s^4 + 2(ac^2 + a^2c)s^3 + a^2c^2s^2$$

$$\Rightarrow\left[(s^4-2(c+a)s^3+(4ac-c^2)s^2+2(ac^2+a^2c)s+a^2c^2\right]\cdot -2bs$$

$$* = -2bs^5+4b(c+a)s^4-(8abc+2bc^2)s^3-4(abc^2+a^2bc)s^2-2a^2bc^2s$$

$$\Rightarrow\left[(s^4-2(c+a)s^3+(4ac-c^2)s^2+2(ac^2+a^2c)s+a^2c^2\right]\cdot -b^2$$

$$* = -b^2s^4+2(b^2c+ab^2)s^3-(4ab^2-b^2c^2)s^2-2(ab^2c^2+a^2b^2c)s-a^2b^2c^2$$

$$\Rightarrow s^6+\left[-2(c+a)-2b\right]s^5+\left[(4ac-c^2)+4b(c+a)-b^2\right]s^4+\left[2(ac^2+a^2c)-(8abc+2bc^2)+2(b^2c+ab^2)\right]s^3$$

$$+\left[a^2c^2-4(abc^2+a^2bc)-(4ab^2-b^2c^2)\right]s^2+\left[-2a^2bc^2-2(ab^2c^2+a^2b^2c)\right]s-a^2b^2c^2$$

$$= s^6-2(c+a-b)s^5+(4ac-c^2+4bc+4ab-b^2)s^4+(2ac^2+2a^2c-8abc-2bc^2+2b^2c+2ab^2)s^3$$

$$+(a^2c^2-4abc^2-4a^2bc-4ab^2+b^2c^2)s^2+(-2a^2bc^2-2ab^2c^2-2a^2b^2c)s-a^2b^2c^2$$

$$I_{Tot}s^6+(I_{Tot}q+C_F)s^5+(I_{Tot}u+C_Fq)s^4+(I_{Tot}p+C_Fu)s^3+(C_Fp+k_dp)s^2+k_pps+k_Ip$$

$$= s^6+\frac{I_{Tot}q+C_F}{I_{Tot}}s^5+\frac{(I_{Tot}u+C_Fq)}{I_{Tot}}s^4+\frac{(I_{Tot}p+C_Fu)}{I_{Tot}}s^3+\frac{C_Fp+k_dp}{I_{Tot}}s^2+\frac{k_pp}{I_{Tot}}s+\frac{k_Ip}{I_{Tot}}$$

$$\frac{C_Fp+k_dp}{I_{Tot}}=(a^2c^2-4abc^2-4a^2bc-4ab^2+b^2c^2)$$

$$k_d=\frac{(a^2c^2-4abc^2-4a^2bc-4ab^2+b^2c^2)I_{Tot}}{p}-C_F$$

$$\frac{k_pp}{I_{Tot}}=-2a^2bc^2-2ab^2c^2-2a^2b^2c$$

$$k_p=\frac{(-2a^2bc^2-2ab^2c^2-2a^2b^2c)I_{Tot}}{p}$$

$$\frac{k_Ip}{I_{Tot}}=a^2b^2c^2$$

$$k_I = \frac{a^2 b^2 c^2 I_{Tot}}{p}$$