

Vertical Axis - Solve for numeric values

$$I_{\text{rot},v} = (I_{\text{cm},o} + m_o L_o^2) + I_{\text{cm}} + m(L_c^2 + (\sin(\theta_c)L)^2)$$

Model payload moment of inertia as solid sphere for simplicity, with a radius of 30 cm (parameterize in firmware to allow real values to be entered)

L_m = length of motor

$$I_{\text{cm}} = \frac{2}{5} m \cdot (0.3 \text{ m})^2 = 0.036 m [\text{kg} \cdot \text{m}^2]$$

$$\text{Let } m = 6 \text{ kg} \Rightarrow 0.036 \cdot 6 \text{ kg} = 0.216 [\text{kg} \cdot \text{m}^2]$$

$$I_{\text{cm},o} = \frac{1}{3} m_o \cdot L_m^2 = \frac{1}{3} \cdot 5.9 \text{ kg} \cdot (0.259 \text{ m})^2 = 0.132 [\text{kg} \cdot \text{m}^2]$$

$$I_{\text{rot},v} = [0.132 [\text{kg} \cdot \text{m}^2] + 5.9 [\text{kg}] \cdot (0.02 [\text{m}])^2] + 0.216 [\text{kg} \cdot \text{m}^2] + 6 [\text{kg}] (0.01 [\text{m}]^2 + (\sin(\theta_c) \cdot 0.41 [\text{m}])^2)$$

$$\text{for } \theta_c = \frac{\pi}{2}: I_{\text{rot},v} = 1.360 [\text{kg} \cdot \text{m}^2]$$

$$\text{for } \theta_c = 0, \pi: I_{\text{rot},v} = 0.351 [\text{kg} \cdot \text{m}^2]$$

$$\text{for } \theta_c = \frac{\pi}{6}, \frac{5\pi}{6}: I_{\text{rot},v} = 0.603 [\text{kg} \cdot \text{m}^2]$$

$$p = 3789395, q = 321.3, u = 48495$$

$$a, b, c = \sin\left(\frac{\pi}{4}\right) \cdot 15.707(1+j) = 11.11 + 11.11j$$

$$R_d = \frac{(a^2 c^2 - 4abc^2 - 4a^2 bc - 4ab^2 + b^2 c^2) I_{\text{rot}}}{p} - C_F$$

$$= \frac{[(11.11 + 11.11j)^4 - 4(11.11 + 11.11j)^4 - 4(11.11 + 11.11j)^4 - 4(11.11 + 11.11j)^3 + (11.11 + 11.11j)^4] I_{\text{rot}}}{3789395} - C_F$$

$$= \frac{[-7(11.11 + 11.11j)^4 - 4(11.11 + 11.11j)^3] \cdot 1.360}{3789395} - C_F$$

$$\frac{[426594 + (10971 - 10971j)] 1.360}{3789395} - C_F$$

$$k_p = \frac{(-2a^2bc^2 - 2ab^2c^2 - 2a^3b^2c)I_{Tot}}{\rho} \leftarrow \text{also will be complex}$$

$$k_I = -\frac{a^2b^2c^2I_{Tot}}{\rho}$$