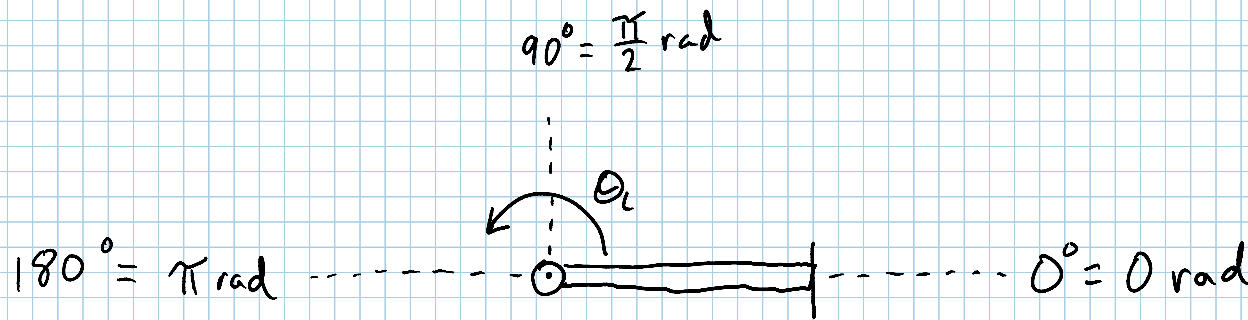
Dynamic Modeling

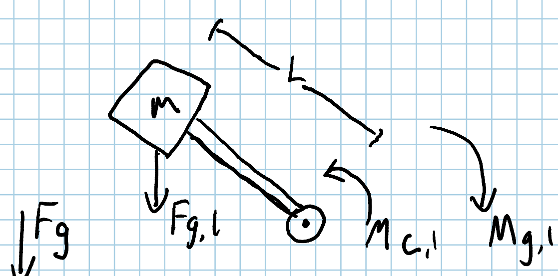
Tuesday, February 11, 2014
2:45 PM

RTx - Control position of each axis
vertAxis
latAxis

latAxis - Coordinate system

Model assuming vertical axis is plumb, then scale by vertical axis position





M_{Ri}: Rotational Force (moment) due to component of F_g

$$L \cdot m \ddot{\theta} = M_{c,1} - M_{g,1}$$

Kinematic helpers:

$M_{g,1}$: At $\theta_L = 0^\circ$, $F_{g,1} = F_g$

At $\theta_L = 180^\circ$, $F_{g,1} = -F_g$

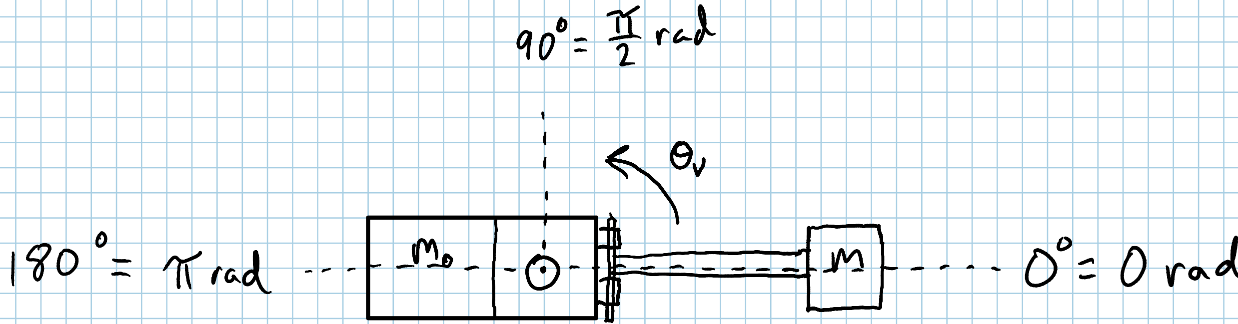
At $\theta = 90^\circ$ $F = 0$

At $\theta_L = 180°$, $F_{g,1} = -F_g$

At $\theta_L = 90°$, $F_{g,1} = 0$

$$M_{g,1} = \cos(\theta_L) \cdot mg \cdot L$$

Scale by position of vertical axis



Scaling of $M_{g,1}$: At $\theta_v = 0°$, $F'_{g,1} = 0$

At $\theta_v = 90°$, $F'_{g,1} = F_{g,1}$

At $\theta_v = 180°$, $F'_{g,1} = 0$

$$M'_{g,1} = \sin(\theta_v) \cdot \cos(\theta_L) \cdot mg \cdot L$$

$$\Rightarrow L \cdot m \ddot{\theta} = M_{c,1} - M'_{g,1} = M_{c,1} - \sin(\theta_v) \cdot \cos(\theta_L) \cdot mg \cdot L$$

