Box-Müller

$$g_1 = \sqrt{-2\ln u_0} \cdot \sin(2\pi u_1) \qquad u_1 \sim U,\ u_2 \sim U$$
$$g_2 = \sqrt{-2\ln u_0} \cdot \cos(2\pi u_1)$$

Algorithm:

Inputs: $u_0, u_1 \sim U$ $\quad f = \sqrt{-2\ln(u_0)}$ $\quad g_0 = f \cdot \sin(\tau u_0)$ $\quad g_1 = f \cdot \cos(\tau u_1)$

Implementation:

Inputs: u_0 : unsigned(47 downto 0)
u_1 : unsigned(15 downto 0)

```
---------------- Generate u0 and u1 ----------------
a  = taus(); b = taus();
u0 = concat(a,b[31:16]);
u1 = b[15:0];

-------------- Evaluate e = -2ln(u0) --------------

# Range Reduction
exp_e = LeadingZeroDetector(u0)+1;
x_e   = u0 << exp_e;

# Approximate -ln(x_e) where x_e = [1,2)
# Degree-2 piecewise polynomial
y_e = ((C2_e[x_e_B]*x_e)+C1_e[x_e_B])*x_e_B
      +C0_e[x_e_B];

# Range Reconstruction
ln2 = ln(2);
e'  = exp_e*ln2;
e   = (e'-y_e)<<1;

--------------- Evaluate f = sqrt(e) ---------------

# Range Reduction
exp_f = 5-LeadingZeroDetector(e);
x_f'  = e >> exp_f;
x_f   = if(exp_f[0], x_f'>>1, x_f');

# Approximate sqrt(x_f) where x_f = [1,4)
# Degree-1 piecewise polynomial
y_f = C1_f[x_f_B]*x_f_B+C0_f[x_f_B];

# Range Reconstruction
exp_f' = if(exp_f[0], exp_f+1>>1, exp>>1);
f      = y_f << exp_f';

------------ Evaluate g0=sin(2*pi*u1) ------------
------------          g1=cos(2*pi*u1) ------------

# Range Reduction
quad  = u1[15:14];
x_g_a = u1[13:0];
x_g_b = (1-2^-14)-u1[13:0];

# Approximate cos(x_g_a*pi/2) and cos(x_g_b*pi/2)
# where x_g_a, x_g_b = [0,1-2^-14]
# Degree-1 piecewise polynomial
y_g_a = C1_g[x_g_a_B]*x_g_a_B+C0_g[x_g_a_B];
y_g_b = C1_g[x_g_b_B]*x_g_b_B+C0_g[x_g_b_B];

# Range Reconstruction
switch(seg)
  case 0:   g0 =  y_g_b; g1 =  y_g_a;
  case 1:   g0 =  y_g_a; g1 = -y_g_b;
  case 2:   g0 = -y_g_b; g1 = -y_b_a;
  case 3:   g0 = -y_g_a; g1 =  y_g_b;

---------------- Compute x0 and x1 ----------------
x0 = f*g0; x1 = f*g1;
```

Symbol	Range	I	F
u_0	$0 ; 1-2^{-48}$	0	48
u_1	$0 ; 1-2^{-16}$	0	16
exp_e	$1 ; 49$	6	0
x_e	$[1 ; 2)$	0	47
$Ln2$	$\ln(2)$	0	32
e'	$\ln(2) \cdot [1 ; 49]$	6	28
e		7	24
y_e		0	27
g_0 g_1	$-1 ; 1$	1	15
y_{ga} y_{gb}		0	15

