1. Let n be a positive integer $a_1, \ldots, a_k$ $(k \geq 2)$ be distinct integer in the set $\{1, \ldots, n\}$ such that n divides $a_i\,(a_{i+1} - 1)$ for $i = 1, \ldots, k-1$. Prove that n does not divide $a_k\,(a_i - 1)$.

2. Let ABC be a triangle with circumcentre O. The points P and Q are interior points of the sides $CAandAB$, respectively. Let K, L and M be the midpoints of the segments BP, CQ and PQ, respectively, and let $\lceil$ bethe circle passing through K, L and M. Suppose that the line PQ is tangent to the circle $\lceil$. Prove that $OP = OQ$.

3. Suppose that $s_1, s_2, s_3, \ldots$ is a strictly increasing sequence of positive integers such that the subsequences
$s_{s1}, s_{s2}, s_{s3}, \ldots$ and $s_{s1+1}, s_{s2+1}, s_{s3+1}, \ldots$
are both arithmetic progressions. Prove that the sequence s_1, s_2, s_3,...is itself an arithmetic progession.

4. Let ABC be a triangle with $AB = AC$. The angle bisectors of $\angle CAB$ and $\angle ABC$ meet the sides BC and CA at D and E, respectively. Let K be the incentre of triangle ADC.Suppose that $\angle BEK$=45°. Find all possible values of $\angle CAB$.

5. Determine all functions f from the set of positive integers to the set of positive integers such that, for all positive integers a and b, there exists a non-degenerate triangle with sides of lengths
$a, f(b)$ and $f(b + f(a) - 1)$.
$(Atriangleisnon-degenerateifitsverticesarenotcollinear)$.

6. Let a_1, a_2,..., a_n be distinct positive integers and let M be a set of $n-1$ positive integers not containing $s = a_1 + a_2 + \ldots + a_n$. A grasshopper is to jump along the real axis, starting at the point 0and making n jumps to the right with lengths $a_1, a_2 \ldots, a_n$ in some order. Prove that the order can be chosen in such a way that the grasshopper never lands on any point in M.