

ASSIGNMENT-8

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1 QUESTION No-2.38(B) (QUADRATIC FORMS)

we have,

Find the coordinates of the foci and the vertices, the eccentricity, the length of the latus rectum of the hyperbola $\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -16 \end{pmatrix} \mathbf{x} = 16$.

$$\mathbf{F} = \pm \begin{pmatrix} \sqrt{17} \\ 0 \end{pmatrix}. \quad (2.0.10)$$

Eccentricity of the hyperbola is given by,

$$e = \frac{\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u})(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}}}{\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}}} \quad (2.0.11)$$

substituting the values in (2.0.11), we have

$$e = \frac{\sqrt{17}}{4}. \quad (2.0.12)$$

Length of the latus rectum is given by,

$$l = \frac{2 \left(\sqrt{\frac{f - \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u}}{\lambda_2}} \right)^2}{\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}}} \quad (2.0.13)$$

substituting the values in (2.0.13), we have

$$l = \frac{1}{2} \quad (2.0.14)$$

The plot of the hyperbola is given below

2 SOLUTION

Given equation of the hyperbola,

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -16 \end{pmatrix} \mathbf{x} = 16 \quad (2.0.1)$$

we have,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & -16 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f = 16 \quad (2.0.3)$$

$$\mathbf{c} = -\mathbf{V}^{-1} \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.4)$$

$$\lambda_1 = 1, \lambda_2 = -16 \quad (2.0.5)$$

Axes of hyperbola is given by

$$\sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} = 4 \quad (2.0.6)$$

$$\sqrt{\frac{f - \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u}}{\lambda_2}} = 1 \quad (2.0.7)$$

The vertices are given as

$$\pm \begin{pmatrix} 4 \\ 0 \end{pmatrix} \text{ and } \pm \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.8)$$

Coordinates of foci are given by,

$$\mathbf{F} = \pm \left(\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \right) \mathbf{p}_1 \quad (2.0.9)$$

where, $\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ since the equation of hyperbola is in standard form. Substituting the values in (2.0.9)

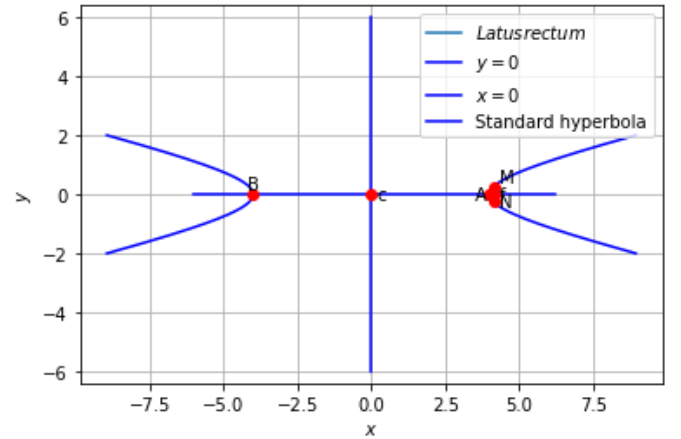


Fig. 2.1: Plot of standard hyperbola