ASSIGNMENT-6

R.YAMINI

1 QUESTION No-2.82 (Quadratic forms)

Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $\begin{pmatrix}4 & -2\end{pmatrix}\mathbf{x}+5 = 0$.

2 Solution

Given equation

$$\mathbf{x}^\top\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x}+\begin{pmatrix}-3 & 0\end{pmatrix}\mathbf{x}+2=0 \tag{2.0.1}$$

we have

$$\therefore \mathbf{u}=\begin{pmatrix}\frac{-3}{2}\\0\end{pmatrix} \tag{2.0.2}$$

$$\mathbf{V}=\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} \tag{2.0.3}$$

$$\implies \left|\mathbf{V}\right|=0 \tag{2.0.4}$$

Thus the curve is a parabola.Now we find the eigen values corresponding to $\mathbf{V}$

$$\left|\mathbf{V}-\lambda\mathbf{I}\right|=0 \tag{2.0.5}$$

$$\begin{vmatrix}-\lambda & 0\\0 & 1-\lambda\end{vmatrix}=0 \tag{2.0.6}$$

$$\implies \lambda=0,1 \tag{2.0.7}$$

Now we find the eigen vectors corresponding to $\lambda = 0,1$ respectively.

$$\mathbf{V}\mathbf{x}=\lambda\mathbf{x} \tag{2.0.8}$$

$$\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\mathbf{x}=0 \implies \mathbf{p_1}=\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.9}$$

$$\begin{pmatrix}-1 & 0\\0 & 0\end{pmatrix}\mathbf{x}=\mathbf{x} \implies \mathbf{p_2}=\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.10}$$

Now by eigen decomposition on $\mathbf{V}$

$$\mathbf{V}=\mathbf{P}\mathbf{D}\mathbf{P}^\top \tag{2.0.11}$$

where

$$\mathbf{P}=(\mathbf{p_1}\mathbf{p_2})=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{2.0.12}$$

$$\mathbf{D}=\begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix}=\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} \tag{2.0.13}$$

Now (2.0.11) becomes

$$\mathbf{V}=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{2.0.14}$$

$$\mathbf{V}=\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} \tag{2.0.15}$$

Given parallel line equation

$$\begin{pmatrix}4 & -2\end{pmatrix}\mathbf{x}+5=0 \tag{2.0.16}$$

Now the tangent to the parabola is parallel to the line equation (2.0.16), The normal vector $\mathbf{n}$ and the direction vector $\mathbf{m}$ is given by

$$\mathbf{n}=\begin{pmatrix}4\\-2\end{pmatrix} \tag{2.0.17}$$

$$\mathbf{m}=\begin{pmatrix}-2\\-4\end{pmatrix} \tag{2.0.18}$$

Now the equation for the point of contact for the parabola is given as,

$$\begin{pmatrix}\mathbf{u}+\kappa\mathbf{n}^\top\\\mathbf{V}\end{pmatrix}\mathbf{q}=\begin{pmatrix}-\mathbf{f}\\\kappa\mathbf{n}-\mathbf{u}\end{pmatrix} \tag{2.0.19}$$

$$\kappa=\frac{\mathbf{p_1}^\top\mathbf{u}}{\mathbf{p_1}^\top\mathbf{n}}=\frac{\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}\frac{-3}{2}\\0\end{pmatrix}}{\begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}4\\2\end{pmatrix}}=\frac{-3}{8} \tag{2.0.20}$$

Thus we have,

$$\implies \begin{pmatrix}-3 & \frac{3}{4}\\0 & 0\\0 & 1\end{pmatrix}\mathbf{q}=\begin{pmatrix}-2\\0\\\frac{3}{4}\end{pmatrix} \tag{2.0.21}$$

Now solving for $\mathbf{q}$ by removing the zero row and representing as augmented matrix and then converting it to echelon form.

$$\begin{pmatrix} -3 & \frac{3}{4} & -2 \\ 0 & 1 & \frac{3}{4} \end{pmatrix} \xleftrightarrow{R_1 \rightarrow \left(\frac{-1}{3}\right) R_1} \begin{pmatrix} 1 & \frac{-1}{4} & \frac{2}{3} \\ 0 & 1 & \frac{3}{4} \end{pmatrix} \tag{2.0.22}$$

$$\begin{pmatrix} 1 & \frac{-1}{4} & \frac{2}{3} \\ 0 & 1 & \frac{3}{4} \end{pmatrix} \xleftrightarrow{R_1 \rightarrow R_1 + \frac{1}{4} R_2} \begin{pmatrix} 1 & 0 & \frac{41}{48} \\ 0 & 1 & \frac{3}{4} \end{pmatrix} \tag{2.0.23}$$

Hence from the (2.0.23) we get the point of contact to be

$$\mathbf{q} = \begin{pmatrix} \frac{41}{48} \\ \frac{3}{4} \end{pmatrix} \tag{2.0.24}$$

Now $\mathbf{q}$ is the point on the tangent.Hence the equation of the line can be expressed as

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{q}) = 0 \tag{2.0.25}$$

where,

$$\mathbf{n}^\top \mathbf{q} = \begin{pmatrix} 4 & -2 \end{pmatrix} \begin{pmatrix} \frac{41}{48} \\ \frac{3}{4} \end{pmatrix} \tag{2.0.26}$$

$$= \frac{23}{12} \tag{2.0.27}$$

Hence the equation of the tangent to the curve (2.0.1) parallel to the line (2.0.16) is given by substituting the value of $\mathbf{n}^\top \mathbf{q}$ and $\mathbf{n}$ from (2.0.27) and (2.0.17) respectively to the equation (2.0.25)

$$\begin{pmatrix} 4 & -2 \end{pmatrix} \mathbf{x} = \frac{23}{12} \tag{2.0.28}$$

Hence (2.0.28) is the equation of the tangent. The plot of the figure is given below

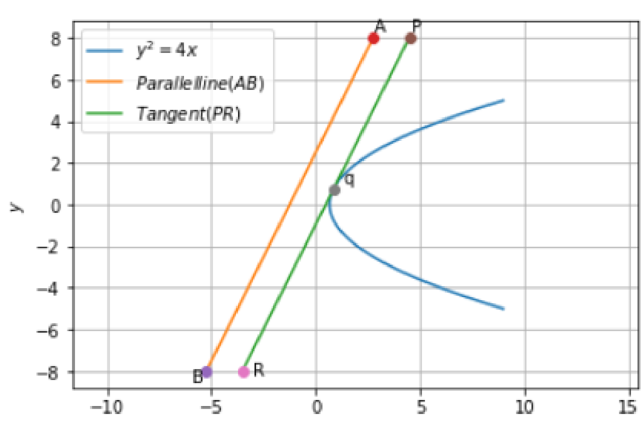

Fig. 2.1: Plot of curve and the lines