In [42]:
```
# import necessary libraries
import yfinance as yf
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from arch import arch_model
from sklearn.metrics import mean_absolute_error, mean_squared_error
from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.stats.diagnostic import acorr_ljungbox
from statsmodels.tsa.stattools import adfuller
```

In [ ]:
```
# Data retrieval
ticker_symbol = '^GSPC'  # S&P 500 Index
start_date = '2016-01-01'
end_date = '2025-05-31' # Or current date if you want the most recent dat

data = yf.download(ticker_symbol, start=start_date, end=end_date)

# Preview the data
print(data.head())
```

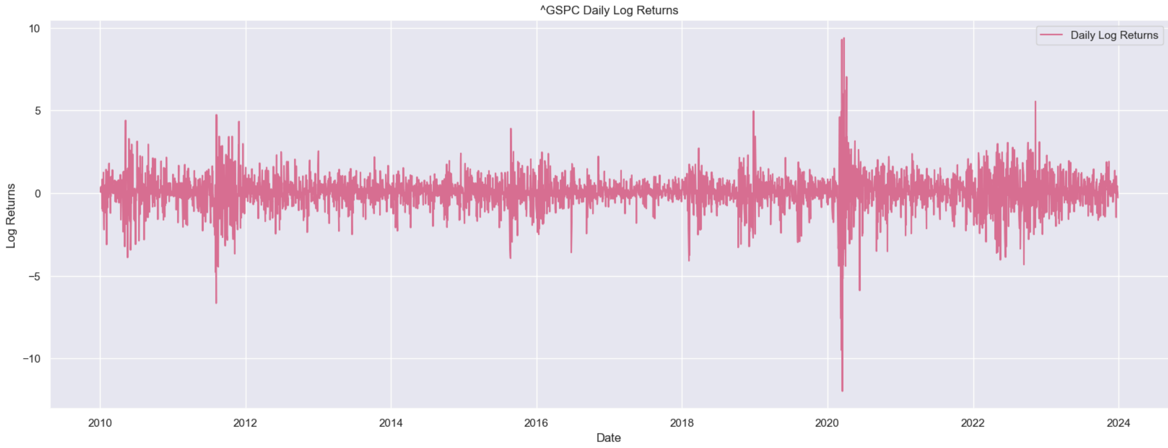
```
/var/folders/xl/175qw0912j98h8914p56tcd00000gn/T/ipykernel_1351/344994787
6.py:7: FutureWarning: YF.download() has changed argument auto_adjust defa
ult to True
  data = yf.download(ticker_symbol, start=start_date, end=end_date)
[*********************100%***********************]  1 of 1 completed
Price             Close         High          Low         Open       Volume
Ticker            ^GSPC        ^GSPC        ^GSPC        ^GSPC        ^GSPC
Date
2016-01-04  2012.660034  2038.199951  1989.680054  2038.199951   4304880000
2016-01-05  2016.709961  2021.939941  2004.170044  2013.780029   3706620000
2016-01-06  1990.260010  2011.709961  1979.050049  2011.709961   4336660000
2016-01-07  1943.089966  1985.319946  1938.829956  1985.319946   5076590000
2016-01-08  1922.030029  1960.400024  1918.459961  1945.969971   4664940000
```

In [40]:
```
# Calculate daily logarithmic returns

# Choose the correct price column
price_col = 'Adj Close' if 'Adj Close' in data.columns else 'Close'

returns = np.log(data[price_col] / data[price_col].shift(1)).dropna()

# Display the first few returns
print("\n First few returns:")
print(returns.head())
```

```
 First few returns:
Ticker         ^GSPC
Date
2016-01-05  0.002010
2016-01-06 -0.013202
2016-01-07 -0.023986
2016-01-08 -0.010898
2016-01-11  0.000853
```

In [39]:
```
# Plot the returns to observe volatility clustering
plt.figure(figsize=(20, 7))
plt.plot(returns.index, returns,color = "palevioletred", label='Daily Log
```

```
plt.title(f'{ticker_symbol} Daily Log Returns')
plt.xlabel('Date')
plt.ylabel('Log Returns')
plt.grid(True)
plt.legend()
plt.show()
```



In [43]:
```
# Optional: Check for stationarity (e.g., using Augmented Dickey-Fuller t

adf_result = adfuller(returns)
print(f'ADF Statistic: {adf_result}')
print(f'P-value: {adf_result[5]}')
# (A low p-value indicates stationarity, which is generally expected for
```

