1. An acute-angled triangle ABC has orthocentre H. The circle passing through H withcentre the midpoint of BC intersects the line BC at $A1$ and $A2$. Similarly, the circle passing through H with centre the midpoint of CA intersects the line CA at $B1$ and $B2$,and the circle passing throughH with centre the midpoint of AB intersects the line AB at $C1$ and $C2$. Show that $A1$, $A2$, $B1$, $B2$,$C1$, $C2$ lie on a circle.

2. a Prove that $\frac{x^2}{(x-1)^2}+\frac{y^2}{(y-1)^2}+\frac{z^2}{(z-1)^2} \geq 1$
   for all real numbers x, y, z, each different from 1, andsatisfying $xyz = 1$.

   b Prove that equality holds above for infinitely many triples of rational numbers x, y, z, each different from 1, and satisfying $xyz = 1$.

3. Prove that there exist infinitely many positive integers n such that $n^2 + 1$ has a prime divisor which is greater than $2n + \sqrt{2}n$.

4. Find all functions $f : 0, \infty \to 0, \infty$ so, (f is a function from the positive real numbers to the positive real numbers) such that
   $\frac{(f(w))^2+(f(x))^2}{f(y)^2+f(z)^2}$ for all positive real numbers w, x, y, z, satisfying $wx = yz$.

5. Let n and k be positive integers with $k \geq n$ and $k - n$ an even number. Let $2n$ lamps labelled $1, 2, \ldots, 2n$ be given, each of which can be either on or off. Initially all the lamps are off.

   We consider sequences of steps: at each step one of the lamps is switched (from on to off or from offto on).

   Let N be the number of such sequences consisting of k steps and resulting in the state where lamps 1 through n are all on, and lamps $n+1$ through $2n$ are all off.

   Let M be the number of such sequences consisting of k steps, resulting in the state where lamps 1 through n are all on, and lamps $n+1$ through $2n$ are all off, but where none of the lamps $n + 1$ through $2n$ is ever switched on.
   Determine the ratio $\frac{N}{M}$.

6. Let $ABCD$ be a convex quadrilateral with $|BA| \neq |BC|$.Denote the incircles of triangles $ABC and ADC$ by $\omega_1 and \omega_2$ respectively.
   Suppose that there exists a circle ω tangent to the ray BA beyond A and tothe ray BC beyond C, which is also tangent to the lines AD and CD.Prove that the common external tangents of ω_1 and ω_2 intersect on ω.