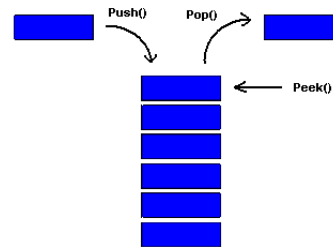


Data Structures and Algorithms

Stack



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Outlines

- Introduction to Stack
- Stack ADT
- Stack Operations
- Stack Applications
- Stack Implementation
- Stack Implementation using :Array & Linked List

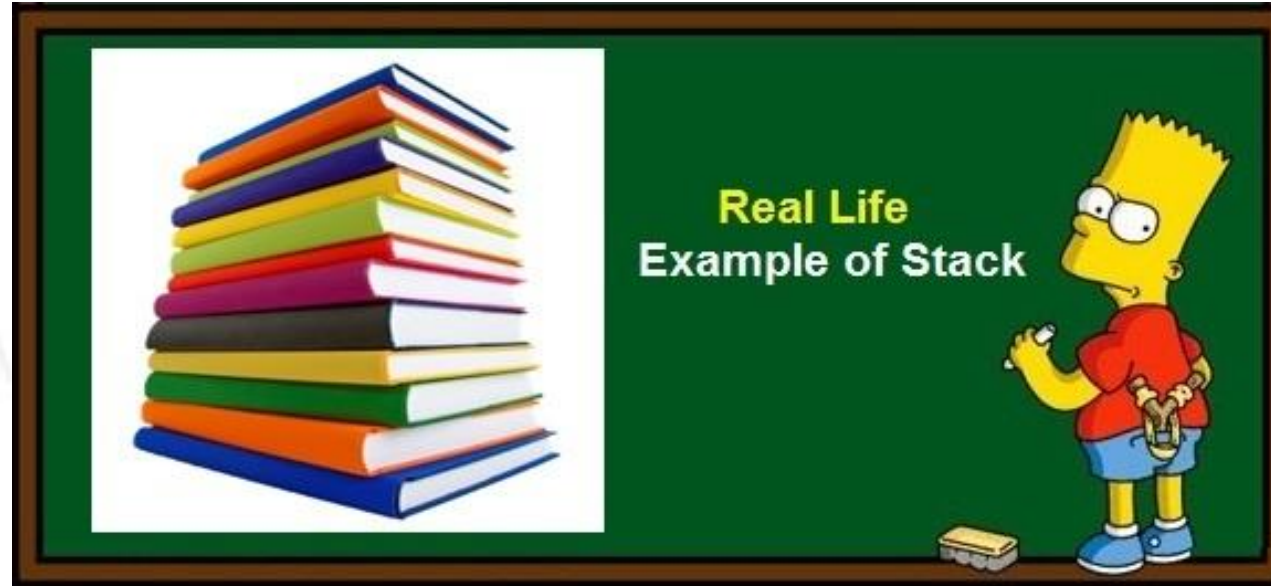


WHAT IS Stack ?



What is Stack?

- Stack is a data structure.
 - Data structures is away to store and organize data.
 - Arrays.
 - Linked Lists.
- We will define a stack as ADT.
 - Features and operations without implementation details.



A stack of plates



A stack of tennis balls



What is Stack?

- Stack is basically a collection with a property that an item in the stack must be inserted or removed from the same end.
- This end is called the Top of the stack.
- The Stack also called Last-In First-Out (LIFO) Data Structure.

Stack ADT

- Formal definition of a stack ADT:

A list with the restriction that insertion and deletion can be performed only from one end, called the top.

Stack ADT

- The main two operations are:
 - Insertion an element, called **Push**.
 - Removing an element, called **Pop**.
- **Top**, returns the element at the top of stack.
- **IsEmpty**, Check if stack is empty or not.
- Push and Pop One element at a time.
- **All operations should be performed in constant time -> $O(1)$.**

<u>Operations</u>	
(1) Push(x)	} Constant time or $O(1)$
(2) Pop()	
(3) Top()	
(4) IsEmpty()	

Logical Representation of Stack

- Logical representation of a stack is a **Container opened from one side.**



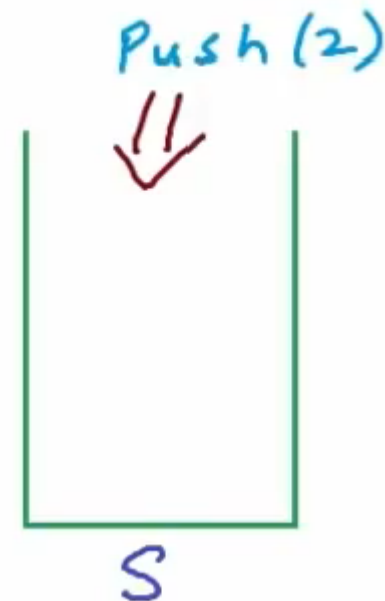
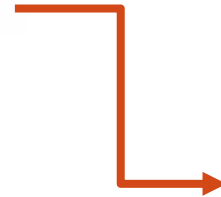
Stack

Stack Operations (1/4)

- This is an empty stack called S, and we will insert number “2” in it.

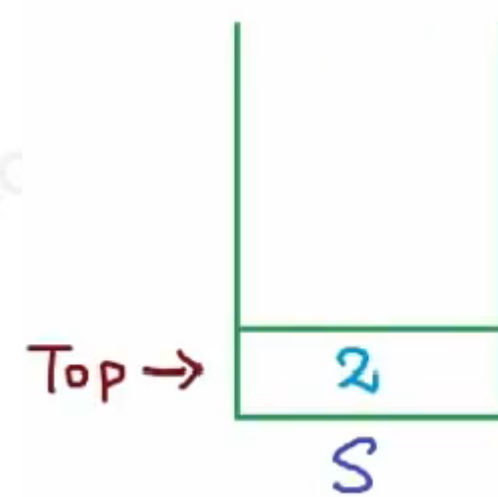


Push(2)



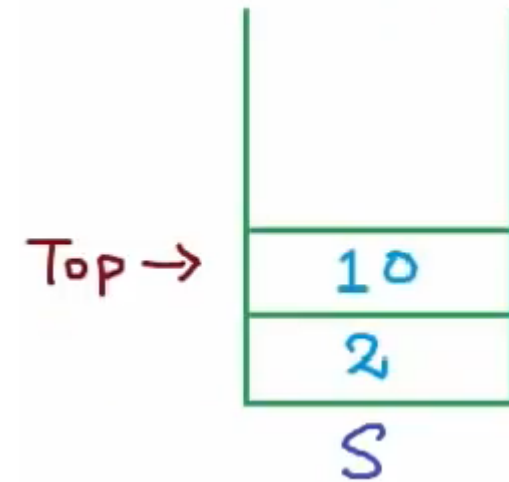
Stack Operations (2/4)

- After the Push, the stack will look like:
 - It contains one integer at the top.



