User's Manual for the code in Matlab

The code in `Matlab` and the User's Manual can be found at GitHub with the user's name `YalinR`.

1 Generating Random Variables from 8 UARS Distributions

In the code, α, β and γ specifying the location parameter $\boldsymbol{S}$ are indicated by `a,b,g` respectively. $\boldsymbol{S}$ is indicated by `S`. The concentration parameter κ is indicated by `k`.

The ranges for α, β, γ and κ are as follows: $\alpha \in [0, 2\pi], \beta \in [0, \pi], \gamma \in [0, 2\pi], \kappa \in (0, \infty)$.

(1) Function used: `randraw.m vmUARS.m`
(`randraw.m` is downloaded online: https://www.mathworks.com/matlabcentral /fileexchange/7309-randraw/content/randraw.m by Alex Bar-Guy Copyright (c) 2005, Alex Bar-Guy All rights reserved.)
`[O,S]=vmUARS(k,a,b,g,n)` generates n random variables from the Von Mises UARS distribution with location parameter `S (a, b, g)` and concentration parameter `k`. `S` is the calculated parameter from `a, b, g`. `O` are the generated `n` number of 3 by 3 matrix. The dimension of `O` is $3n$ by 3.

(2) Function used: `randraw.m`,
`densityMF.m`, `MFUARS.m`, `MFUARSv.m`, `MFUARSfinal.m`
(`randraw.m` is downloaded online,see above)
`[O,S]=MFUARSfinal(k,a,b,g,n)` generates `n` random variables from the Matrix Fisher UARS distribution with location parameter `S (a, b, g)` and concentration parameter `k`. `S` is the calculated parameter from `a, b, g`. `O` are the generated `n` number of 3 by 3 matrix. The dimension of `O` is $3n$ by 3.

(3) Function used: `wnUARS.m`
`[O,S]=wnUARS(k,a,b,g,n)` generates `n` random variables from the wrapped normal UARS distribution with location parameter `S (a, b, g)` and concentration parameter `k`. `S` is the calculated parameter from `a, b, g`. `O` are the generated `n` number of 3 by 3 matrix. The dimension of `O` is $3n$ by 3.

(4) Function used: `rwmb.m WTNUARS.m`
`[O,S]=WTNUARS(k,a,b,g,n)` generates `n` random variables from the wrapped

trivariate normal UARS distribution with location parameter `S (a, b, g)` and concentration parameter `k`. `S` is the calculated parameter from `a, b, g`. `O` are the generated `n` number of 3 by 3 matrix. The dimension of `O` is $3n$ by 3.

(5) Function used: `densitywmb.m rwmb.m BungeUARS.m`
`[O,S]=BungeUARS(k,a,b,g,n)` generates `n` random variables from the Bunge UARS distribution with location parameter `S (a, b, g)` and concentration parameter `k`. `S` is the calculated parameter from `a, b, g`. `O` are the generated `n` number of 3 by 3 matrix. The dimension of `O` is $3n$ by 3.

(6) Function used: `densitylorentz.m LorentzianUARS.m`
`[O,S]=LorentzianUARS(k,a,b,g,n)` generates `n` random variables from the Lorentzian UARS distribution with location parameter `S (a, b, g)` and concentration parameter `k`. `S` is the calculated parameter from `a, b, g`. `O` are the generated `n` number of 3 by 3 matrix. The dimension of `O` is $3n$ by 3.

(7) Function used: `densitycayley.m densitywmb.m rwmb.m CayleyUARS.m`
`[O,S]=CayleyUARS(k,a,b,g,n)` generates `n` random variables from the Cayley UARS distribution with location parameter `S (a, b, g)` and concentration parameter `k`. `S` is the calculated parameter from `a, b, g`. `O` are the generated `n` number of 3 by 3 matrix. The dimension of `O` is $3n$ by 3.

(8) Function used: `densityig.m densitywmb.m rwmb.m igUARS.m`
`[O,S]=igUARS(k,a,b,g,n)` generates `n` random variables from the isotropic Gaussian UARS distribution with location parameter `S (a, b, g)` and concentration parameter `k`. `S` is the calculated parameter from `a, b, g`. `O` are the generated `n` number of 3 by 3 matrix. The dimension of `O` is $3n$ by 3.

2 Bayesian Inference for Parameters in Matrix Fisher UARS Distribution

(1) Bayesian estimate and credible set for the location parameter `S` and the concentration parameter `k`
Function used: `SUMO.m posteriordensityMFUARS.m mcmcburnin.m`
`[Shat,S95,Khat,K025,K975,rate1,rate2]`
`=mcmcburnin(k,a,b,g,delta,sigma,n,OS,m,burnin)` returns a Bayesian estimate `Shat`, a 95% credible set for `S`, a Bayesian estimate of `k`, a 95% credible set for `k` after one simulates `m` number of `S`s and `k`s from the posterior distribution and discards the first `burnin` number of `S`s and `k`s. rate1

and rate2 are the acceptance rates for `S` and `k`, respectively. In the function, `k` is the starting value for κ. `a, b, g` can decide the starting value for $\boldsymbol{S}$. `delta` is the parameter for r in Metropolis-Hastings within Gibbs algorithm. `sigma` is the parameter for the proposal of κ. `OS` is the data set used for the Bayesian inference. `n` is the number of 3 by 3 matrices included in the data set `OS`. The dimension of `OS` is $3n$ by 3. The credible set for $\boldsymbol{S}$ is viewed in terms of the set of 3 cones of constant angle `S95` centered at `Shat`. (I used the method of Humbert et al.(1996) to get `Shat`).

(2) Trace plots
need to be fixed
Function used: `mcmc.m getvector.m plots.m`
`[h,kj,Shatm1,Shatm2,Shatm3,Shatm4,Shatm5,`
`Shatm6,Shatm7,Shatm8,Shatm9]`
`=getvector(k,a,b,g,delta,sigma,n,OS,m)` generates the results needed for the trace plots. The explanation for `k, a, b, g, delta, sigma, n, OS` is the same as that in (1). `m` is the number of Bayesian estimates one can obtain.