Eigenmode Truncation Exploration for Thermal Treatments on Brain Tissue

For this exploration, we focused primarily on improving the computation characteristics of the linear system. First we explored how well simply selecting the slowest modes performed for various truncation sizes; using Forward Euler as the primary comparison of the full model versus the shortened model over a 10 second simulation period:

| q | Time (seconds) | Error in Last Solve REF-RED | |
|-------------------|----------------|-------------------------------|--|
| Full system solve | 14.3 | | |
| 2210 (no loss) | 13.0 | 5.7e-12 | |
| 1000 | 1.78 | 66 | |
| 100 | 0.29 | 223 | |
| 10 | 0.23 | 829 | |
| 2 | 0.23 | 1310 | |

So obviously just choosing the g slowest nodes is not sufficient.

We subsequently experimented with several other options: q fastest nodes, q nodes of smallest residue, q nodes of largest residue, etc. We found that choosing the slowest modes performed best; though clearly missed some information lost when reduced further.

We also implemented this code using the Trapezoidal Solver, and noticed similar results. To visualize the actual response of the system, see the different methodologies below, all solved with q=1000.

| Full System | Slow Modes | Fast Modes | Small Residue | Small Ratio |
|-------------|------------|------------|---------------|-------------|
| | | | | |

In this case, we see that the small ratio (picking small values of the residue divided by the eigenvalue of the residue index) isn't bad, but like the slow-modes suffers from blur. We see that the fastest modes are capturing our edge conditions, which is fairly important to us. The slow modes are capturing well what happens outside of those transitions.