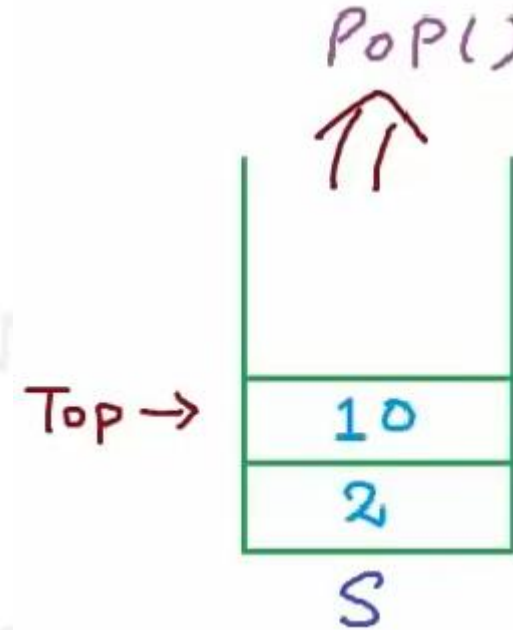
- Inserting number 10.

Push(10)



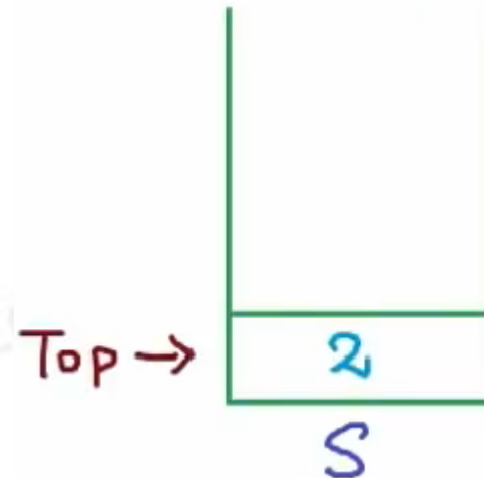
Stack Operations (3/4)

➡ Removing an element.



➡ After the Pop operation.

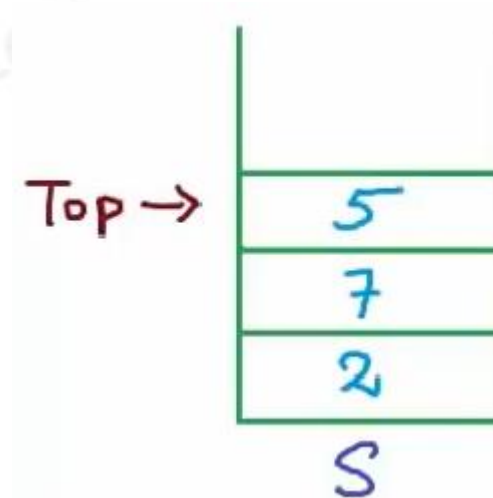
POP()



Stack Operations (4/4)

- After another Two Push operations.

Push(7)
Push(5)



- At this stage of the stack:

- Top will return 5.

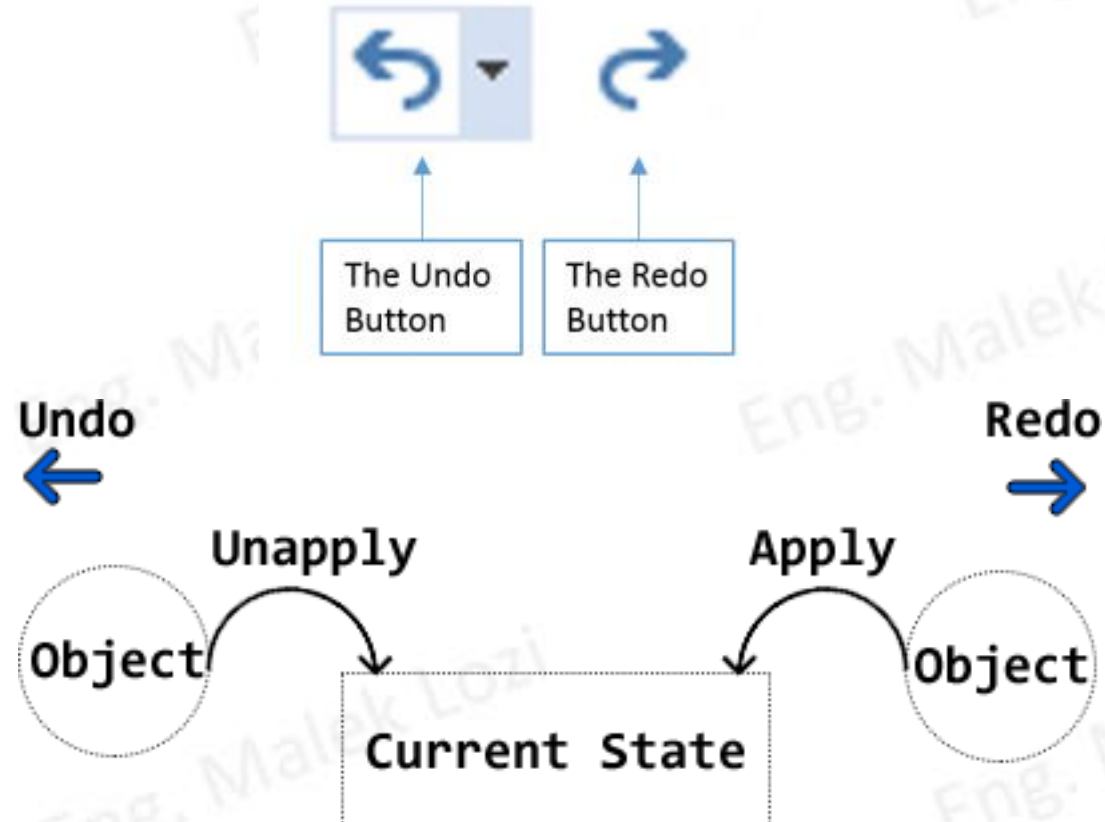
- IsEmpty will return False.