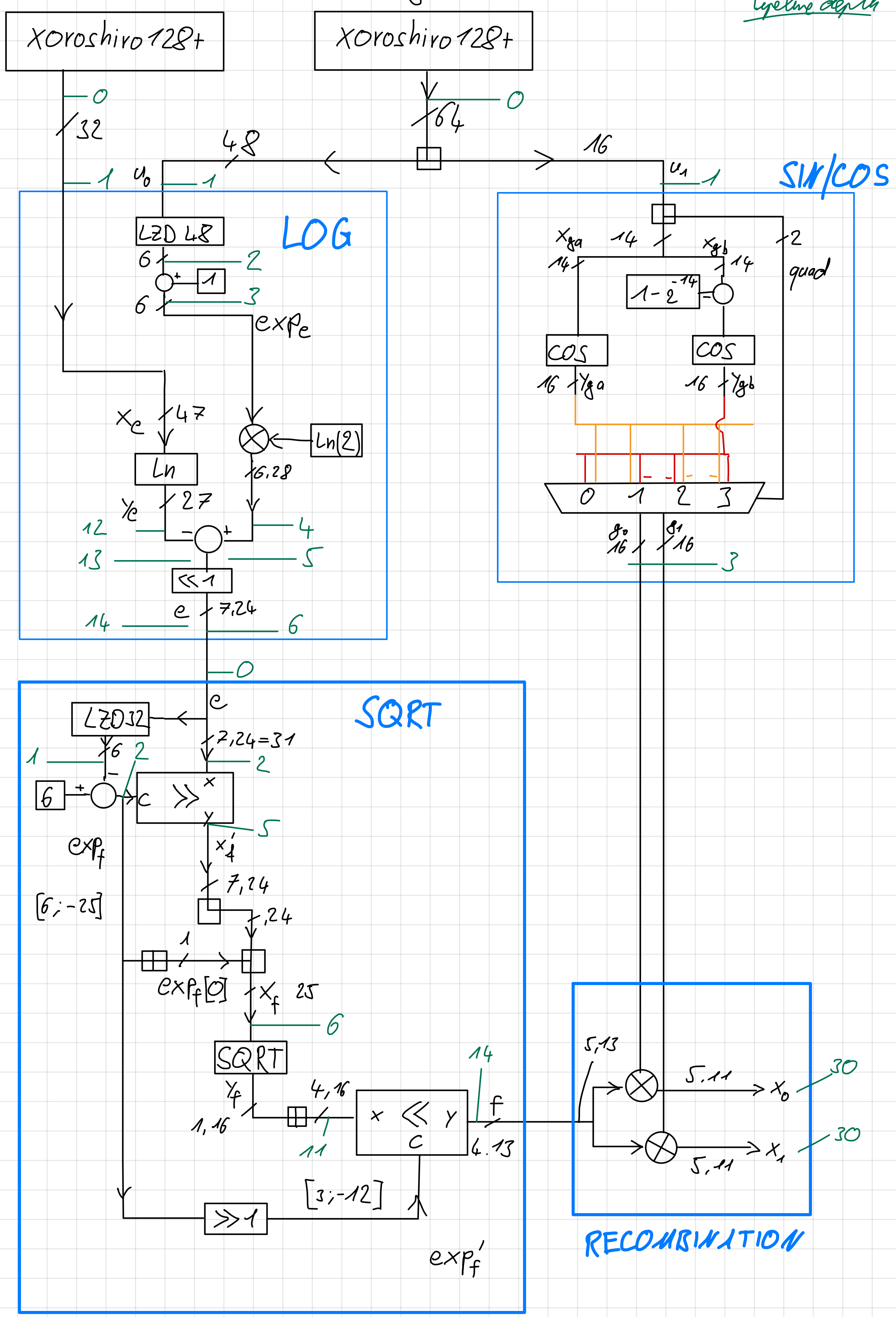
Symbol	Range	I	F
C_{0g} D_g	$0 ; 1$	0	18
C_{1g}	$0 ; \sim 1{,}6$	1	18
C_{1f}	$0 ; 0{,}5$	0	19
C_{0f} D_{0f}	$1 ; 2$	0	19
$C2_e$			13
$C1_e$			23
$C0_e$			30
f		4	13
exp_f	$-25 ; 5$	6	0
y_f		1	16
x_0 x_1		5	11

