ASSIGNMENT-16

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1 QUESTION No-8.6 (GATE Probability)

Suppose X and Y are two random variables such that $aX + bY$ is a normal random variable for all $a, b \in \mathbb{R}$. Consider the following statements P,Q,R and S:
(P): X is a standard normal random variable.
(Q): The conditional distribution of X given Y is normal.
(R): The conditional distribution of X given $X + Y$ is normal.
(S): X - Y has mean 0.
Which of the above statements ALWAYS hold TRUE?

1) both P and Q
2) both Q and R
3) both Q and S
4) both P and S

2 Solution

Definition 1. *Two random variables X and Y are said to be bivariate normal, or jointly normal, if $aX + bY$ has a normal distribution for all $a, b \in \mathbb{R}$.*

(P) X is a standard normal random variable.
By taking $a = 1, b = 0$ we see that X must be normal. Similarly by setting $a = 0, b = 1$ we have Y to be normal. The given information is not sufficient to conclude whether X is a standard normal random variable. Thus this statement does not hold true always.
(Q) The conditional distribution of X given Y is normal.
The conditional distribution $X|Y$ follows normal distribution with mean $\mu_x + \rho\frac{\sigma_x}{\sigma_y}(y - \mu_x)$ and standard deviation $\sigma_x\left(\sqrt{1-\rho^2}\right)$, where μ_x and σ_x denote the mean and standard deviation of random variable X similarly μ_y and σ_y denote the mean and standard deviation of random variable Y and thus $X|Y \sim N\left(\mu_x + \rho\frac{\sigma_x}{\sigma_y}(y - \mu_x), \sigma_x\left(\sqrt{1-\rho^2}\right)\right)$. Thus the statement holds true always.

(R) The conditional distribution of X given $X + Y$ is normal.
By taking $a = 1, b = 1$ the linear combination $X + Y$ is also normal. Then from the previous statement it directly follows that X given $X + Y$ is normal. Thus the statement holds true always.
(S) X - Y has mean 0.
This is possible only when X and Y are independent standard normal variable.Thus the given information is not sufficient to conclude whether $X - Y$ has mean 0. Thus this statement does not hold true always.