Top() ⇒ 5
IsEmpty() ⇒ false

Stack Applications (1/2)

► Real applications of Stack.

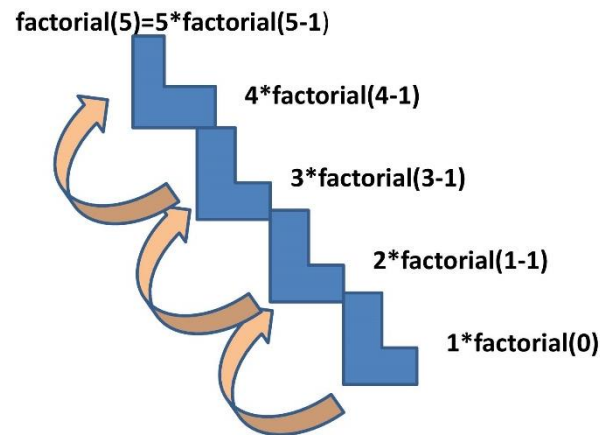
1- Undo operation in a text editor.



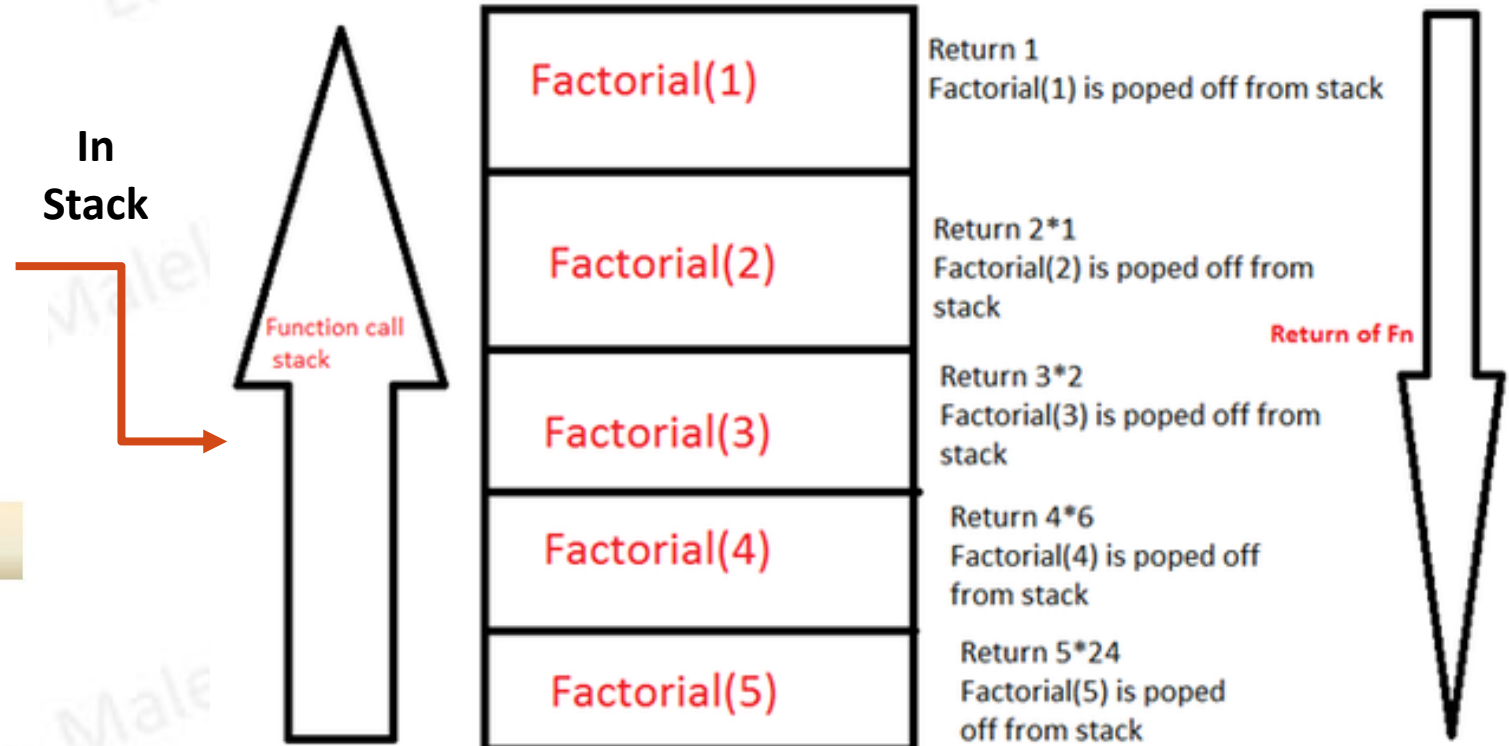
Stack Applications (2/2)

➡ Real applications of Stack.

2- Function calls / recursion.



* Recursion expands when $n > 0$
* Its starts winding up on hitting the base





**Think about other
applications for a Stack?**

Stack Implementation (1/2)

- From the definition of the Stack:

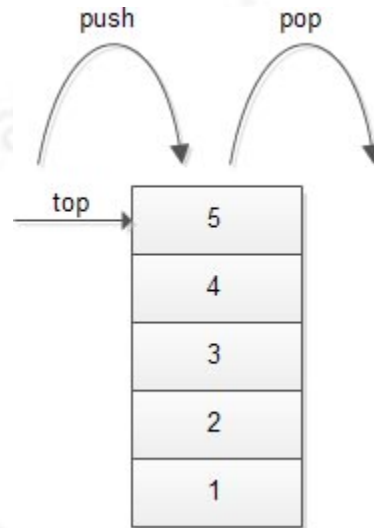
A list with the restriction that insertion and deletion can be performed only from one end, called the top.

