Dynamic Modeling VIII

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4:05 PM

$$p = 3789395,\ q = 321.3,\ u = 48495$$

Roots:

$$a,b,c = \sin\left(\frac{\pi}{4}\right)\cdot 15.707(1+j) = 11.11 + 11.11j$$

$$(s-a)(s-a^*)(s-b)(s-b^*)(s-c)(s-c^*)$$

$$= (s-a)^3(s-a^*)^3 = (s^2-2as+a^2)(s-a)(s-a^*)^3$$

$$= (s^3-2as^2+a^2s-as^2+2a^2s-a^3)(s-a^*)^3$$

$$= (s^3-3as^2+3a^2s-a^3)(s^2-2a^*s+(a^*)^2)(s-a^*)$$

$$= (s^3-3as^2+3a^2s-a^3)(s^3-2a^*s^2+(a^*)^2s-a^*s^2+2(a^*)^2s-(a^*)^3)$$

$$= (s^3-3as^2+3a^2s-a^3)(s^3-3a^*s^2+3(a^*)^2s-(a^*)^3)$$

$$= s^6-3as^5+3a^2s^4-a^3s^3-3a^*s^5+9aa^*s^4-9a^2a^*s^3+3a^3a^*s^2$$
$$+3(a^*)^2s^4-9a(a^*)^2s^3+9a^2(a^*)^2s^2-3a^3(a^*)^2s-(a^*)^3s^3+3a(a^*)^3s^2-3a^2(a^*)^3s+a^3(a^*)^3$$

$$= s^6-3(a+a^*)s^5+3(a^2+3aa^*+(a^*)^2)s^4-(a^3+9a^2a^*+9a(a^*)^2+(a^*)^3)s^3$$
$$+3(a^3a^*+3a^2(a^*)^2+a(a^*)^3)s^2-3(a^3(a^*)^2+a^2(a^*)^3)s+a^3(a^*)^3$$

$$\frac{C_F p + k_d p}{I_{Tot}} = 3(a^3a^*+3a^2(a^*)^2+a(a^*)^3)$$

$$k_d = \frac{3(a^3a^*+3a^2(a^*)^2+a(a^*)^3)\cdot I_{Tot}}{p} - C_F$$

$$k_d = 0.14474 \cdot I_{Tot} - C_F$$

$$\frac{k_p \rho}{I_{Tot}} = 3(a^3(a^*)^2 + a^2(a^*)^3)$$

$$k_p = \frac{3(a^3(a^*)^2 + a^2(a^*)^3) \cdot I_{Tot}}{\rho}$$

$$k_p = 1.07204165 \cdot I_{Tot}$$

$$\frac{k_I \rho}{I_{Tot}} = a^3(a^*)^3$$

$$k_I = \frac{a^3(a^*)^3 \cdot I_{Tot}}{\rho}$$

$$k_I = 3.97013 \cdot I_{Tot}$$

for $\theta_L = \frac{\pi}{2}$: $I_{Tot,V} = 1.360\ [kg \cdot m^2]$

for $\theta_L = 0, \pi$: $I_{Tot,V} = 0.351\ [kg \cdot m^2]$

for $\theta_L = \frac{\pi}{6}, \frac{5\pi}{6}$: $I_{Tot,V} = 0.603\ [kg \cdot m^2]$

Assume $C_F = 0$

for $\theta_L = \frac{\pi}{2}$:

$$k_{d,V} = 0.14474 \cdot I_{Tot} - C_F = 0.1968464$$

$$k_{p,v} = 1.07204165 \cdot I_{Tot} = 1.45797664$$

$$k_{I,v} = 3.97013 \cdot I_{Tot} = 5.3993768$$

for $\theta_L = 0, \pi$:

$$k_{d,v} = 0.14474 \cdot I_{Tot} - C_F = 0.050804$$

$$k_{p,v} = 1.07204165 \cdot I_{Tot} = 0.37628662$$

$$k_{I,v} = 3.97013 \cdot I_{Tot} = 1.393516$$

for $\theta_L = \frac{\pi}{6}, \frac{5\pi}{6}$:

$$k_{d,v} = 0.14474 \cdot I_{Tot} - C_F = 0.087278$$

$$k_{p,v} = 1.07204165 \cdot I_{Tot} = 0.6464411$$

$$k_{I,v} = 3.97013 \cdot I_{Tot} = 2.3939884$$

for lat axis:

$$I_{Tot,l} = I_{cm} + m L^2$$

$$I_{cm} = \frac{2}{5} m \cdot (0.3\,m)^2 = 0.036\,m\ [kg \cdot m^2]$$

$m = 6kg$; $L = 0.41m$;

$$\Rightarrow I_{cm} = 0.036 \cdot 6kg = 0.216\ [kg \cdot m^2]$$

$$\Rightarrow I_{Tot,l} = 0.216 + 6kg \cdot (0.41)^2 = 1.2246\ [kg \cdot m^2]$$

$$\Rightarrow I_{Tot,I} = 0.216 + 6kg \cdot (0.41)^2 = 1.2246\,[kg \cdot m^2]$$

$$k_{d,v} = 0.14474 \cdot I_{Tot} - C_F = 0.1772486$$

$$k_{p,v} = 1.07204165 \cdot I_{Tot} = 1.31128222$$

$$k_{I,v} = 3.97013 \cdot I_{Tot} = 4.8618212$$