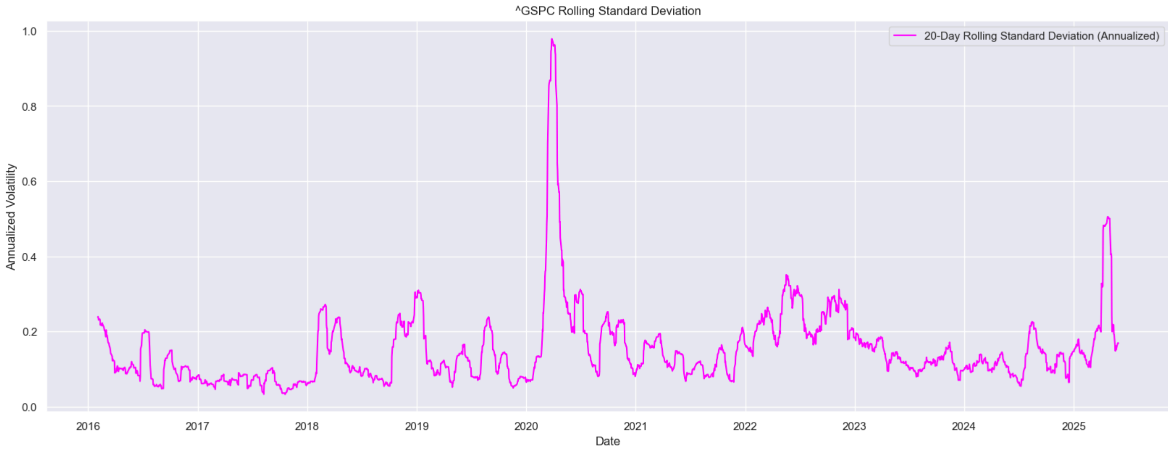
```
ADF Statistic: (np.float64(-15.501836862462824), np.float64(2.410073622861
824e-28), 8, 2356, {'1%': np.float64(-3.433128624404979), '5%': np.float64
(-2.8627675485259805), '10%': np.float64(-2.5674234771965376)}, np.float64
(-14327.675405444457))
P-value: -14327.675405444457
```

In [45]:
```
# Define a rolling window size (e.g., 20 days for approximate monthly vol
window_size = 20

# Calculate rolling standard deviation and annualize it
# Annualization factor is sqrt(252) for daily returns
rolling_std = returns.rolling(window=window_size).std() * np.sqrt(252)

# Display the first few rolling standard deviations
print("\nFirst few rolling standard deviations (annualized):")
print(rolling_std.head())
```

```
First few rolling standard deviations (annualized):
Ticker      ^GSPC
Date
2016-01-05    NaN
2016-01-06    NaN
2016-01-07    NaN
2016-01-08    NaN
2016-01-11    NaN
```

In [46]:
```
# Plot rolling standard deviation
plt.figure(figsize=(20, 7))
plt.plot(rolling_std.index, rolling_std, label=f'{window_size}-Day Rollin
plt.title(f'{ticker_symbol} Rolling Standard Deviation')
plt.xlabel('Date')
plt.ylabel('Annualized Volatility')
plt.grid(True)
```

```
plt.legend()
plt.show()
```