- We can add only this one extra constraint to an implementation of a List, then we can get a Stack.
- The constraint is that insertion and deletion must be performed only from one end.

Stack Implementation (2/2)

- There are two popular ways of creating lists.
 - Arrays.
 - Linked Lists.
- We can use any of them to create a Stack.

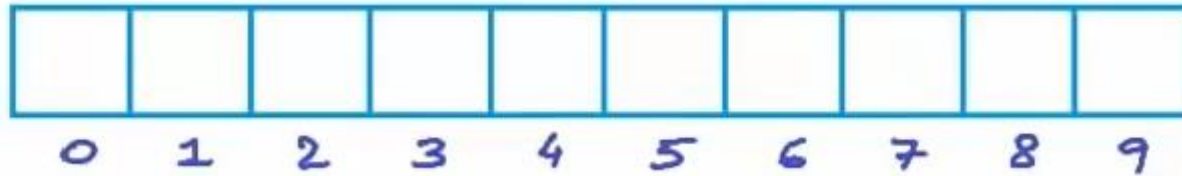
Stack Implementation Using Array



Stack Implementation Using Array (1/19)

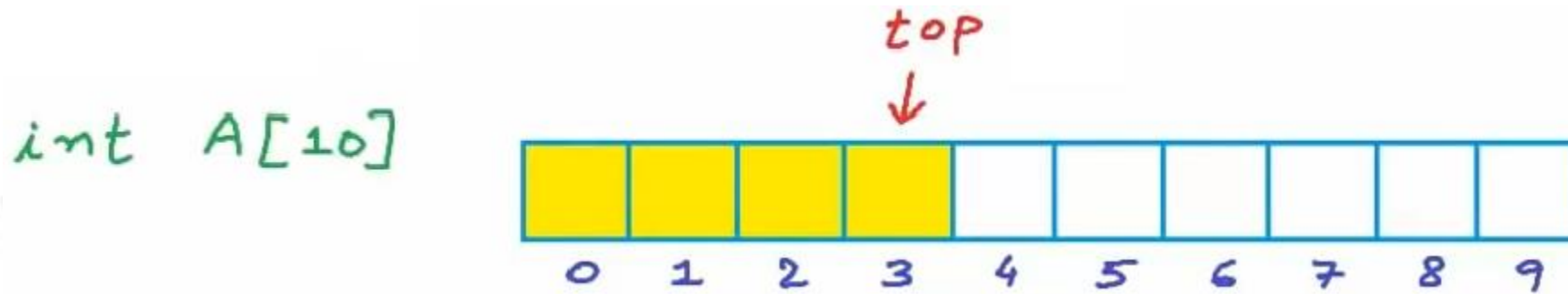
- We want to create a Stack of integers.
- First, we can create an array of integers.
 - Array of Ten integers.

`int A[10]`



Stack Implementation Using Array (2/19)

- At any point, some part of this array starting index Zero to index marked as **top** will be our stack.

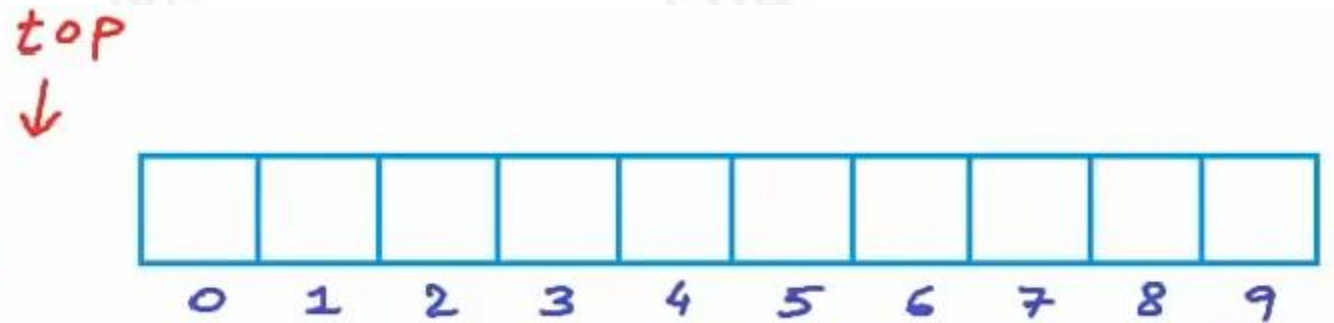


- We can create a variable named **top** to store the index of top of the stack.

Stack Implementation Using Array (3/19)

- For an empty stack top will set to -1.

```
int A[10]
top ← -1 //empty stack
```



Stack Implementation Using Array (4/19)

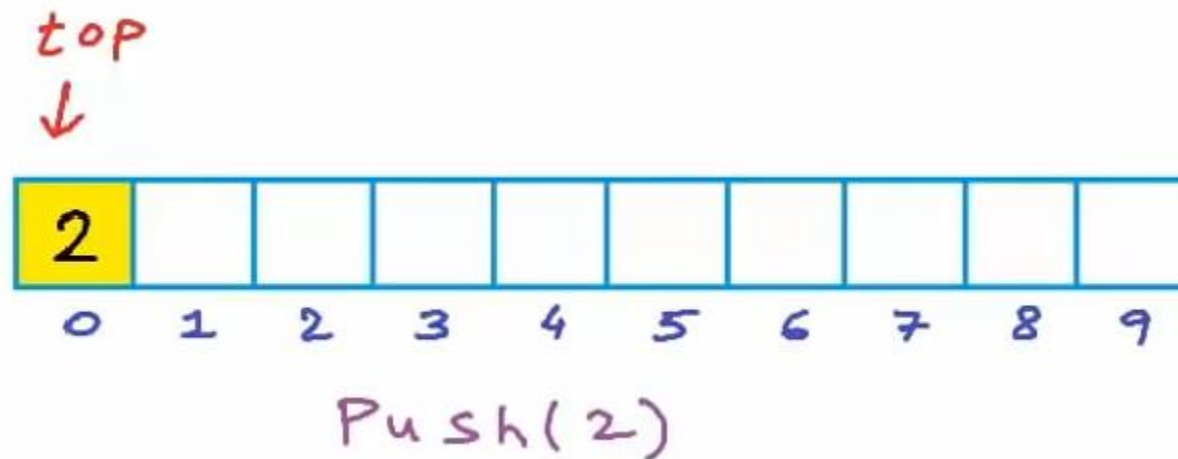
- For Push operation, we can write a function named **push** that will take an integer x as an argument.

```
int A[10]
top ← -1 //empty stack

push(x)
{
    top ← top + 1
    A[top] ← x
}
```

