

ASSIGNMENT-5

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1 QUESTION No-2.98 (QUADRATIC FORMS)

Find the area lying above x-axis and included between the circle $\mathbf{x}^T \mathbf{x} - 8 \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0$ and inside of the parabola $y^2 = 4x$.

2 SOLUTION

Given equation of the circle

$$\mathbf{x}^T \mathbf{x} - 8 \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0. \quad (2.0.1)$$

We know that the general equation of a circle is given by

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.2)$$

We have $\mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$ and $f = 0$. Thus we have the center and radius as

$$\mathbf{c} = -\mathbf{u} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (2.0.3)$$

and

$$r = \sqrt{\mathbf{u}^T \mathbf{u} - f} = 4 \quad (2.0.4)$$

respectively. Given equation of the parabola

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - 4 \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.5)$$

The plot of the above two curves is

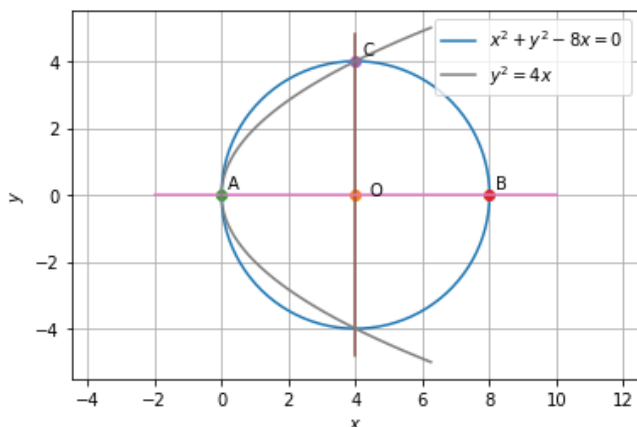


Fig. 2.1: Plot of the curves

Now to find the area bounded above the x-axis, the parabola and the circle. From fig.2.1 the area to be calculated is $AOBCA$.

$$Ar(AOBCA) = Ar(ACOA) + Ar(OCBO) \quad (2.0.6)$$

$$= A_1 + A_2 \quad (2.0.7)$$

To calculate A_1 : A_1 is the area enclosed by the parabola $y^2 = 4x$ and the line OC . Thus

$$A_1 = \frac{2}{3} (AO)(OC) \quad (2.0.8)$$

$$= \frac{2}{3} (4)(4) = \frac{32}{3} \quad (2.0.9)$$

To calculate A_2 : A_2 is one fourth of the area of the circle.

$$A_2 = \frac{1}{4} (\pi r^2) \quad (2.0.10)$$

$$= \frac{1}{4} (16\pi) \quad (2.0.11)$$

$$= 4\pi \quad (2.0.12)$$

Now substituting (2.0.5) and (2.0.8) in (2.0.3) we get

$$A_1 + A_2 = \frac{32}{3} + 4\pi \quad (2.0.13)$$

$$= 4 \left(\frac{8}{3} + \pi \right) \quad (2.0.14)$$

Thus (2.0.10) is the required area.