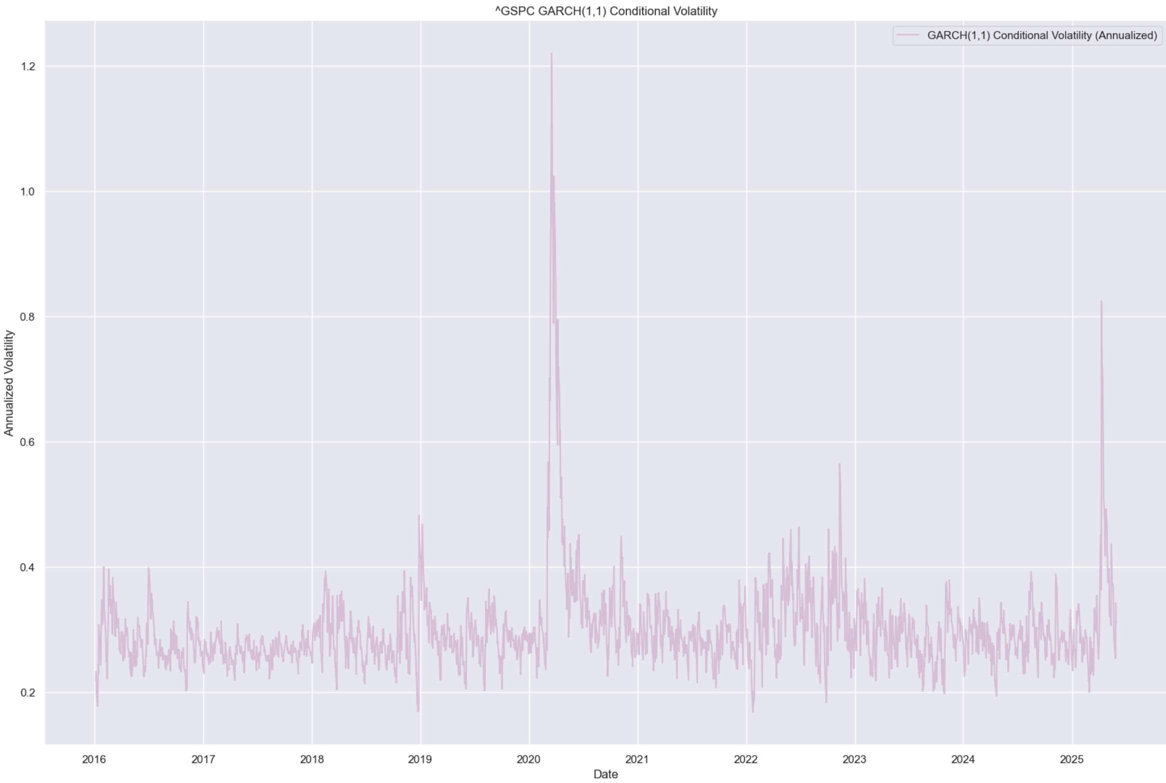


In [52]:

```
'''
The mean return of the S&P 500 is statistically significant but very clos
The volatility of the S&P 500 returns is not constant but exhibits volati
volatility. This is evidenced by the highly significant alpha[1] and beta
Both past unexpected market movements (alpha[1]) and past volatility (bet
The sum of alpha[1] and beta[1] being 0.98 indicates very persistent vola
'''

# Specify the GARCH(1,1) model
# mean='Constant' assumes a constant mean for returns
# vol='Garch' specifies the GARCH volatility model
# p=1, q=1 for GARCH(1,1)
# dist='normal' for Gaussian errors, 'skewt' or 't' for fat tails (more r
garch_model = arch_model(returns, mean='Constant', vol='Garch', p=1, q=1,

# Fit the model
# update_freq=10 prints progress every 10 iterations
garch_results = garch_model.fit(disp='off') # disp='off' to suppress iter

# Print the model summary
print("\nGARCH(1,1) Model Summary:")
print(garch_results.summary())

# Extract conditional volatility from the fitted model
garch_conditional_volatility = garch_results.conditional_volatility * np.

# Plot conditional volatility
plt.figure(figsize=(20, 13))
plt.plot(garch_conditional_volatility.index, garch_conditional_volatility
plt.title(f'{ticker_symbol} GARCH(1,1) Conditional Volatility')
plt.xlabel('Date')
plt.ylabel('Annualized Volatility')
plt.grid(True)
plt.legend()
plt.show()
```

```
GARCH(1,1) Model Summary:
                     Constant Mean - GARCH Model Results
==========================================================================
====
Dep. Variable:                  ^GSPC   R-squared:
0.000
Mean Model:             Constant Mean   Adj. R-squared:
0.000
Vol Model:                      GARCH   Log-Likelihood:                587
2.56
Distribution:                  Normal   AIC:                          -117
37.1
Method:            Maximum Likelihood   BIC:                          -117
14.0
                                        No. Observations:
2365
Date:                Tue, Jun 24 2025   Df Residuals:
2364
Time:                        15:46:43   Df Model:
1
                                  Mean Model
==========================================================================
====
                 coef    std err          t      P>|t|       95.0% Conf.
Int.
--------------------------------------------------------------------------
----
mu            -0.0160  4.368e-04    -36.599 2.986e-293 [-1.684e-02,-1.513e
-02]
                              Volatility Model
==========================================================================
==
                 coef    std err          t      P>|t|     95.0% Conf. In
t.
--------------------------------------------------------------------------
--
omega      2.7946e-06  2.946e-07      9.487  2.368e-21 [2.217e-06,3.372e-0
6]
alpha[1]       0.2001  6.132e-02      3.263  1.103e-03   [7.989e-02,  0.32
0]
beta[1]        0.7799  7.020e-02     11.110  1.126e-28     [  0.642,  0.91
8]
==========================================================================
==

Covariance estimator: robust
```

```
/opt/anaconda3/envs/yaman/lib/python3.13/site-packages/arch/univariate/bas
e.py:309: DataScaleWarning: y is poorly scaled, which may affect convergen
ce of the optimizer when
estimating the model parameters. The scale of y is 0.0001364. Parameter
estimation work better when this value is between 1 and 1000. The recommen
ded
rescaling is 100 * y.

This warning can be disabled by either rescaling y before initializing the
model or by setting rescale=False.

  warnings.warn(
```