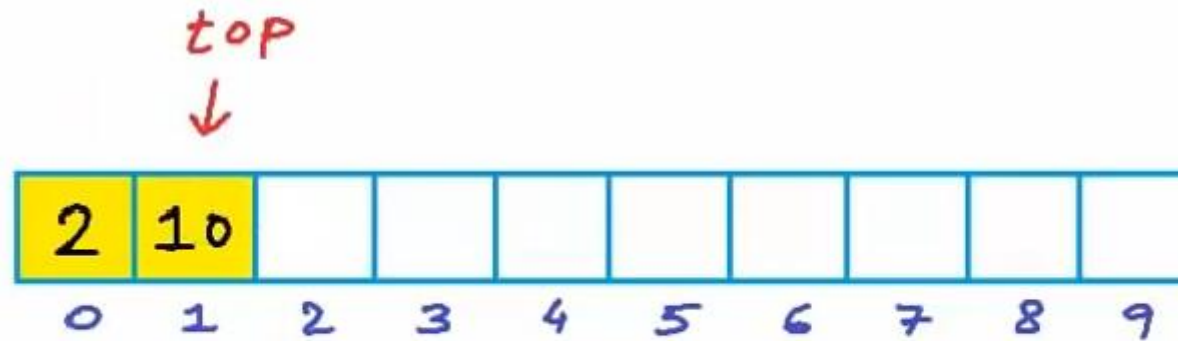
Stack Implementation Using Array (5/19)

- The stack is still empty.
- Let us insert data into the stack.
- We will call the **push** function.



Stack Implementation Using Array (6/19)

- We will push another item onto the stack.

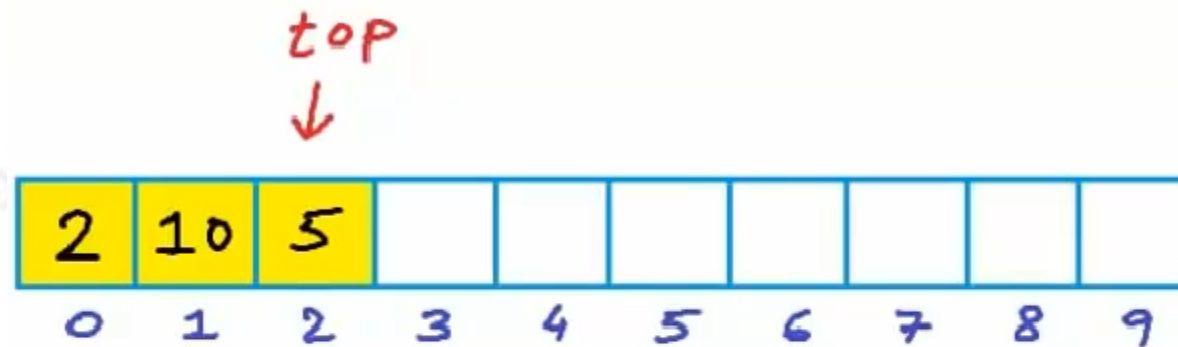


Push(2)

Push(10)

Stack Implementation Using Array (7/19)

- With each push, the stack will expand towards higher indices in the array.



Push(2)

Push(10)

Push(5)

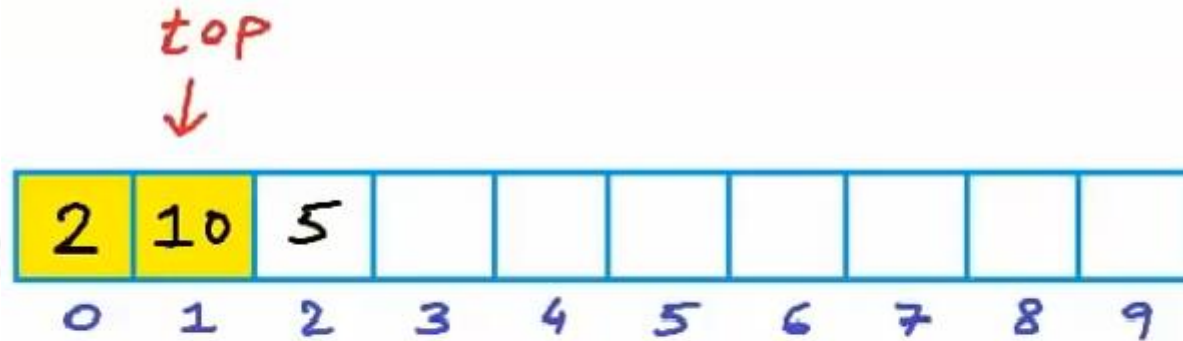
Stack Implementation Using Array (8/19)

- Now, we will write a **pop** function, to pop an element from the stack.
- All we need to do is decrementing the top by One.

```
POP()  
{  
    top ← top - 1  
}
```

Stack Implementation Using Array (9/19)

- Let us make a call to the **pop** function.



