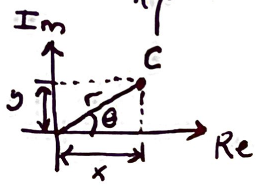
BLOCH Sphere

A complex number $C = x + yi$ can also be expressed in polar coordinates:



$x = r\cos\theta$
$y = r\sin\theta$

$$C = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$$\left(\bar{C} = re^{-i\theta} \rightarrow |c|^2 = c \cdot \bar{c}\right)$$

{ if we restrict $|c| = 1$ then every complex number of length 1 can be represented by the angle it makes with the x-axis (i.e. θ)

A qubit has the form

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle \quad \text{where} \quad |c_0|^2 + |c_1|^2 = 1$$

<u>Claim</u>: $c_0 = e^{i\gamma}\cos\frac{\theta}{2}$ and $c_1 = e^{i(\gamma+\varphi)}\sin\frac{\theta}{2}$ $\left(\begin{matrix} 0 \le \theta \le \pi \\ 0 \le \varphi \le 2\pi \end{matrix}\right)$

$$|c_0|^2 = e^{i\gamma}\cos\frac{\theta}{2}\, e^{-i\gamma}\cos\frac{\theta}{2} = \cos^2\frac{\theta}{2}$$

$$|c_1|^2 = e^{i(\gamma+\varphi)}\sin\frac{\theta}{2}\, e^{-i(\gamma+\varphi)}\sin\frac{\theta}{2} = \sin^2\frac{\theta}{2}$$

$$\rightarrow |c_0|^2 + |c_1|^2 = 1 \checkmark$$

Hence, we can also express

$$|\psi\rangle = e^{i\gamma}\cos\frac{\theta}{2}|0\rangle + e^{i(\gamma+\varphi)}\sin\frac{\theta}{2}|1\rangle$$

$$|\psi\rangle = e^{i\gamma}\left[\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right]$$

since this is invariant under measurement*

$$|\psi\rangle \cong \left[\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right]$$

* $e^{i\gamma}|\psi\rangle \longrightarrow$ using the measurement matrix M, and making a measurement

$$\langle\psi|\underbrace{e^{-i\gamma}}_{}M^{+}M\underbrace{e^{i\gamma}}_{\text{scalars}}|\psi\rangle = \langle\psi|M^{+}M|\psi\rangle$$