HI 4 電気磁気学 演習課題 (2024/10/22)

[課題15]

(a)(a0)

$$\oint_S D\,ds = \sum_i Q_i$$

(a1) ガウスの法則より.

(左辺) $= D\cdot 4\pi r^2$

(右辺) $= Q$

(左辺) = (右辺)

$$D\cdot 4\pi r^2 = Q$$

$$D = \frac{Q}{4\pi r^2}$$

(a2) $D=\varepsilon E$ より.

$$E = \frac{Q}{4\pi\varepsilon r^2}\ [V/m]$$

(a3)

$$\int_\infty^r \frac{Q}{4\pi\varepsilon r^2}$$

$$= \frac{Q}{4\pi\varepsilon}\left[-\frac{1}{r}\right]_\infty^r$$

$$= \frac{Q}{4\pi\varepsilon r}\ [V]$$

(a4)

$$P = \varepsilon_0(\varepsilon - 1)E$$

$$= (\varepsilon - \varepsilon_0)E$$

$$= \frac{Q}{4\pi\varepsilon r^2}(\varepsilon - \varepsilon_0)\ (C/m)$$

(b)(b1) ガウスの法則より $Q = q_l l =$ (右辺)

(左辺) $= D\cdot 2\pi r l$

(左辺) = (右辺)

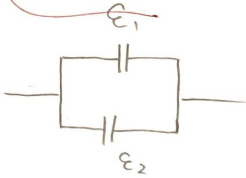
$$D\cdot 2\pi r l = q_l l$$

$$D = \frac{q_l}{2\pi r}\ (C/m^2)$$

(b2) $D=\varepsilon E$, $\varepsilon = \varepsilon_0\varepsilon_r$ より.

$$E = \frac{q_l}{2\pi\varepsilon_0\varepsilon_r r}\ [V/m]$$

(c)(c1)



(c2) $C = \dfrac{\varepsilon_0 S}{d}$ より.

$$C_1 = \frac{\varepsilon_1 S}{2d},\quad C_2 = \frac{\varepsilon_2 S}{2d}$$ となり.

したがって

$$C = C_1 + C_2 \rightarrow \frac{\varepsilon_1 S}{2d} + \frac{\varepsilon_2 S}{2d} = \frac{S(\varepsilon_1+\varepsilon_2)}{2d}$$

H2 4 電気磁気学 演習課題 (2024/10/19)

[課題16]

(a)(a1) $\varepsilon = \varepsilon_0$, $\sigma = \frac{Q}{S} = D$ より,

$$E = \frac{D}{\varepsilon_0} = \frac{Q}{\varepsilon S},$$

$$V = \frac{Qd}{\varepsilon S}, \quad Q = \frac{\varepsilon S}{d} V$$

$$C = \frac{Q}{V} = \frac{\varepsilon S}{d}$$

(a2) ε_0 をE_1, とtての t の極板をE_2とする.

$$\varepsilon_1 = \varepsilon_0, \quad \varepsilon_2 = \varepsilon_0 \varepsilon_r$$

$$d_1 = (d-t), \quad d_2 = t$$

$$E_1 = \frac{D}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 S},$$

$$E_2 = \frac{Q}{\varepsilon_0 \varepsilon_r S}$$

$$V = E_1 (d-t) + E_2 t$$

$$= \frac{Q}{\varepsilon_0 S}(d-t) + \frac{Q}{\varepsilon_0 \varepsilon_r S} t$$

$$= \frac{Q}{\varepsilon_0 S}\left(d - t + \frac{t}{\varepsilon_r}\right)$$

$$Q = \frac{SV}{\frac{d-t}{\varepsilon_0} + \frac{t}{\varepsilon_0 \varepsilon_r}} \text{ より},$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{S}\left(\frac{d-t}{\varepsilon_0} + \frac{t}{\varepsilon_0 \varepsilon_r}\right)}$$