Push(2)

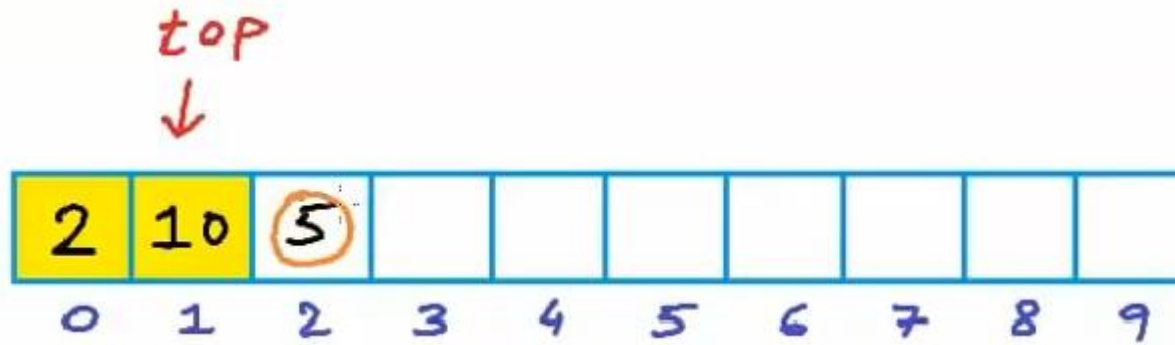
Push(10)

Push(5)

Pop()

Stack Implementation Using Array (10/19)

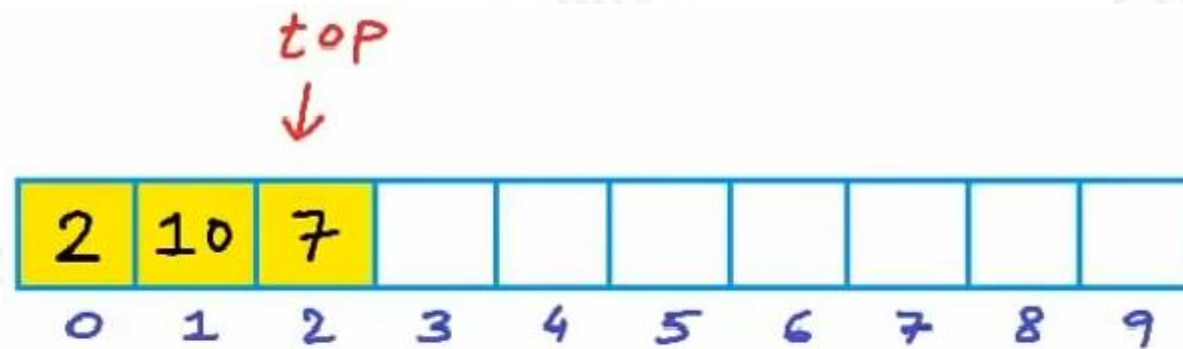
- The cells in Yellow in the figures are part of the stack.
- We don't need to reset this cell before or after popping.



- If a cell is not part of the stack anymore, we don't care what garbage lies there.
 - Next time when we will push, we will modify it anyway.

Stack Implementation Using Array (11/19)

- After the pop operation, let us perform a push operation again.



Push(2)

Push(10)

Push(5)

Pop()

Push(7)

Stack Implementation Using Array (12/19)

- The Two function that we have written (push and pop), will take a constant time.
 - We have simple operations in these two functions.
 - The execution time will not depend on the size of the stack.
- When we defined a Stack ADT, we had said that all the operations must take constant time.
 - The time complexity should be $O(1)$.

Stack Implementation Using Array (13/19)

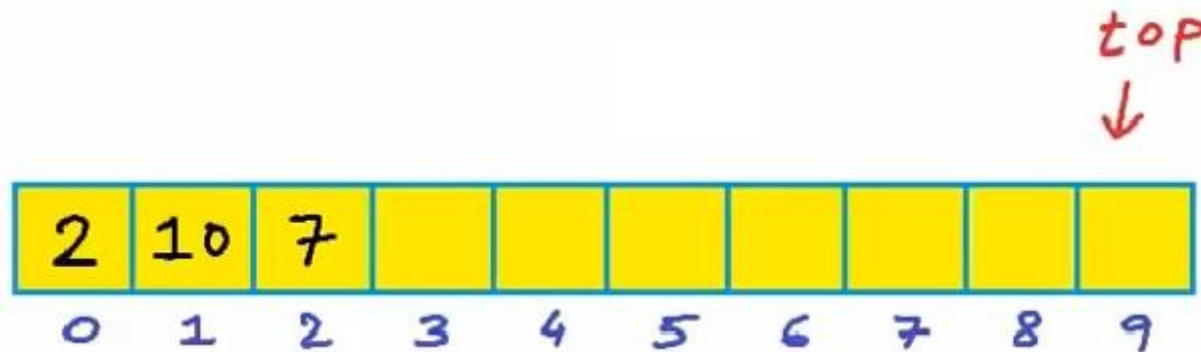
```

Push(x)
{
    top ← top + 1
    A[top] ← x
}
Pop()
{
    top ← top - 1
}
    
```

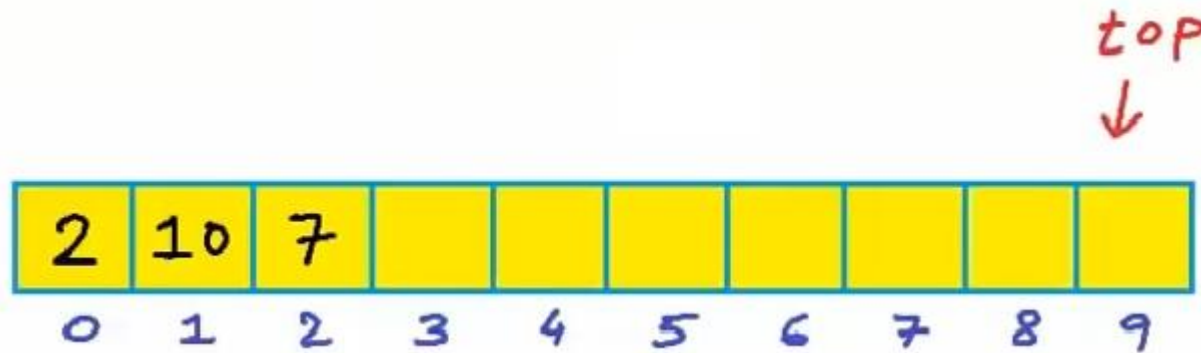
→ Constant time
 or
 → $O(1)$

Stack Implementation Using Array (14/19)

- One important thing here, we can push onto the stack only till array is not exhausted (all elements are used).
- We can have a situation where stack would consume the whole array.
 - Top will equal the highest index of the array.