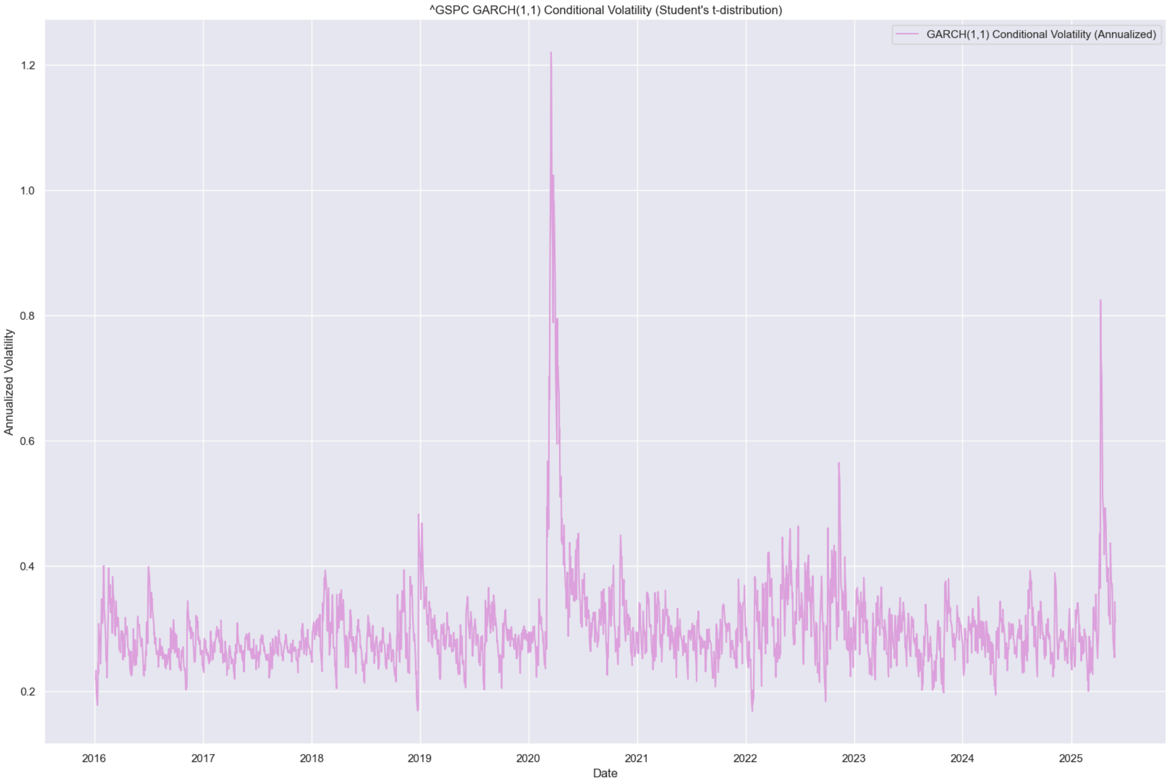
In [50]:

```python
import numpy as np
import pandas as pd
from arch import arch_model
import matplotlib.pyplot as plt

# Assuming 'returns' and 'ticker_symbol' are already defined
# For demonstration purposes, let's create some dummy data:
# np.random.seed(42)
# dummy_returns = np.random.normal(loc=0.0005, scale=0.01, size=1000)
# returns = pd.Series(dummy_returns, index=pd.date_range(start='2018-01-0
# ticker_symbol = "DUMMY_STOCK"

# Specify the GARCH(1,1) model with 't' distribution
# mean='Constant' assumes a constant mean for returns
# vol='Garch' specifies the GARCH volatility model
# p=1, q=1 for GARCH(1,1)
# dist='t' for Student's t-distribution, which accounts for fat tails
garch_model = arch_model(returns, mean='Constant', vol='Garch', p=1, q=1,

# Fit the model
# disp='off' to suppress iteration output
garch_results = garch_model.fit(disp='off')

# Print the model summary
print("\nGARCH(1,1) Model Summary (with Student's t-distribution):")
print(garch_results.summary())

# Extract conditional volatility from the fitted model
garch_conditional_volatility = garch_results.conditional_volatility * np.

# Plot conditional volatility
plt.figure(figsize=(20, 13))
plt.plot(garch_conditional_volatility.index, garch_conditional_volatility
plt.title(f'{ticker_symbol} GARCH(1,1) Conditional Volatility (Student\'s
plt.xlabel('Date')
plt.ylabel('Annualized Volatility')
```

```
plt.grid(True)
plt.legend()
plt.show()
```

