# Integral

## Terminology：

* Riemann integral (黎曼积分 / 定积分)
* Improper integral （反常积分 / 不定积分）
* Singularity (奇点)
* Divergence (发散)
* Convergence (收敛)
* Polar coordinate system (极坐标)
* Symmetry (对称性)
* Even Function / odd function (偶函数 / 奇函数)
* [Differentiable](https://en.wikipedia.org/wiki/Derivative) (可微的)
* Remainder (余项)

## Taylor's formula

If is k times differentiable at point then to any we could use the following equation to approximates :

is reminder here, for partial expansion (), we use [Peano](https://en.wikipedia.org/wiki/Peano) form of the remainder

For global expansion, we use [Lagrange](https://en.wikipedia.org/wiki/Joseph_Louis_Lagrange) form of the remainder (k+1 times differentiable)

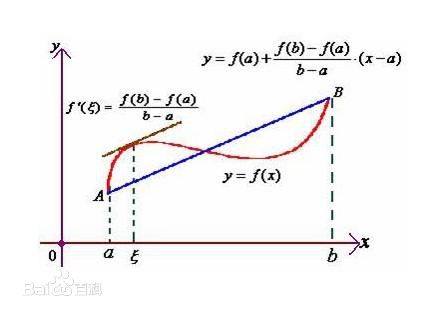
### e.g. Prove:

### e.g. is 2 times differentiable when , Prove:

1. let

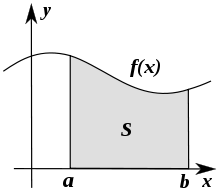
## Lagrange Mean Value Theorem

if is a continuous function on the closed interval, and differentiable on the open interval , then there exists a point in such that:



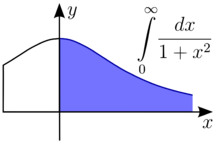
e.g. Assume : , n=1,2,3…, Prove is bounded, and calculate

## Riemann integral (definite integral)



Has two limits: [a, b] is bounded, f(x) in [a, b.] is bounded

## Improper integral



Break those two limits: [a, b] is bounded, f(x) in [a, b.] is bounded

Principle: we only allow one Singularity exist in one integral, So:

## Tips:

## Examples

### e.g. Assume a, b >0, improper integral is bounded, we could get:

(A) a < 1 and b > 1 (B) a > 1 and b > 1

(C) a < 1 and a + b > 1 (D) a > 1 and a + b < 1

### e.g. Assume a > b >0, improper integral is bounded, we could get:

(A) a > 1 and b > 1 (B) a > 1 and b < 1

(C) a < 1 and a + b > 1 (D) a < 1 and a + b < 1

### e.g. Assume a >0, improper integral is bounded, we could get a + b > 3

Hint: When , ,

### e.g. Prove:

When p >1, I is Convergence, when p , I is Divergence

### e.g. Calculate the result of (Symmetry)

### e.g. Calculate the result of