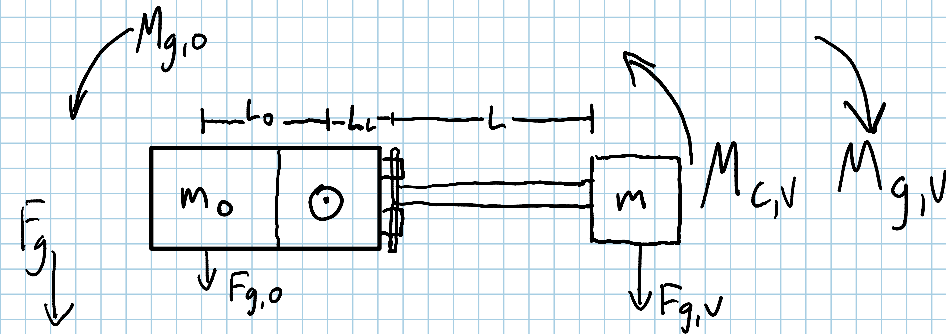
$$\Rightarrow \quad \ddot{\theta} = \frac{M_{c,1}}{mL} - \sin(\theta_v) \cdot \cos(\theta_L) g$$

$$\mathcal{L}\{\sin(\theta_v) \cdot \cos(\theta_L) \cdot g\} = g \cdot \mathcal{L}\{\sin(\theta_v) \cdot \cos(\theta_L)\}$$

$$= g \cdot \frac{\cos(\theta_L)}{s^2 + 1}$$

vertAxis - use established coordinate system.

model assuming lateral axis is straight,
then scale by latAxis position.



$$(m_o L_o + (L_L + L) m)\ddot{\theta} = M_{c,v} - M_{g,v}$$

kinematic helper:

$M_{g,v}$: At $\theta_v = 0°$, $M_{g,v} = (L_L + L) mg - M_{g,o}$

At $\theta_v = 90°$, $M_{g,v} = 0$

At $\theta_v = 90°$, $M_{g,v} = 0$

At $\theta_v = 180°$, $M_{g,v} = M_{g,o} - (L_L + L)mg$

$M_{g,o}$: At $\theta_v = 0°$, $M_{g,o} = m_o L_o g$

At $\theta_v = 90°$, $M_{g,o} = 0$

At $\theta_v = 180°$, $M_{g,o} = -m_o L_o g$

$$M_{g,o} = \cos(\theta_v) \cdot m_o L_o g$$

$$M_{g,v} = \cos(\theta_v) \cdot (M_{g,o} - (L_L + L)mg)$$

$$= \cos(\theta_v) \cdot (\cos(\theta_v) \cdot m_o L_o g - (L_L + L)mg)$$

$$= \cos(\theta_v) \cdot g(\cos(\theta_v) \cdot m_o L_o - (L_L + L)m)$$

Scale by position of lateral axis

At $\theta_L = 0°$, $M'_{g,v} = \cos(\theta_v) \cdot g(\cos(\theta_v) \cdot m_o L_o - L_L m)$

At $\theta_L = 90°$, $M'_{g,v} = M_{g,v}$

At $\theta_L = 180°$, $M'_{g,v} = \cos(\theta_v) \cdot g(\cos(\theta_v) \cdot m_o L_o - L_L m)$

$$M'_{g,v} = \cos(\theta_v) \cdot g(\cos(\theta_v) \cdot m_o L_o - (L_L + \cos(\theta_L) \cdot L)m)$$

$$\Rightarrow g(m_o L_o + (L_L + L)m)\ddot{\theta} = M_{c,v} - \cos(\theta_v) \cdot g(\cos(\theta_v) \cdot m_o L_o - (L_L + \cos(\theta_L) \cdot L)m)$$

$$\Rightarrow \ddot{\theta} = \frac{M_{c,v}}{g(m_o L_o + (L_L + L)m)} - \frac{\cos(\theta_v) \cdot (\cos(\theta_v) \cdot m_o L_o - (L_L + \cos(\theta_L) \cdot L)m)}{m_o L_o + (L_L + L)m}$$