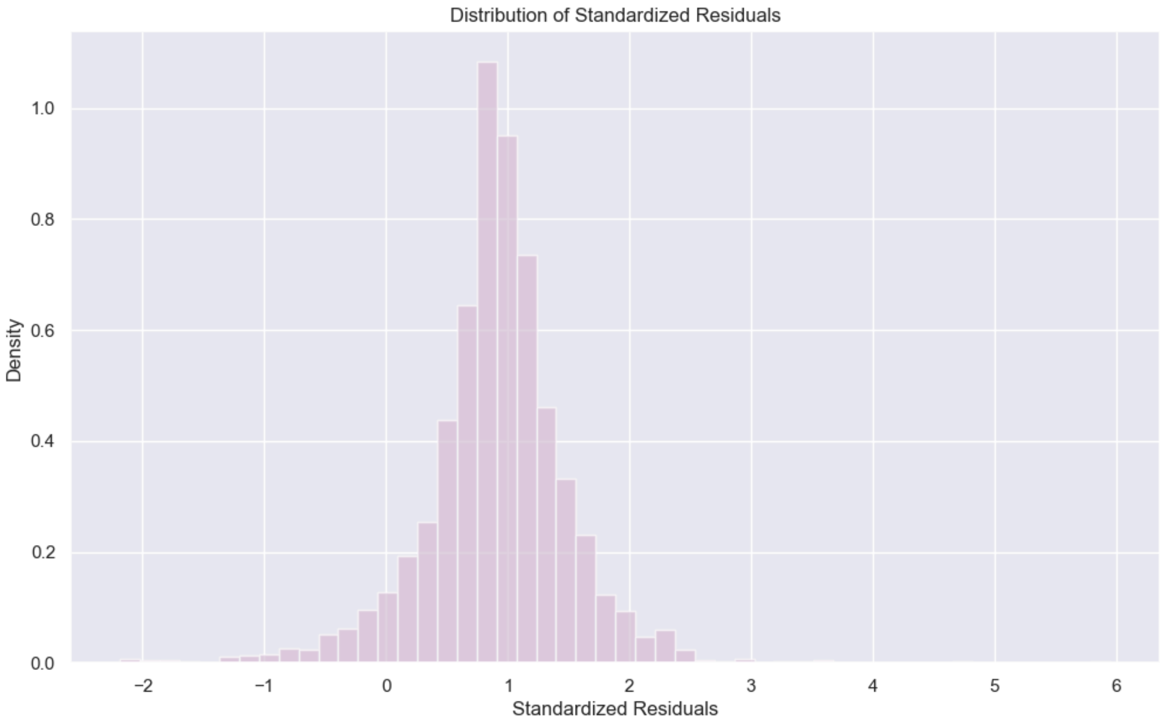
```
GARCH(1,1) Model Summary (with Student's t-distribution):
                     Constant Mean - GARCH Model Results
==========================================================================
====
Dep. Variable:                  ^GSPC   R-squared:
0.000
Mean Model:             Constant Mean   Adj. R-squared:
0.000
Vol Model:                      GARCH   Log-Likelihood:                587
2.56
Distribution:                  Normal   AIC:                          -117
37.1
Method:            Maximum Likelihood   BIC:                          -117
14.0
                                        No. Observations:
2365
Date:                Tue, Jun 24 2025   Df Residuals:
2364
Time:                        15:46:16   Df Model:
1
                                 Mean Model
==========================================================================
====
                 coef    std err          t      P>|t|       95.0% Conf.
Int.
--------------------------------------------------------------------------
----
mu            -0.0160  4.368e-04    -36.599 2.986e-293 [-1.684e-02,-1.513e
-02]
                             Volatility Model
==========================================================================
==
                 coef    std err          t      P>|t|     95.0% Conf. In
t.
--------------------------------------------------------------------------
--
omega      2.7946e-06  2.946e-07      9.487  2.368e-21 [2.217e-06,3.372e-0
6]
alpha[1]       0.2001  6.132e-02      3.263  1.103e-03   [7.989e-02,  0.32
0]
beta[1]        0.7799  7.020e-02     11.110  1.126e-28     [  0.642,  0.91
8]
==========================================================================
==

Covariance estimator: robust
```

```
/opt/anaconda3/envs/yaman/lib/python3.13/site-packages/arch/univariate/bas
e.py:309: DataScaleWarning: y is poorly scaled, which may affect convergen
ce of the optimizer when
estimating the model parameters. The scale of y is 0.0001364. Parameter
estimation work better when this value is between 1 and 1000. The recommen
ded
rescaling is 100 * y.

This warning can be disabled by either rescaling y before initializing the
model or by setting rescale=False.

  warnings.warn(
```