Stack Implementation Using Array (15/19)



- A further push will not be possible because it will result an overflow.
- This is one limitation with array-based implementation.
- We must handle overflow in our implementation.
 - In push function, check whether array is exhausted or not.
 - A) Throw an error in case of an overflow, push operation will not succeed.
 - B) We can use the concept of dynamic array.

Stack Implementation Using Array (16/19)

- If we used the dynamic array, the time complexity of push with this strategy:

Overflow
 ↳ create a larger array. copy all elements in new array.
 twice the
 ↓ smaller
 Cost - $O(n)$
 where n = no. of elements in stack

Stack Implementation Using Array (17/19)

- The time complexity of push operation with this strategy:

Push - $O(1)$ - best case
 $O(n)$ - worst case

Stack Implementation Using Array (18/19)

- We still have two more operations in the definition of stack ADT:
 - Top.
 - IsEmpty.
- Top will simply return the element at the top:

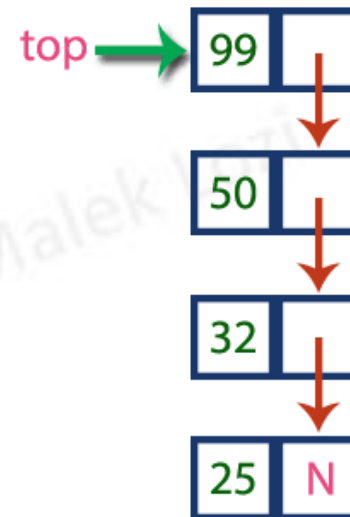
```
Top()
{
    return A[top]
}
```

Stack Implementation Using Array (19/19)

- To verify the stack IsEmpty or not:

```
IsEmpty()  
{  
    if (top == -1)  
        return true  
    else  
        return false  
}
```

Stack Implementation Using Linked List



Stack Implementation Using Linked List (1/7)

► Remember:

Stack ADT (LIFO)

A list with the restriction that insertion and deletion can be performed only from one end, called the top.

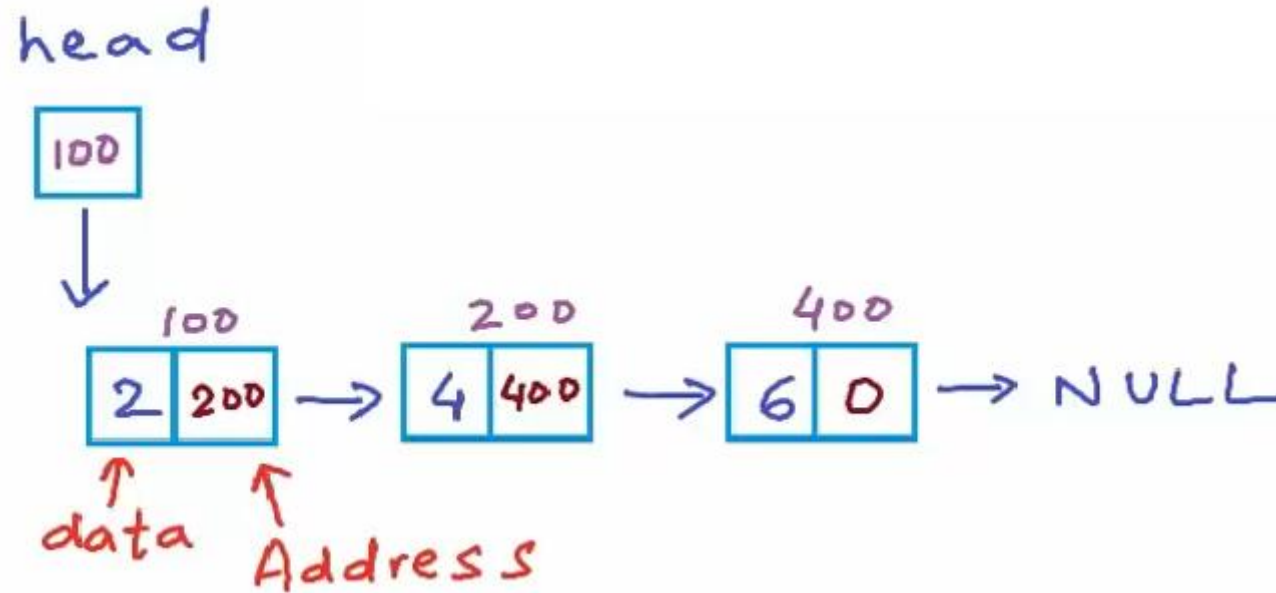
Operations

- (1) Push(x)
- (2) Pop()
- (3) Top()
- (4) IsEmpty()

Constant
time
or
 $O(1)$

Stack Implementation Using Linked List (2/7)

- We have a linked list of integers here:



Stack Implementation Using Linked List (3/7)

- Unlike arrays, linked lists are not fixed size and elements are not stored in a continuous block of memory.
- To use linked list for a stack, we want the insertion and deletion must always happen from the same end.
- We have Two options:

Insert/delete

- at end (tail)
- at beginning (head)

Stack Implementation Using Linked List (4/7)

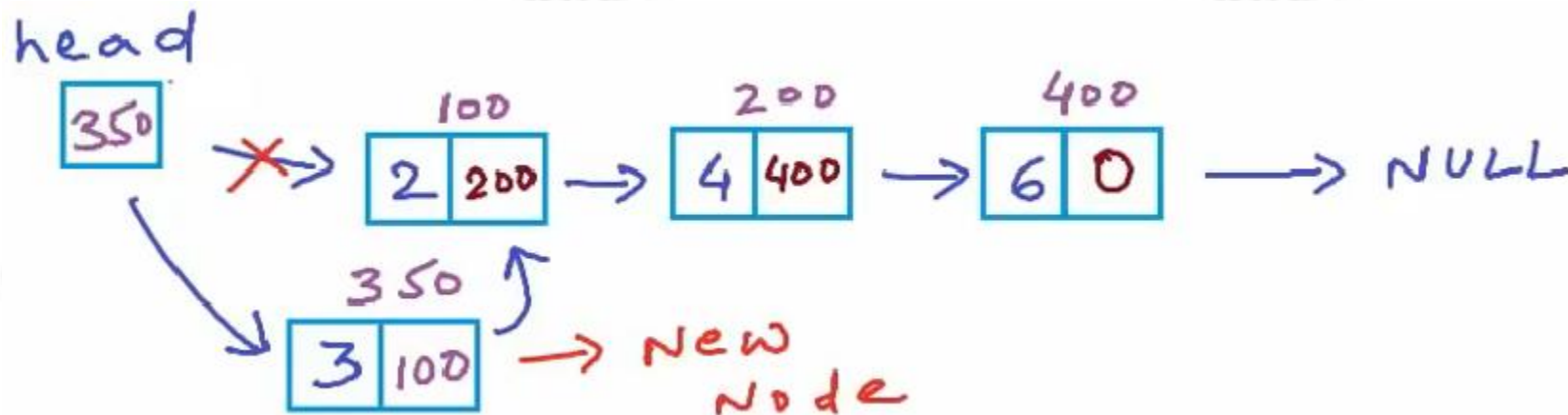
- The cost of insertion at the end of a linked list is $O(n)$.
- It is not an option for us because we will not be able to do push and pop in constant time.