(a3)

$$\frac{Q}{\frac{Q}{S}\left(\frac{d-t}{\varepsilon_0} + \frac{t}{\varepsilon_0 \varepsilon_r}\right)} \div \frac{d}{\varepsilon_0 S}$$

$$= \frac{\cancel{Q} d}{\cancel{Q}\left(d - t + \frac{t}{\varepsilon_r}\right)}$$

$$= \frac{d}{d - t + \frac{t}{\varepsilon_r}}$$

$$= \frac{d}{d + t\left(\frac{1}{\varepsilon_r} - 1\right)}$$

$$= \frac{1}{1 + \frac{t}{d}\left(\frac{1}{\varepsilon_r} - 1\right)} \text{ [倍]}$$

(b)(b1) ガウスの法則より,

(左辺) $= D \cdot \Delta S = D 4\pi r^2$.

(右辺) $= Q$

(左辺) = (右辺)

$$D = \frac{Q}{4\pi r^2} \text{ [C/m}^2\text{]}$$

(b2) ($a < r < R$) のとき,

$$E_1 = \frac{Q}{4\pi r^2 \varepsilon_1} \text{ [V/m}^2\text{]}$$

($R < r < b$) のとき

$$E_2 = \frac{Q}{4\pi r^2 \varepsilon_2} \text{ [V/m}^2\text{]}$$

[課題16] 続き

(b3)

$$-\int_b^R E_2\,dr - \int_R^a E_1\,dr$$

$$= -\int_b^R \frac{Q}{4\pi r^2\varepsilon_2} - \int_R^a \frac{Q}{4\pi\varepsilon_1 r^2}dr$$

$$= -\frac{Q}{4\pi\varepsilon_2}\left[-\frac{1}{r}\right]_b^R - \frac{Q}{4\pi\varepsilon_1}\left[-\frac{1}{r}\right]_R^a$$

$$= \frac{Q}{4\pi\varepsilon_2}\left(\frac{1}{R}-\frac{1}{b}\right)+\frac{Q}{4\pi\varepsilon_1}\left(\frac{1}{a}-\frac{1}{R}\right)$$

$$= \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_2}\left(\frac{1}{R}-\frac{1}{b}\right)+\frac{1}{\varepsilon_1}\left(\frac{1}{a}-\frac{1}{R}\right)\right)\ [V]$$

(C)

$$\begin{cases} D_1\cos\theta_1 = D_2\cos\theta_2 & ① \\ E_1\sin\theta_1 = E_2\sin\theta_2 & ② \end{cases}$$

①, $D = \varepsilon E$ より,

$$E_0\varepsilon_0\cos\theta_0 = E\,\varepsilon_0\varepsilon_r\cos\theta \quad ③$$

②より,

$$E_0^2\sin^2\theta_0 = E^2\sin^2\theta \cdot \varepsilon_r^2$$

$$E_0^2\varepsilon_r^2\sin^2\theta_0 = E^2\varepsilon_r\sin^2\theta \quad ④$$

③より

$$E_0\cos\theta_0 = E\,\varepsilon_r\cos\theta$$

$$E_0^2\cos^2\theta_0 = E^2\varepsilon_r\sin^2\theta \quad ⑤$$

④+⑤

$$E_0^2(\varepsilon_r^2\sin^2\theta_0 + \cos^2\theta_0$$

$$= E^2\varepsilon^2(\sin^2\theta + \cos^2\theta)$$

↓

$$E^2\varepsilon_r^2 = E_0^2(\varepsilon_r^2\sin^2\theta_0 + \cos^2\theta_0)$$

$$E^2 = E_0^2\left(\sin^2\theta_0 + \frac{1}{\varepsilon_r^2}\cos^2\theta\right)$$

$$E = E_0\sqrt{\sin^2\theta + \frac{\cos^2\theta}{\varepsilon_r^2}}\ [V/m]$$