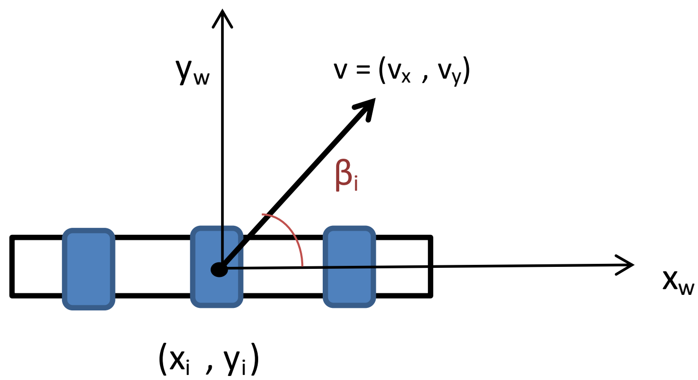
Finding the kinematic equations for the three-omniwheel robot, meaning that we have to find a relation between the rotation speed of each wheel, and the speed for the center of gravity of the robot.



where w:{x_w,y_w} is the wheel frame, fixed to the center of the wheel "I". Every wheel have a static coordinates, depends on the shape of the robot's chassis, "v" represent the velocity of the robot written in frame W, and β_i is the angle between velocity vector and x_w .

Omnidirectional wheel have two types of movement –ignoring slipping- movement on the X_w axis v_{drive}, driven by the wheel's motor, and a movement on the Y_w axis caused by the free sliding rollers v_{slide}.

In this type of omnidirectional wheels, movement on Y_w (v_y) is isolated from the main wheel movement which is caused by motor "ω_i", in another way, v_y can take value from free sliding movement only .

"in omnidirectional wheel, the angle between free sliding vector and y axis " Υ_i "is always 0"

So now we can write :

$$u_i = \omega_i = \frac{vdrive}{r} = \frac{vx}{r} \qquad (1.1)$$

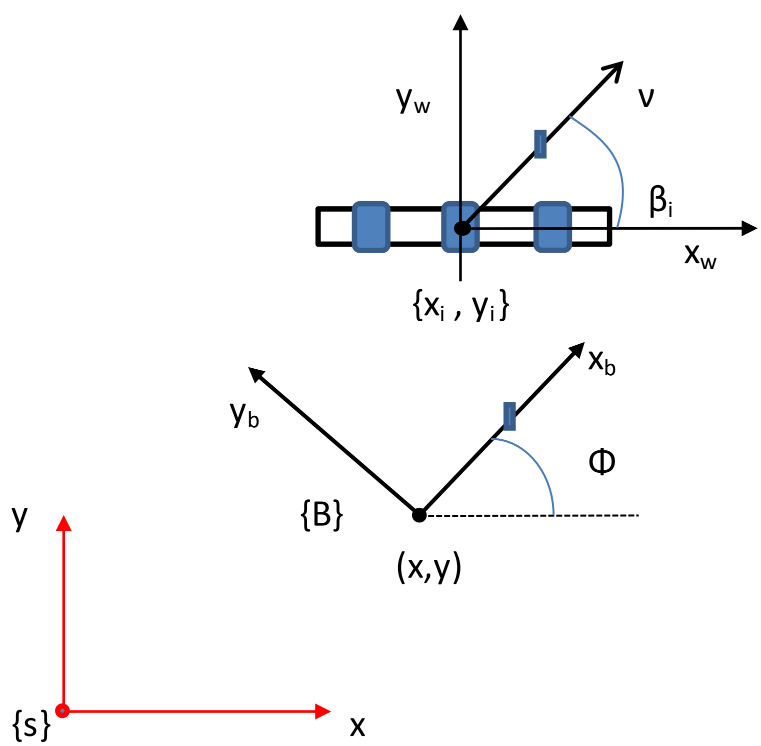
$$u_i = \frac{1}{r}[1\ 0]\begin{bmatrix} v_{wx} \\ v_{wy} \end{bmatrix} \qquad (1.2)$$

where “r” is the radius of the wheel, we can apply this to all wheels.

But we need the relation between the rotation speed for each wheel, with the velocity of the robot, based on that, we have to define a new frame.

S:{x,y} is space frame or inertial frame, and B:{x_b,y_b} is a frame with origin at the centroid of the robot, the angle of x_b with respect to x_s

is Φ , the rotation for the chassis with respect to the inertial frame is rotation on axis z with angle Φ.



