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Course: CS - 608 - 21141

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1) Find the Time Complexity of the recurrence relation

$T(n) = T(n-1) + n.$

$$\Rightarrow \quad T(n) = \begin{cases} T(n-1) + n, & \text{if } n > 0, \\ 1, & n = 0. \end{cases}$$

Solution:

$T(n) = T(n-1) + n$ ——— (i)

$T(n) = [T(n-2) + n-1] + n$ ——— From (i) Substitute value of $T(n)$

$T(n) = T(n-2) + (n-1) + n$ ——— (ii)

$T(n) = [T(n-3) + (n-2)] + (n-1) + n$ ——— For (ii) Substitute value of $T(n)$.

$T(n) = T(n-3) + (n-2) + (n-1) + n$ ——— (iii)

$\vdots$

$\vdots$

$T(n) = T(n-k) + T(n-(k-1)) + (n-(k-2)) + \cdots + (n-1) + n$

Assume $n-k = 0$

$n = k$

$T(n) = T(n-n) + (n-n+1) + (n-n+2) + \cdots + (n-1) + n$

$T(n) = T(0) + \underbrace{1 + 2 + \cdots + (n-1) + n}$

Sum of n natural numbers.

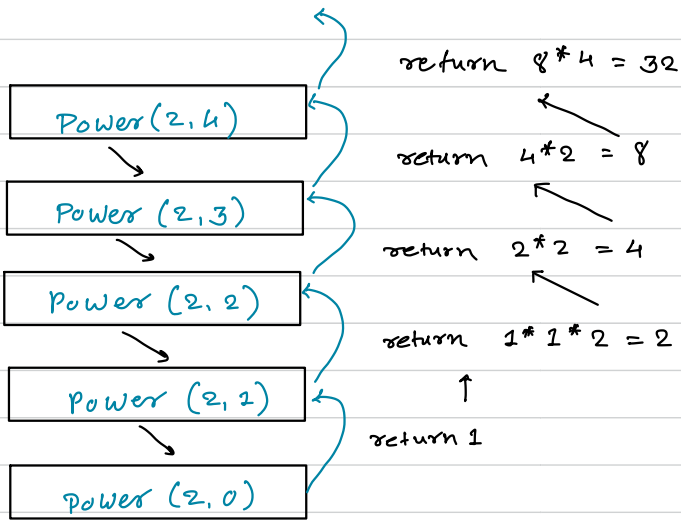
$T(n) = 1 + \frac{n(n+1)}{2} = \frac{n^2 + n}{2}$

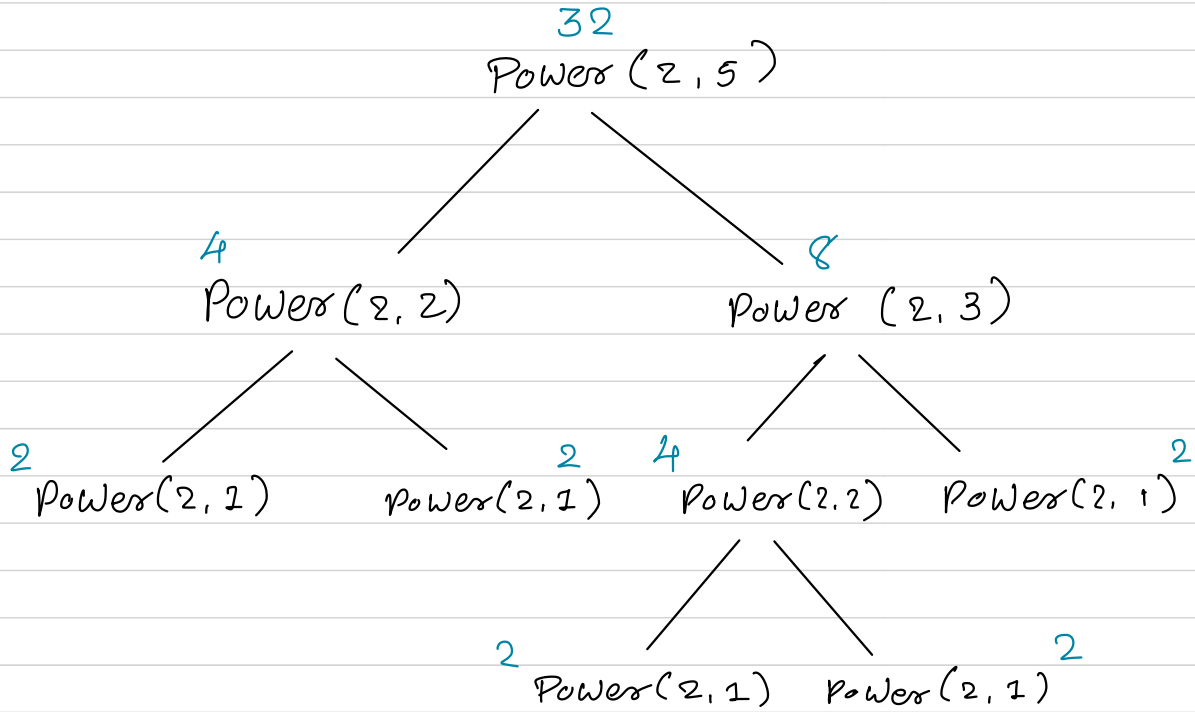
Time Complexity is $O(n^2)$.

2) Recursive tree for the computation of Power (2,5) and give O(n).

Answer: input size : n
Function calls: n - 1
Time taken: O(n)

Represent the Recurrence Tree for the Previous function $T(n) = \begin{cases} 1 & \text{if } n = 0 \\ \text{Power}(2,5), & \text{if } n > 0 \end{cases}$





3) Recursive algorithm for finding the maximum element in a sequence, S, of an n element. Give Pseudocode. What is the running time and space usage of this Algorithm?

Answer:

Algorithm max(A, n)

Input:
Array A, of integers
element n such that $0 \leq n \leq |A|$

Output:
max value of the n element in A.

Pseudocode:

```
if n = 0 then
    return 0
else
    if (max(A, n-1) < A[n-1])
        return A[n-1]
    else return (max(A, n-1)
```

⇒ the running time is $O(n)$.

⇒ the space usage is $O(n)$.

4) Arrange the sequences according to the order of complexity:

A. n^3, $10 \log n$, $57\sqrt{n}$, $49 \log\log n$, 2^n.

Answer: $49 \log\log n < 10 \log n < 57\sqrt{n} < n^3 < 2^n$.

B. $6897 n \log n$, $5n^5$, $2n^n$, $85 \log n$, $10000n$.

Answer: $85 \log n < 10000n < 6897 n \log n < 5n^5 < 2n^n$.

C. $2n\sqrt{n}$, 36485, $73\lceil n/2 \rceil$, 2^{18}, $5n^2$.

Answer: $36485 < 2^{18} < 73\lceil n/2 \rceil < 2n\sqrt{n} < 5n^2$.