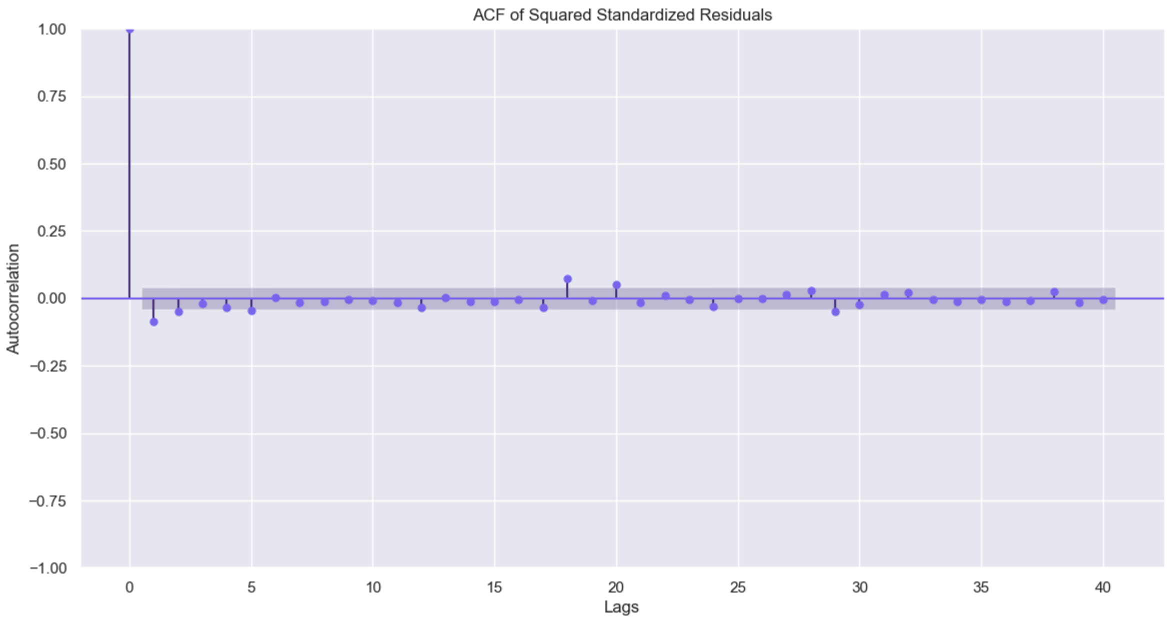
In [53]:
```python
# Get standardized residuals
standardized_residuals = garch_results.resid / garch_results.conditional_

# Plot distribution of standardized residuals
plt.figure(figsize=(12, 7))
plt.hist(standardized_residuals, bins=50, density=True, alpha=0.7, color=
plt.title('Distribution of Standardized Residuals')
plt.xlabel('Standardized Residuals')
plt.ylabel('Density')
plt.grid(True)
plt.show()

# Plot ACF of standardized residuals
plt.figure(figsize=(20, 10))
plot_acf(standardized_residuals, lags=40, alpha=0.05 , color = 'mediumpur
plt.title('ACF of Standardized Residuals')
plt.xlabel('Lags')
plt.ylabel('Autocorrelation')
plt.show()

# Plot ACF of squared standardized residuals (to check for remaining ARCH
plt.figure(figsize=(20, 10))
plot_acf(standardized_residuals**2, lags=40, alpha=0.05, color = 'mediums
```

```
plt.title('ACF of Squared Standardized Residuals')
plt.xlabel('Lags')
plt.ylabel('Autocorrelation')
plt.show()
```




```
<Figure size 2000x1000 with 0 Axes>
```


```
<Figure size 2000x1000 with 0 Axes>
```



