MA122 — Series and Multivariable Calculus

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As the word "and" in our official title suggests, this course focuses on two important topics within Calculus. But they will have unequal status. Our dominant concern will be to extend the ideas you learned in your first calculus course to the multivariable setting. In order to do that, we will frequently have to remember and rethink the ideas in single-variable calculus, finding the right way to understand them, the point of view that can be generalized to several variables. This part of the course will have a strongly *geometrical* flavor, because the most striking effects of going to several variables have to do with the obvious fact that geometry in two or three dimensions is more interesting (and more complicated) than one-dimensional geometry.

Before we do that, however, we will explore the theory of series in a single-variable context. The most important aspect of this is the representation of functions as infinite sums of simpler functions. (Just as the number 1/3 can also be written as an infinite decimal 0.3333..., functions can be represented as infinite sums.) Since the simpler functions that make up the sum are easy to understand and to compute with, this representation is one of the fundamental methods of applied mathematics. We will focus on those aspects that are most useful. Even with this applied focus, however, this part of the course will have a much more theoretical flavor, simply because the issues are much more subtle.

We begin with series for two reasons: first, because it is still single-variable calculus, continuing on from what you have already done; second, because it is harder, so we want to get it out of the way. The key theme of this part of the course are *approximation* and *alternative representations*.

All the while we will be concentrating on those ideas that have actually been useful in applied settings. We won't, however, merely focus on recipes for doing things: we want to *understand* the Calculus, and not just to know how to use it. Modern computer technology can do computations better than any human being, so knowing how to "just do it" is not really a marketable skill. The key thing, rather, is to *know what needs to be done*. That requires understanding your tools rather than just being able to use them.

In particular, I expect you to develop the ability to use your understanding of the theory to apply it in new practical situations. This might well be the most important skill you can learn in this course: the ability to understand a theory well enough to put it into practice.

Where to find me: My office is Davis 208; my phone extension is 5836; my email address is `fqgouvea@colby.edu`. If you need to reach me when I'm not in my office, email is the best method.

Office hours: Personal interactions are probably the aspect of our lives most affected by COVID-19. I'm afraid the best way to do office hours will be via Zoom. I will set up my available times using Calendly.com and encourage you to book me for any conversations you might want to have.

This semester I have set up Calendly.com to allow you to book meetings lasting either 15 or 30 minutes on Mondays, Wednesdays, and Fridays between 2:30 and 4:00 PM.

Please do not hesitate to talk to me — in fact, I strongly encourage you to do that. It is part of your education, and one of your privileges as a Colby student.

You can always reach me by email, and if necessary we can make an appointment. Notice, however, that I am usually not on campus on Thursdays.

Course web site: I will use Colby's Moodle system to post assignments, solutions, and other information. Please visit `moodle.colby.edu`.

TA help sessions: The class TAs will hold help sessions via Zoom. I will announce the schedule soon after the beginning of classes.

How the class will be organized: I will be trying to run the course using a mixture of lectures and discussion and in-class activity as far as social distancing rules permit. At any time (and especially in classes where I'm doing most of the talking) you should feel free to interrupt with questions and comments. I will try to prompt this participation by asking you questions too!

Text: Our basic text is *Calculus* by Taalman and Kohn. It is a fairly standard calculus textbook that also includes all the material you have already learned in single-variable calculus, so it may be useful for review as well.

For the initial portion of the course, we will use notes that I wrote instead of the textbook. The main reason is that I don't like how the book does this part, but I hope my notes are also easier to read and understand. I will post the notes on Moodle. Since these are a work in progress, I will be grateful for any comments, suggestions, or corrections

I want you to *read* the notes and the book, and not to simply use them as a source of examples and problems. I know it is hard to make sense of mathematical text, but if you can learn to do that you will have acquired a skill that may prove useful in the future. Read slowly, re-do the computations in the book, come up with variations of your own. On the other hand, the book does contain a great many examples, many more than I will do in class, so do look at those.

As a supplement, you will also be asked to read a selection of mathematical blogs from the internet. We will use this as a way to not lose sight of the forest despite spending most of our time paying attention to the trees. See the section called "A Broader View" for what we will be doing with these.

But don't stop there. Colby has a library, and it contains a great many books about calculus and even more books about mathematics. Do check them out!

Technology: Computers are becoming ever more useful to people who need to use mathematics, and it is important to learn how to use these tools. My general philosophy is that one should be able to do simple calculations by hand, but complex ones should be done on a computer. This applies to calculus computations (derivatives, integrals) just as much as to ordinary arithmetic.

We will make use of a powerful computer program called *SageMath*. I recommend learning to use *SageMath* because it is a free program you will be able to use in other courses and after you leave Colby. You really do need to *learn* to use it, because sometimes things go wrong and you need to be able to recognize that when it happens. I will prepare a handout with the basics of *SageMath* and will often use it in class as well.

