ASSIGNMENT-5

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1 QUESTION NO-2.11

If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$ then prove that $\frac{(1+(y_1)^2)^{\frac{3}{2}}}{y_2}$ is a constant independent of a and b.

2 Solution

Given that $(x-a)^2+(y-b)^2=c^2$, for some $c>0$.In order to prove $\frac{(1+(y_1)^2)^{\frac{3}{2}}}{y_2}$ is a constant independent of a and b. Consider,

$$(x-a)^2+(y-b)^2=c^2 \tag{2.0.1}$$

Now differentiating equation (2.0.1) on both sides with respect to x we get

$$2(x-a)+2(y-b)\frac{dy}{dx}=0 \tag{2.0.2}$$

$$y_1=\frac{dy}{dx}=\frac{a-x}{y-b} \tag{2.0.3}$$

Differentiating (2.0.3) again with respect to x we have,

$$\frac{d^2y}{dx^2}=\frac{-(y-b)-(a-x)y_1}{(y-b)^2} \tag{2.0.4}$$

$$y_2=\frac{-(y-b)-(a-x)\left(\frac{a-x}{y-b}\right)}{(y-b)^2} \tag{2.0.5}$$

$$=\frac{-(x-a)^2-(y-b)^2}{(y-b)^3} \tag{2.0.6}$$

$$=-\left(\frac{(x-a)^2+(y-b)^2}{(y-b)^3}\right) \tag{2.0.7}$$

$$y_2=\frac{-c^2}{(y-b)^3} \tag{2.0.8}$$

Now substituting the values of y_1 from equation (2.0.3) and y_2 from equation (2.0.8) in $\frac{(1+(y_1)^2)^{\frac{3}{2}}}{y_2}$

$$\frac{(1+(y_1)^2)^{\frac{3}{2}}}{y_2}=\frac{(1+\frac{(x-a)^2}{(y-b)^2})^{\frac{3}{2}}}{\frac{-c^2}{(y-b)^3}} \tag{2.0.9}$$

$$=\frac{((x-a)^2+(y-b)^2)^{\frac{3}{2}}}{(y-b)^3}\left(\frac{-(y-b)^3}{c^2}\right) \tag{2.0.10}$$

From equation (2.0.1), we get

$$=\frac{-(c^2)^{\frac{3}{2}}}{c^2} \tag{2.0.11}$$

$$=-c \tag{2.0.12}$$

Hence, we have $\frac{(1+(y_1)^2)^{\frac{3}{2}}}{y_2}=-c$.
Therefore, we have proved that, if $(x-a)^2+(y-b)^2=c^2$, for some $c>0$ then $\frac{(1+(y_1)^2)^{\frac{3}{2}}}{y_2}$ is a constant independent of a and b.