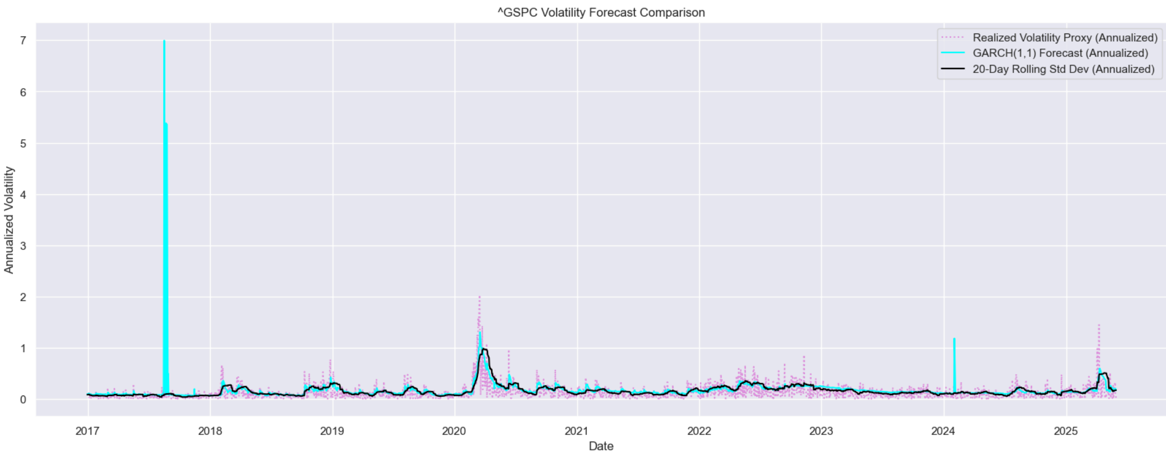
In [54]:
```
# Ljung-Box test on standardized residuals
# H0: The data is independently distributed (white noise)
# If p-value < 0.05, reject H0, indicating remaining autocorrelation

lb_test_residuals = acorr_ljungbox(standardized_residuals, lags=[1, 13],
print("\nLjung-Box Test on Standardized Residuals:")
print(lb_test_residuals)

# Ljung-Box test on squared standardized residuals
lb_test_squared_residuals = acorr_ljungbox(standardized_residuals**2, lag
print("\nLjung-Box Test on Squared Standardized Residuals:")
print(lb_test_squared_residuals)

# Interpretation: the given p-values are high (e.g., > 0.05), it suggests
```

```
Ljung-Box Test on Standardized Residuals:
      lb_stat     lb_pvalue
1   20.602436  5.652415e-06
13  69.076151  1.186958e-09

Ljung-Box Test on Squared Standardized Residuals:
      lb_stat  lb_pvalue
1   17.278624   0.000032
13  36.127976   0.000567
```

