ASSIGNMENT-12

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1 QUESTION No-2.28(Optimization)

Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively.They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table: How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

Transportation cost per quintal (in rupees)		
From/To	A	B
D	6	4
E	3	2
F	2.50	3

TABLE 1.1: Transportation table

2 Solution

Let A supply x quintals to D and y quintals to E. From the data given we have

$$x + y \le 100 \tag{2.0.1}$$

$$x \le 60 \tag{2.0.2}$$

$$y \le 50 \tag{2.0.3}$$

$$x + y \ge 60 \implies -x - y \le -60 \tag{2.0.4}$$

Now our aim is to minimize the transportation cost,we obtain the minimizing function as,

$$\min_{\mathbf{x}} Z = \begin{pmatrix} 2.5 & 1.5 \end{pmatrix} \mathbf{x} + 410. \tag{2.0.5}$$

Subject to the constraints,

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & -1 \\ -1 & -1 \end{pmatrix} \mathbf{x} \le \begin{pmatrix} 100 \\ 60 \\ 50 \\ -60 \end{pmatrix} \tag{2.0.6}$$

We solve the optimization problem by the method of Lagrange multipliers, thus the Lagrangian function is given as follows,

$$\begin{aligned} &L(\mathbf{x}, \lambda) \\ &= \begin{pmatrix} 2.5 & 1.5 \end{pmatrix} \mathbf{x} + 410 + \left[\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} - 100 \right] \lambda_1 \\ &+ \left[\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 60 \right] \lambda_2 + \left[\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} - 50 \right] \lambda_3 \\ &+ \left[\begin{pmatrix} -1 & -1 \end{pmatrix} \mathbf{x} + 60 \right] \lambda_4 + \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] \lambda_5 \\ &+ \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \lambda_6 \end{aligned} \tag{2.0.7}$$

Now, we have

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = \begin{pmatrix} 2.5 + \begin{pmatrix} 1 & 1 & 0 & -1 & -1 & 0 \end{pmatrix} \boldsymbol{\lambda} \\ 1.5 + \begin{pmatrix} 1 & 0 & 1 & -1 & 0 & -1 \end{pmatrix} \boldsymbol{\lambda} \\ \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} - 100 \\ \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} - 60 \\ \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} - 50 \\ \begin{pmatrix} -1 & -1 \end{pmatrix} \mathbf{x} + 60 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{2.0.8}$$

where $\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix}$. Therefore, Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -2.5 \\ -1.5 \\ 100 \\ 60 \\ 50 \\ -60 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.9}$$

We see that λ_3, λ_4 are the only active multiplier and hence we proceed by considering them,

$$\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -2.5 \\ -1.5 \\ 50 \\ -60 \end{pmatrix} \tag{2.0.10}$$

Now we have,

$$\begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -2.5 \\ -1.5 \\ 50 \\ -60 \end{pmatrix} \tag{2.0.11}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2.5 \\ -1.5 \\ 50 \\ -60 \end{pmatrix} \tag{2.0.12}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 10 \\ 50 \\ 1 \\ 2.5 \end{pmatrix} \tag{2.0.13}$$

$\because \boldsymbol{\lambda} = \begin{pmatrix} 1 \\ 2.5 \end{pmatrix} \succ 0$

Thus, the Optimal solution is given as,

$$\mathbf{x} = \begin{pmatrix} 10 \\ 50 \end{pmatrix} \tag{2.0.14}$$

Substituting in (2.0.5) (2.0.15)

$$Z = \begin{pmatrix} 2.5 & 1.5 \end{pmatrix} \mathbf{x} + 410 \tag{2.0.16}$$

$$= \begin{pmatrix} 2.5 & 1.5 \end{pmatrix} \begin{pmatrix} 10 \\ 50 \end{pmatrix} \tag{2.0.17}$$

$$= 510 \tag{2.0.18}$$

Hence, in order to minimize the transportation cost we see that A must supply 10 quintals to D and 50 quintals to E, and the obtained minimum cost is ₹510.

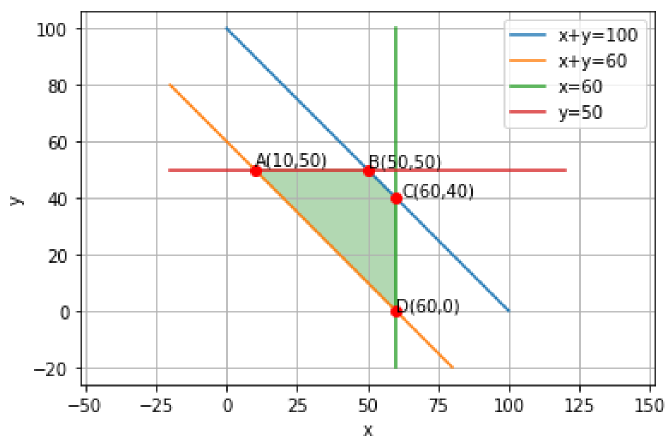

Fig. 2.1: Graphical Solution