

ASSIGNMENT-13

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1 QUESTION No-2.7(PROBABILITY)

A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

2 SOLUTION

$X \in \{1, 2\}$ represent the machines A and B and $Y \in \{0, 1\}$ represents the quality of the item, 0 if it is defective and 1 if it has no defects. From the given information we have,

$$Pr(X = 1) = \frac{60}{100} \quad (2.0.1)$$

$$Pr(X = 2) = \frac{40}{100} \quad (2.0.2)$$

$$Pr(Y = 0|X = 1) = \frac{2}{100} \quad (2.0.3)$$

$$Pr(Y = 0|X = 2) = \frac{1}{100} \quad (2.0.4)$$

The probability that the randomly chosen defective item is produced by machine B is given by,

$$Pr(X = 2|Y = 0) = \frac{Pr(Y = 0|X = 2)Pr(X = 2)}{\sum_{i=1}^2 Pr(Y = 0|X = i)Pr(X = i)} \quad (2.0.5)$$

Substituting the given values in (2.0.5),

$$Pr(X = 2|Y = 0) = \frac{\frac{1}{100} \frac{40}{100}}{\frac{2}{100} \frac{60}{100} + \frac{40}{100} \frac{1}{100}} = \frac{1}{4} \quad (2.0.6)$$

Thus, the probability that the chosen defective item was produced by machine B is 0.25 i.e., $Pr(X = 2|Y = 0) = \frac{1}{4}$.