ASSIGNMENT-5

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1 QUESTION No-2.98 (Quadratic forms)

Find the area lying above x-axis and included between the circle $\mathbf{x}^T\mathbf{x} - 8\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = 0$ and inside of the parabola $y^2 = 4x$.

2 Solution

Given equation of the circle $\mathbf{x}^T\mathbf{x} - 8\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = 0$.We know that the general equation of a circle is given by

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

We have $\mathbf{u} = \begin{pmatrix}-4\\0\end{pmatrix}$ and $f = 0$.Thus we have the center and radius as $\mathbf{c} = -\mathbf{u} = \begin{pmatrix}4\\0\end{pmatrix}$ and $r = \sqrt{\mathbf{u^T u} - f} = 4$ respectively. Given equation of the parabola $\mathbf{x}^T\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} - 4\begin{pmatrix}0 & 1\end{pmatrix} = 0$. The plot of the above two curves is

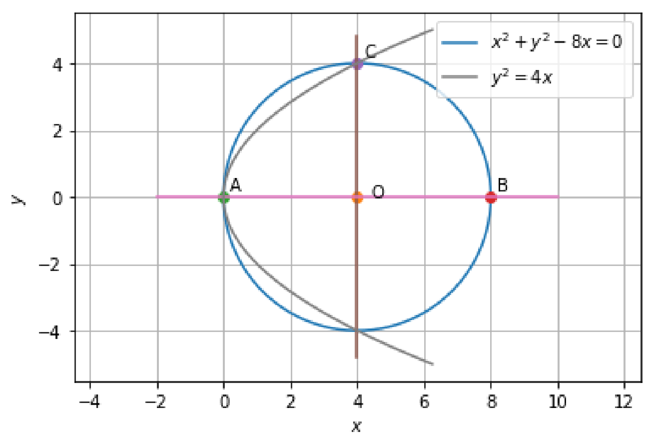

Fig. 2.1: Plot of the curves

Now to find the area bounded above the x-axis,the parabola and the circle. From fig.2.1 the area to be calculated is $AOBCA$.

$$Ar(AOBCA) = Ar(ACOA) + Ar(OCBO) \tag{2.0.2}$$
$$= A_1 + A_2 \tag{2.0.3}$$

To calculate A_1: A_1 is the area enclosed by the parabola $y^2 = 4x$ and the line OC.Thus

$$A_1 = \frac{2}{3}(AO)(OC) \tag{2.0.4}$$
$$= \frac{2}{3}(4)(4) = \frac{32}{3} \tag{2.0.5}$$

To calculate A_2: A_2 is one fourth of the area of the circle.

$$A_2 = \frac{1}{4}\left(\pi r^2\right) \tag{2.0.6}$$
$$= \frac{1}{4}(16\pi) \tag{2.0.7}$$
$$= 4\pi \tag{2.0.8}$$

Now substituting (2.0.5) and (2.0.8) in (2.0.3) we get

$$A_1 + A_2 = \frac{32}{3} + 4\pi \tag{2.0.9}$$
$$= 4\left(\frac{8}{3} + \pi\right) \tag{2.0.10}$$

Thus (2.0.10) is the required area.