The use of technology in assignements is encouraged: just indicate what you did (e.g., "I asked *SageMath* to compute the integral; the answer is…"). This extends to exams: I will have one or two computers with these programs running at the front of the room so that you can use them. Of course, trying to learn how to use a program the day before the exam is unlikely to work, so if you want to take advantage of this you should practice using it routinely.

email: I like email. Your zeroth assignment will involve sending me an email message, and I will occasionally use email to communicate with you either individually or as a class. So do check your inbox every once in a while.

Prerequisites: I will assume that you know the basics about limits, continuity, derivatives, and integrals of functions of one variable. I will expect you to be familiar with the logarithm and the exponential function in addition to the basic trigonometric functions. As topics arise during the semester, I can offer quick reviews of the necessary background material, but if you need extensive review you'll need to work on your own. If you have doubts about your background please come talk to me.

Our textbook includes all of these topics (and much more) in the first six chapters, so it is the obvious place to look if you need to review. There are also less formal options. I like a book called *How to Ace Calculus*, by Adams, Hass, and Thompson; it's short, practical, and has a sense of humor. (There's also a *How to Ace the Rest of Calculus*, which covers the material in *this* course. It's worth a look too.) For a more conceptual take, I like Körner's *Calculus for the Ambitious*.

A Broader View: Short writing assignments. Calculus is only a small part of mathematics, and these days it may no longer be the most useful part. So I want to make sure to give you a glimpse of the wider mathematical world. To do that, we have an easily available source: the internet.

Beginning with the second week of class, I will ask you to read an online article on mathematics and respond to it. Your response should be a short (one-page maximum!) paper telling me what you read and considering the questions it raised in your mind. You might write about whether you understood the ideas, whether

they are related to what we are doing in class, and whether they seem useful. You might want to tell me whether you find the ideas under discussion interesting or not. Whatever it is, your paper should demonstrate that you have heard or read something and that you have *given it some thought*. These papers will be due on most **Mondays**: check the Moodle page for specific assignments and dates.

Homework: One can't learn mathematics without doing mathematics, and doing mathematics, in the context of the calculus, consists in solving non-trivial problems. At the same time, learning a new subject often requires doing a number of "five finger exercises": relatively simple problems that basically drill you on what you have just learned. Your textbook has a lot of drill problems; I will sometimes suggest some for you to do in your problem sets, but I won't ask you to turn those in: it is *up to you* to do as many of them as you feel is necessary.

I will ask you to write up and turn in your solutions to problems that require some thought and understanding. You will receive a problem set every **Friday**, and it will be due *before class the following Friday*. These assignments will often contain problems that require a little creativity to solve. It is by solving this type of problem that you will really begin to understand what the mathematics is about, and it is also this kind of problem that you will meet in the "real world" when it comes time to use your knowledge. So, while these problems will be difficult, solving them will be worth the effort.

Since the problem sets are difficult, I will separate some class time to discuss them. We will do this on **Tuesdays**. This means you should make sure to take a first stab at the problems before the Tuesday class so that you know what questions you need to ask.

Please don't leave your assignments for the last day, because you will probably not be able to do them in one day. The problems I'll assign will require time for their solution, and you should plan to put in that time. (*Read this paragraph again. You have been warned!*)

Since homework is so important, it will have a relatively heavy weight in your grade. On the other hand, since homework is also the place to make mistakes and learn from them, I will give ample room for that: I will discard the worst score from the first half of the semester, and also the worst score from the second half. **On the other hand, I will not accept late homework** unless some terrible catastrophe happens.

The goal of homework is learning. I know that as you work there will be a desire to simply "get it done," but if you don't learn from your work you will be wasting your time. Learning from your assignments is the best way to prepare for exams.

Outside of class: You should expect to have to study two to three hours outside of class for every hour you spend in class. The best use of this time is to spend a part of it working with a group of friends, even if remotely. This will add a social dimension to your study, and will also help you resolve difficulties by using

the differing strengths of people in your group. The ideal sequence seems to be to work the problems on your own first, then to work in a group to resolve any difficulties and reconcile any differences (when two people in the group get different answers, it can be a great learning opportunity).

I will ask my teaching assistants to set up regular help sessions. The goal is to provide a good place for you to ask questions. These will only work well for you if you come to them having already spent some time with the problems. Of course, you should also book me for office hours. I'm happy to answer questions.

While you are free to work on homework with a group, *you should write your final draft by yourself*, so that it reflects your state of understanding of the material. In other words, getting help from others to understand, then writing a solution that reflects that understanding is completely acceptable, and even desirable. Copying someone else's solution without understanding is *not* acceptable, and will be treated as a form of academic dishonesty. See the discussion below on what this entails.

When you write up your solutions, be careful: use good grammar and full sentences, explain your reasoning, justify your approach, make it readable. Writing well requires real understanding; by requiring yourself to write well you will make sure that you really do understand what is going on. I will tell the TAs to grade accordingly. If you have questions about what kind of writing I'll be looking for, please feel free to ask.

Tests: We will have two midterm exams, on **March 15** and **April 12**. The tests will consist of problems similar in style (but not in difficulty!) to the problems assigned for homework. I will provide more information as we get closer, but **please mark your calendars.** There will also be a final exam; exam groups haven't been assigned yet; check MyColby for information.