In [30]:
```
# Define the size of the training window for rolling forecasts
# A common choice is 250 days (approx. 1 year of trading days)
train_window_size = 250

# Initialize lists to store forecasts and realized volatility
garch_forecasts = []
realized_volatility_proxy = []
# Using squared returns as a simple proxy for daily realized volatility

# Loop through the returns data to perform rolling forecasts
# Start forecasting after the initial training window
for i in range(train_window_size, len(returns)):
    # Define the training data window
    train_data = returns.iloc[i - train_window_size:i]

    # Fit GARCH(1,1) model to the training data
```

```
    # disp='off' to suppress output for each fit
    model = arch_model(train_data, mean='Constant', vol='Garch', p=1, q=1
    res = model.fit(disp='off')

    # Make a one-step-ahead forecast
    # forecast() returns a DataFrame with 'h.1' column for 1-step ahead v
    forecast = res.forecast(horizon=1)
    garch_next_period_variance = forecast.variance.iloc[-1, 0] # Get the

    # Store the forecasted volatility (square root of variance)
    garch_forecasts.append(np.sqrt(garch_next_period_variance) * np.sqrt(

    # Store the actual realized volatility for the next period (squared r
    # For simplicity, we use the squared return of the next day as a prox
    # In practice, realized volatility is often calculated from intraday
    realized_volatility_proxy.append(np.abs(returns.iloc[i]) * np.sqrt(25

# Create a DataFrame for forecasts and realized volatility
forecast_index = returns.index[train_window_size:]
garch_forecast_series = pd.Series(garch_forecasts, index=forecast_index)
realized_vol_series = pd.Series(realized_volatility_proxy, index=forecast

print("\nRolling GARCH Forecasts (first 5):")
print(garch_forecast_series.head())
print("\nRealized Volatility Proxy (first 5):")
print(realized_vol_series.head())
```

```
# Check if the aligned series are empty before proceeding
if realized_vol_series_aligned.empty:
    print("Error: No common data points for comparison after alignment an
else:
    # Plotting Forecasted vs. Realized Volatility
    plt.figure(figsize=(20, 7))
    plt.plot(realized_vol_series_aligned.index, realized_vol_series_align
    plt.plot(garch_forecast_series_aligned.index, garch_forecast_series_a
    plt.plot(rolling_std_aligned.index, rolling_std_aligned, label=f'{win
    plt.title(f'{ticker_symbol} Volatility Forecast Comparison')
    plt.xlabel('Date')
    plt.ylabel('Annualized Volatility')
    plt.legend()
    plt.grid(True)
    plt.show()

    # Statistical Metrics for GARCH(1,1)
    garch_mae = mean_absolute_error(realized_vol_series_aligned, garch_fo
    garch_mse = mean_squared_error(realized_vol_series_aligned, garch_for
    garch_rmse = np.sqrt(garch_mse)
    garch_correlation = garch_forecast_series_aligned.corr(realized_vol_s

    print("\n--- GARCH(1,1) Forecast Performance ---")
    print(f"Mean Absolute Error (MAE): {garch_mae:.6f}")
    print(f"Mean Squared Error (MSE): {garch_mse:.6f}")
    print(f"Root Mean Squared Error (RMSE): {garch_rmse:.6f}")
    print(f"Correlation (Forecast vs. Realized): {garch_correlation:.4f}"

    # Statistical Metrics for Rolling Standard Deviation
    rolling_std_mae = mean_absolute_error(realized_vol_series_aligned, ro
    rolling_std_mse = mean_squared_error(realized_vol_series_aligned, rol
    rolling_std_rmse = np.sqrt(rolling_std_mse)
    rolling_std_correlation = rolling_std_aligned.corr(realized_vol_serie

    print("\n--- Rolling Standard Deviation Forecast Performance ---")
    print(f"Mean Absolute Error (MAE): {rolling_std_mae:.6f}")
    print(f"Mean Squared Error (MSE): {rolling_std_mse:.6f}")
    print(f"Root Mean Squared Error (RMSE): {rolling_std_rmse:.6f}")
    print(f"Correlation (Forecast vs. Realized): {rolling_std_correlation

    # Summary Comparison Table
    print("\n--- Comparative Performance Summary ---")
    print("| Model | MAE | MSE | RMSE | Correlation |")
    print("|-------------------------|----------|----------|----------|-
    print(f"| GARCH(1,1) | {garch_mae:.6f} | {garch_mse:.6f} | {garch_rms
    print(f"| Rolling Standard Deviation | {rolling_std_mae:.6f} | {rolli
```



```
--- GARCH(1,1) Forecast Performance ---
Mean Absolute Error (MAE): 0.103000
Mean Squared Error (MSE): 0.069568
Root Mean Squared Error (RMSE): 0.263758
Correlation (Forecast vs. Realized): 0.2305

--- Rolling Standard Deviation Forecast Performance ---
Mean Absolute Error (MAE): 0.089855
Mean Squared Error (MSE): 0.017252
Root Mean Squared Error (RMSE): 0.131346
Correlation (Forecast vs. Realized): 0.5470

--- Comparative Performance Summary ---
```

Model	MAE	MSE	RMSE	Correlation
GARCH(1,1)	0.103000	0.069568	0.263758	0.2305
Rolling Standard Deviation	0.089855	0.017252	0.131346	0.5470