Now we need to rewrite the equation (1.2) with respect to frame B, frame B should achieves tow conditions, the angle and the position.
The type of equation we will have is like :

$$u_i = H(0)v_b$$

$$u_i = H(0)\begin{bmatrix}\omega_{bz}\\ v_{bx}\\ v_{by}\end{bmatrix}$$

$H(0)$ should have two steps, to describe the velocity of the robot for all wheels in frame B:

1. **rotate the B frame with the same angle β_i for each wheel:**
   for rotation, we use *orthogonal rotation matrix* around z axis, This rotation transforms the robot's desired directional movements from the robot frame (b) to the wheel frame (w), where the angle (β_i) between the x-axis of the robot frame (b) and the x-axis of the wheel frame (w)

$$R(\beta) = \begin{bmatrix}\cos\beta_i & \sin\beta_i\\ -\sin\beta_i & \cos\beta_i\end{bmatrix}$$

2. **translate the axes to the center of each wheel.**
   for this step we want to achieve the second condition "position", we want to represent the position of wheel i relative to the robot's center of mass (centroid), in another way, moving the frame (b) axes to the center of each wheel :

$$Transformation\ matrix = \begin{bmatrix}y_i & 1 & 0\\ x_i & 0 & 1\end{bmatrix}$$

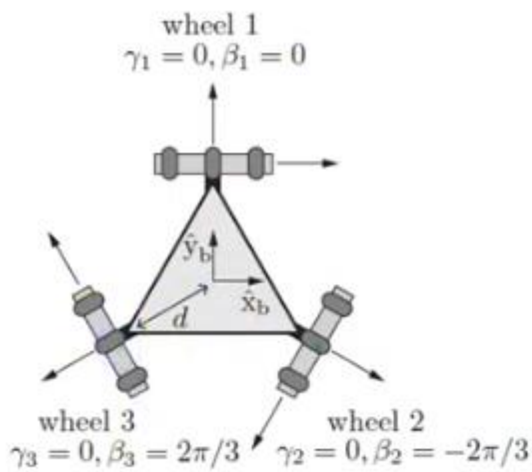
where y_i, x_i is the y-coordinate and the x-coordinate of wheel i relative to the robot's center of mass in the robot frame (b).

Now we can get the final equation that describe the rotation speed for each wheel with represented in frame B :

$$u_i = \frac{1}{r}[1\ 0]\begin{bmatrix}\cos\beta_i & \sin\beta_i\\ -\sin\beta_i & \cos\beta_i\end{bmatrix}\begin{bmatrix}y_i & 1 & 0\\ x_i & 0 & 1\end{bmatrix}\begin{bmatrix}\omega_{bz}\\ v_{bx}\\ v_{by}\end{bmatrix}$$

using this equation in our robot, and after replacing β_i with each wheel angle, and the coordinates for each wheel in the Transformation matrix :

Examples:



$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = H(0)\mathcal{V}_b = \underbrace{\frac{1}{r}\begin{bmatrix} -d & 1 & 0 \\ -d & -1/2 & -\sin(\pi/3) \\ -d & -1/2 & \sin(\pi/3) \end{bmatrix}}_{H(0)} \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix}$$

where the coordinates of the wheel as following :

- wheel 1 : $[x_1 = 0, y_1 = d]$
- wheel 2 : $[x_2 = d\cos 30°, y_2 = -d\sin 30°]$
- wheel 3 : $[x_3 = -d\cos 30°, y_3 = -d\sin 30°]$

To describe robot motion in terms of component motions, it will be necessary to map motion along the axes of the space frame {S} to motion along the axes of the robot's frame{B}. mapping is a function of the current pose of the robot. we can accomplished mapping using the orthogonal rotation matrix to go from frame {S} to frame {B} with respect to the angle Φ :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

this is the rotation matrix around z axis, the rest values is just to deal with $\phi^{\cdot}$

This matrix can be used to map motion in space frame {S}

frame to motion interms of the robot's frame{B} .

Now, get the final equation that describe the rotation speed for each wheel with represented in frame S will be :

$$u_i = \frac{1}{r}[1\ 0]\begin{bmatrix} \cos\beta_i & \sin\beta_i \\ -\sin\beta_i & \cos\beta_i \end{bmatrix}\begin{bmatrix} y_i & 1 & 0 \\ x_i & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_i & \sin\phi_i \\ 0 & -\sin\phi_i & \cos\phi_i \end{bmatrix}\begin{bmatrix} \phi^{\cdot} \\ x^{\cdot} \\ y^{\cdot} \end{bmatrix}$$

where :

- $\phi^{\cdot}$ is the rotation speed around z axis, in frame S .
- $x^{\cdot}$ is the COM velocity on x-coordinate in frame S .
- $y^{\cdot}$ is the COM velocity on y-coordinate in frame S .