HI4 電気磁気学 演習課題 (2024/5/16)

45号 山口 惺司

[課題 5]

(a)

2.10

$$V = k\frac{Q}{r}$$

$$= \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+y^2+z^2}}\ [\mathrm{V}]$$

$\frac{1}{\sqrt{x^2 \cdots}}$ (x^2+1) $-\frac{1}{2}(x^2+1)$

$$E_x = -\frac{dV}{dx}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\cdot -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}\cdot 2x$$

$$= \frac{Qx}{4\pi\varepsilon_0(x^2+y^2+z^2)^{\frac{3}{2}}}\ [\mathrm{V/m}]$$

$$E_y = -\frac{dV}{dy}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\cdot -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}\cdot 2y$$

$$= \frac{Qy}{4\pi\varepsilon_0(x^2+y^2+z^2)^{\frac{3}{2}}}\ [\mathrm{V/m}]$$

$$E_z = -\frac{dV}{dz}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\cdot -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}\cdot 2z$$

$$= \frac{Qz}{4\pi\varepsilon_0(x^2+y^2+z^2)^{\frac{3}{2}}}\ [\mathrm{V/m}]$$

(b)

2.11

$$V = \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{Q}{4\pi\varepsilon_0 l\sqrt{x^2+a^2}}\,dx$$

$$= \frac{Q}{2\pi\varepsilon_0 l}\int_0^{\frac{l}{2}} \frac{1}{\sqrt{x^2+a^2}}\,dx$$

$$= \frac{Q}{2\pi\varepsilon_0 l}\left[\log\left(x+\sqrt{x^2+a^2}\right)\right]_0^{\frac{l}{2}}$$

$$= \frac{Q}{2\pi\varepsilon_0 l}\left(\log\left(\frac{l}{2}+\sqrt{\frac{l^2}{4}+a^2}\right)-\log\sqrt{a^2}\right)$$

$$= \frac{Q}{2\pi\varepsilon_0 l}\log\frac{\frac{l}{2}+\sqrt{\frac{l^2}{4}+a^2}}{a}$$

$$= \frac{Q}{2\pi\varepsilon_0 l}\log\frac{l+\sqrt{l^2+4a^2}}{2a}\ [\mathrm{V}]$$

電気磁気学　演習課題 6　(2024/5/23)

[課題 6]

2.13

出ていく電荷を$-Q_1$とする
入り込む電荷をQ_2とする。

$$\frac{Q_1}{\varepsilon_0} = 1500$$

$$-Q_1 = 1500 \times \varepsilon_0$$

$$Q_1 = -1500 \times \varepsilon_0$$

$$\frac{Q_2}{\varepsilon_0} = 2500$$

$$Q_2 = 2500 \times \varepsilon_0$$

$$Q_1 + Q_2 = -1500 \times \varepsilon_0 + 2500 \times \varepsilon_0$$

$$= 1000 \times \varepsilon_0$$

$$= 8.85 \times 10^{-9}\ \mathrm{C}$$

2.17

$r > a$ の場合
ガウスの法則より

$$(\text{左辺}) = \oint E\, ds = E \oint ds = E \cdot 4\pi r^2$$

$1\mathrm{m}^3$ の $Q = \rho \times \frac{4}{3}\pi a^3$

$$(\text{右辺}) = \frac{Q}{\varepsilon_0} = \frac{\rho \times \frac{4}{3}\pi a^3}{\varepsilon_0}$$

(左辺) = (右辺)

$$E \cdot 4\pi r^2 = \frac{\rho \times \frac{4}{3}\pi a^3}{\varepsilon_0}$$

$$E = \frac{\rho \cdot 4\pi a^3}{3 \cdot 4\pi r^2 \varepsilon_0}$$

$$= \frac{\rho a^3}{3 r^2 \varepsilon_0}\ [\mathrm{V/m}]$$

r^{-2}　$-r^{-1}$

$$V = -\int_\infty^r E\, dr$$

$$= -\int_\infty^r \frac{\rho a^3}{3 r^2 \varepsilon_0} dr$$

$$= -\frac{\rho a^3}{3\varepsilon_0} \int_\infty^r \frac{1}{r^2} dr$$

$$= -\frac{\rho a^3}{3\varepsilon_0} \left[-\frac{1}{r}\right]_\infty^r$$

$$= \frac{\rho a^3}{3 r \varepsilon_0}\ [\mathrm{V}]$$

$r < a$ の場合

$$(\text{左辺}) = \oint E\, ds = E \oint ds = E \cdot 4\pi r^2$$

$1\mathrm{m}^3$ あたりの $Q = \rho \times \frac{4}{3}\pi r^3$

$$(\text{右辺}) = \frac{4\rho\pi r^3}{3} \cdot \frac{1}{\varepsilon_0}$$

(左辺) = (右辺)

$$E \cdot 4\pi r^2 = \frac{4\rho\pi r^3}{3\varepsilon_0}$$

$$E = \frac{4\rho\pi r^3}{3 \cdot 4\pi r^2 \varepsilon_0} = \frac{\rho r}{3\varepsilon_0}\ [\mathrm{V/m}]$$

$$V = -\int_\infty^a \frac{\rho a^3}{3\varepsilon_0 r^2} dr - \int_a^r \frac{\rho r}{3\varepsilon_0} dr$$

$$= -\frac{\rho a^3}{3\varepsilon_0}\int_\infty^a \frac{1}{r^2} dr - \frac{\rho}{3\varepsilon_0}\int_a^r r\, dr$$

$$= -\frac{\rho a^3}{3\varepsilon_0}\left[-\frac{1}{r}\right]_\infty^a - \frac{\rho}{3\varepsilon_0}\left[\frac{1}{2}r^2\right]_a^r$$

$$= \frac{\rho a^3}{3\varepsilon_0 a} - \frac{\rho}{3\varepsilon_0}\left(\frac{1}{2}r^2 - \frac{1}{2}a^2\right)$$

$$= \frac{\rho a^2}{3\varepsilon_0} - \frac{\rho(r^2 - a^2)}{6\varepsilon_0} = \frac{\rho(3a^2 - r^2)}{6\varepsilon_0}\ [\mathrm{V}]$$