PP-Tables

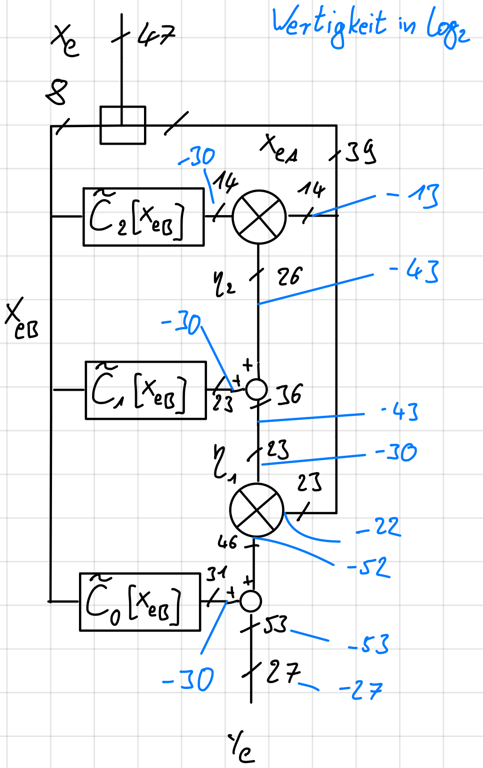
f	Segments	Degree	C_s
$\cos(2\pi x)$	128	1	C_{g0} C_{g1}
$\ln(x)$	256	2	$C2_e$ $C1_e$ $C0_e$
sqrt	$2 \cdot 64$	1	$C0_f$, $C1_f$ $D0_f$

Flowgraph

Pipeline depth



LN



$$x_c = (x_{eB};\ x_{eA};\ null)$$

$$\eta_2 = \tilde C_2 \cdot x_{eA} = -2^{30} \cdot x_{eA} \cdot C_2$$

$$\tilde C_2 = -2^{30} \cdot C_2 \qquad \tilde C_1 = 2^{30} \cdot C_1 \qquad C_0 = 2^{30} \cdot C_0$$

$$\eta_1 = \tilde C_1 - \eta_2 = 2^{30} \cdot (C_1 - x_{eA} \cdot C_2)$$

$$y_e = \tilde C_0 + \tilde C_1 x_{eA} + \tilde C x_{eA}^2 = 2^{30} \cdot (C_0 + C_1 x_{eA} + C_2 x_{eA}^2)$$

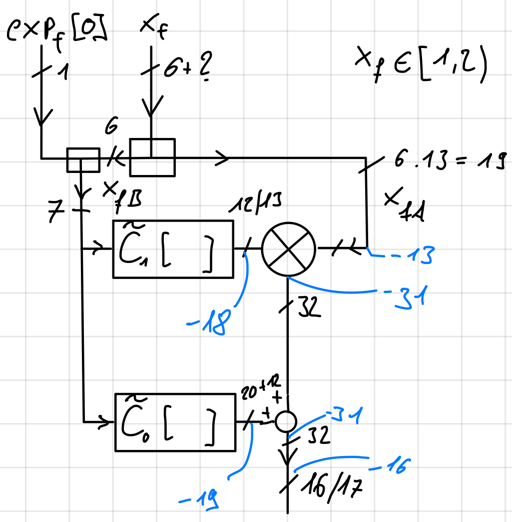
$$= 2^{30} \cdot C_0 + 2^{30} \cdot C_1 \cdot x_{eA} + 2^{30} C_2 x_{eA}^2$$

$$= \tilde C_0 + \underbrace{x_{eA}}_{27} \cdot \underbrace{(\tilde C_1 + x_{eA} \tilde C_2)}_{27}$$

$\tilde C_0$: 30
$x_{eA} \cdot p$: 30
$\tilde C_1$: 22

$$\underbrace{\underbrace{x_{eA}}_{13} \cdot \underbrace{\tilde C_2}_{13}}_{26}$$

SQRT



TRIG