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Final Examination
METU CENG310 Spring 2021
Data Structures and Algorithms with Python

Exam Date: July 9th, 2021
6:00pm - 8:00pm
(120 Minutes)

A Who are you? (2 pts.)

1. What is your full name?
2. What is your student ID?

B True/False Questions (8 pts.)

3. The depth of node u of a general tree data structure is the number of proper ancestors of u. (True/False)
4. An AVL tree is also a binary search tree. (True/False)
5. In a (2,4) Tree, every node can have at most 4 children. (True/False)
6. In a naive implementation of a priority queue with a sorted list, all insertion or deletion would be performed in $O(n)$ time. (True/False)
7. *Open addressing (probing)* is a collision handling approach adopted in hash tables. (True/False)
8. Merge sort is a comparison-based sorting algorithm. (True/False)
9. Bucket sort is not a stable sorting algorithm. (True/False)
10. A connected graph has a path between each pair of its distinct vertices. (True/False)

C Fill in the Blanks Questions (6 pts.)

Please find a suitable word, expression, word, number, etc. to the blanks (denoted as five consecutive underscores _____) in the following questions. Please note that there might be more than one correct answer, any one of them will be accepted, so do not try to enumerate all possible correct answers.

11. A binary tree is said to be _____ if its each node has either zero or two children.
12. In the worst-case scenario, the *search* operation in a binary search tree whose height is h is $O($_____ $)$.
13. The heap sort algorithm sorts a sequence of n items in $O($_____$)$ time.
14. *Expected* complexity of the *get_item* operation for a hash table is $O(1)$, however, the worst-case complexity is $O(n)$.
15. Two sorted sequences S_1 and S_2, whose total number of items is n, can be merged into a single sorted sequence in $O($_____$)$ time.
16. In the context of graph data structures, a _____ is a path that begins and ends at the same vertex.

D Definitions (6 pts.)

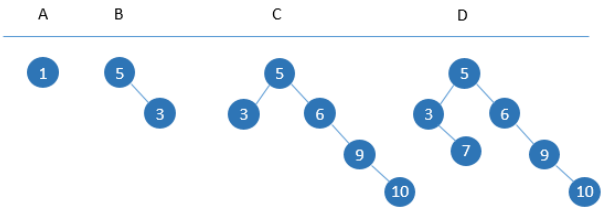
Please provide *formal* definitions for the following terms.

17. What is *complete binary tree property* in the context of heap data structures?
18. In the context of sorting algorithms, what does being *stable* mean?

E Simple Answers (10 pts.)

Answers to these questions are expected to be very brief such as one or two words, or at most one sentence.

19. Which of the trees below represent a binary search tree? Zero, one or multiple of them could be binary search trees. Children are positioned such that ones having keys smaller than the parent should be in the left subtree whilst one with larger keys should be in the right subtree.



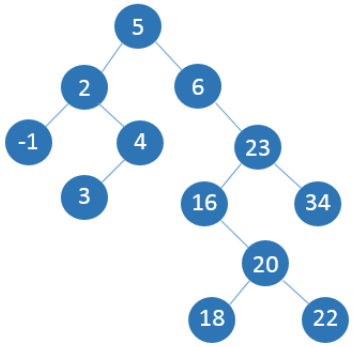
20. List three ADT operations of priority queues except for *is_empty* and *len*.
21. Give an example (only the name) of a non-comparison sorting algorithm.

F Trees (10 pts.)

22. Let T be a general tree, p be an arbitrary node of T, and the $T.children(p)$ be an ADT operation listing children nodes of p. Write a pseudo code, $preorder(T, p)$, that performs a pre-order traversal on tree T. During the traversal, each node should be marked as visited with the $visit(p)$ operation.

G Search Trees (18 pts.)

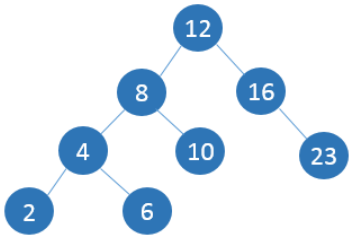
23. (5 pts.) Considering the binary search tree below, how would the insertion of a node whose key is 17 be performed? Please (a) list all the nodes that would be visited during the search operation, and (b) clearly state where exactly the insertion would be made ("as the left/right child of node X").



24. (8 pts.) Answer the following questions pertaining to the AVL tree shown below.

    (a) Is it balanced?

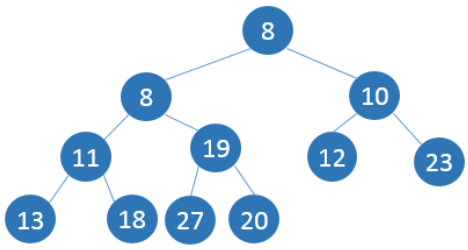
    (b) Insert a new node with key 5 to this AVL tree. Please explain the operation steps. After initial insertion, would the tree become unbalanced? If so, which *case* would the tree be considered in? Which node would be the unbalanced node (the pivot)? Which rotation operation(s) would be performed?



25. (5 pts.) To an initially empty (2,4) tree, show step by step insertion operation of the nodes with the following keys: 5, 10, 15, 20, 16. Please show the state of the tree after each insert operation.

H Heaps (10 pts.)

26. To the heap data structure shown below, insert a new item with key 5. Please provide a step by step explanation for the up-heap bubbling operation.



I Hash Tables (8 pts.)

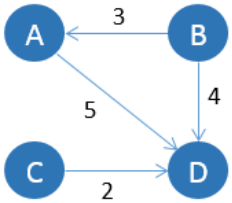
27. Let H be a hash table that handles collisions with quadratic probing. Let the size of the hash table, N be 11. By making use of a hash function $h(x) = xmodN$ and assuming that H is initially empty, insert items with keys 1, 23, 45, 34, 12 to H. Show the state of the hash table after each insertion operation.

J Sorting (17 pts.)

28. (10 pts.) Provide a pseudo code for the merge sort algorithm. You can assume that there exists a function with the signature *merge(S1, S2, S)* and merges two sorted lists (*S1* and *S2*) into a single sorted list (*S*).
29. (7 pts.) Let us assume that we are given a sequence of 1073741824 numbers each of which is known to be in range [0,1023]. If we were to sort the numbers in this sequence, assuming that we have unlimited computing memory, which algorithm would be a better choice: Merge sort or bucket sort? Why? Justify your answer by showing some solid facts. ($1073741824 = 2^{30}$, $1024 = 2^{10}$)

K Graphs (5 pts.)

30. Form the adjacency matrix for the graph below.



Good luck!