Attendance: Class attendance is required. Should you need to miss a class, please talk to me in advance to see if your absence can be excused. Missing too many classes will result in academic warnings, grade penalties, and eventually failing the class.

Quarantined and Isolated Students: Should one of you be required to stay away from class, I will do my best to provide accomodations. But if you *can* come to class, you are expected to be there.

Grading: Your grade will be computed as follows:

response papers	10%
problem sets	30%
midterm 1	20%
midterm 2	20%
final exam	20%

An outline: Rather than giving you a week-by-week schedule (which I'd end up not following), here's an outline of what I'm planning to do, in roughly the order we'll want to do it. In your textbook, this material is in my notes on series and in chapters 10 to 13 in the textbook, but we will not cover everything in those chapters.

1. Tangent line as an approximation to the function, estimating the error using the second derivative. We will spend some time thinking about what it means to "approximate" and what makes an approximation "good."

2. Improving the approximation using more derivatives, Taylor polynomial and error estimate, Taylor expansion for exponential, sine, and cosine.

3. What is convergence, power series and radius of convergence, manipulating power series. How power series are used in applied mathematics.

4. Coordinate geometry in 3D, curves, vectors, dot product, lines and planes. Vectors as a way of doing geometry, translation between vector algebra and geometric meaning.

5. Functions of several variables, graphs and contour plots. Visualization. What do we do when there are three or more variables?

6. Partial derivatives, gradient, differential, directional derivatives. The main theme here is to generalize the "tangent line approximation" from the first week of class to functions of several variables.

7. Optimization, constrained and unconstrained. This is the most famous application of differential calculus, especially in economics.

8. Integration of functions of several variables, some simple applications of integration, integration in polar coordinates.

About Me

Students often wonder about their professors. You can find out more about me by looking at my home page, at

`http://www.colby.edu/~fqgouvea`

But, just for fun, here are some random facts:

- I was born in São Paulo, Brazil.
- I have been at Colby since 1991.
- I have two sons, one of them a Colby alum.
- My main research interests are in the history of mathematics and in number theory.
- I'm also interested in lots of other things: wine, fountain pens, perfume, theology, books, football, politics,...
- I'm a Christian; I am a member of the Lutheran Church of the Resurrection in Waterville.
- I sing, but mostly in church.
- I've written several books. Visit my home page to see some information about them.
- I have had a beard since 1980, but it has only been white for the last few years.
- Some people think my house is the best place to eat in Waterville Alas, it's only open by invitation.
- I own a rather excitable dog called Jelly Bean. She's about twelve years old and is lots of fun. (But scary if you're afraid of dogs.)
- On my bookshelves, you'll find books by (among others): Gene Wolfe, Evelyn Waugh, Dorothy L. Sayers, Barry Mazur, R. A. Lafferty, Larry Hurtado, Robert Jenson, T. W. Körner, G. K. Chesterton, Walt Kelly, Richard Binder, Flannery O'Connor, Arthur Conan Doyle, Spencer Quinn, Jacques Barzun.

Academic Honesty & Consequences for Academic Dishonesty

Honesty, integrity, and personal responsibility are cornerstones of a Colby education and provide the foundation for scholarly inquiry, intellectual discourse, and an open and welcoming campus community. These values are articulated in the *Colby Affirmation* below. I will expect you to behave accordingly. You are expected to demonstrate academic honesty in all aspects of this course. If you are clear about course expectations, give credit to those whose work you rely on, and submit your best work, you are highly unlikely to commit an act of academic dishonesty.

Academic dishonesty includes, but is not limited to: violating clearly stated rules for taking an exam or completing homework; plagiarism (including material from sources without a citation and quotation marks around any borrowed words); claiming another's work or a modification of another's work as one's own; buying or attempting to buy papers or projects for a course; fabricating information or citations; knowingly assisting others in acts of academic dishonesty; misrepresentations to faculty within the context of a course; and submitting the same work, including an essay that you wrote, in more than one course without the permission of the instructors.

Academic dishonesty is a serious offense against the college. Sanctions for academic dishonesty are assigned by an academic review board and may include failure on the assignment, failure in the course, or suspension or expulsion from the College. For more on recognizing and avoiding plagiarism, see the library guide: `libguides.colby.edu/avoidingplagiarism`.

The Colby Affirmation:

Colby College is a community dedicated to learning and committed to the growth and well-being of all its members.

As a community devoted to intellectual growth, we value academic integrity. We agree to take ownership of our academic work, to submit only work that is our own, to fully acknowledge the research and ideas of others in our work, and to abide by the instructions and regulations governing academic work established by the faculty.

As a community built on respect for ourselves, each other, and our physical environment, we recognize the diversity of people that have gathered here and that genuine inclusivity requires active, honest, and compassionate engagement with one another. We agree to respect each other, to honor community expectations, and to comply with college policies.

As a member of this community, I pledge to hold myself and others accountable to these values.