Insert/delete

- at end of list (tail) $\swarrow O(n)$ X
- at beginning (head) $\leftarrow O(1)$

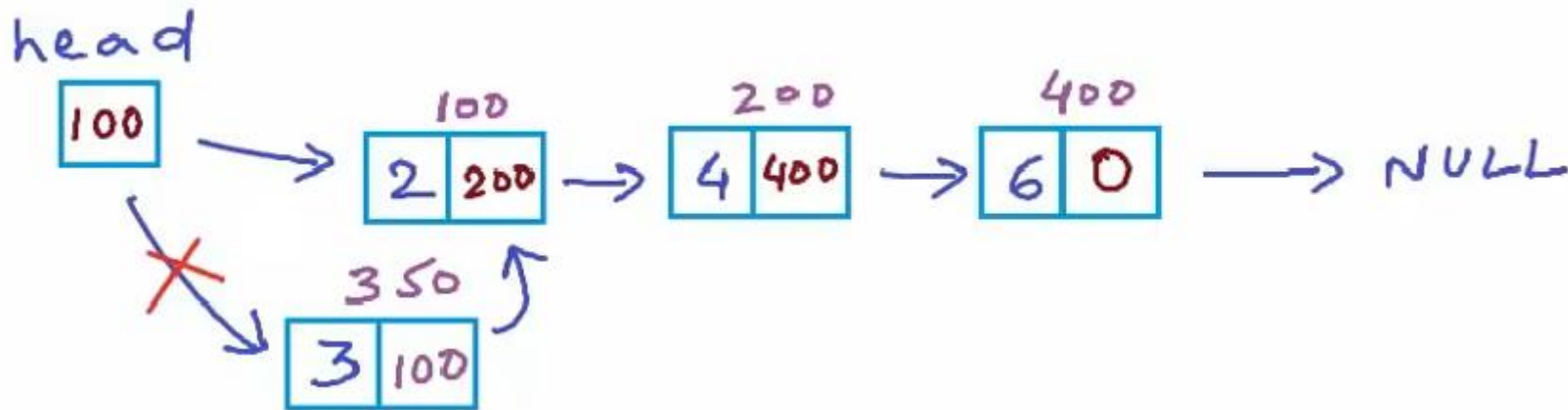
Stack Implementation Using Linked List (5/7)

- To insert a node at the beginning:
 - Create a new node.
 - Build two links.
 - From the new node to the first node.
 - From head to the new node.



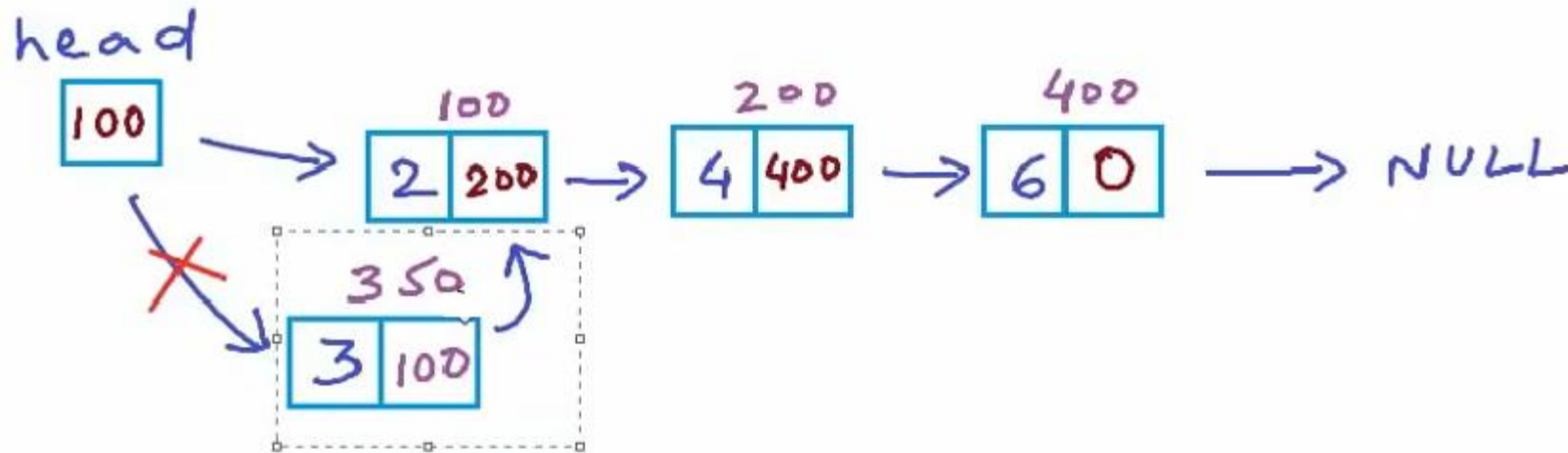
Stack Implementation Using Linked List (6/7)

- To delete a node from the beginning:
 - Cut the link from head to the first node.
 - Build a new link from head to the second Node.
- This means resetting the reference in the head.



Stack Implementation Using Linked List (7/7)

- Do we need to free the memory allocated to the deleted node?



Any Questions???