ASSIGNMENT-8

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1 Question No-2.38(b) (Quadratic forms)

Find the coordinates of the foci and the vertices, the eccentricity,the length of the latus rectum of the hyperbola $\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & -16 \end{pmatrix} \mathbf{x} = 16$.

2 Solution

Given equation of the hyperbola,

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & -16 \end{pmatrix} \mathbf{x} = 16 \tag{2.0.1}$$

we have,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & -16 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f = 16 \tag{2.0.3}$$

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.4}$$

$$\lambda_1 = 1, \lambda_2 = -16 \tag{2.0.5}$$

Axes of hyperbola is given by

$$\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} = 4 \tag{2.0.6}$$

$$\sqrt{\frac{f - \mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u}}{\lambda_2}} = 1 \tag{2.0.7}$$

The vertices are given as

$$\pm \begin{pmatrix} 4 \\ 0 \end{pmatrix} \text{and} \pm \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.8}$$

Coordinates of foci are given by,

$$\mathbf{F} = \pm \left(\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \right) \mathbf{p_1} \tag{2.0.9}$$

where, $\mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ since the equation of hyperbola is in standard form. Substituting the values in (2.0.9) we have,

$$\mathbf{F} = \pm \begin{pmatrix} \sqrt{17} \\ 0 \end{pmatrix}. \tag{2.0.10}$$

Eccentricity of the hyperbola is given by,

$$e = \frac{\sqrt{\frac{(\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u})(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}}}{\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}}} \tag{2.0.11}$$

substituting the values in (2.0.11),we have

$$e = \frac{\sqrt{17}}{4}. \tag{2.0.12}$$

Length of the latus rectum is given by,

$$l = \frac{2\left(\sqrt{\frac{f - \mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u}}{\lambda_2}}\right)^2}{\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}}} \tag{2.0.13}$$

substituting the values in (2.0.13),we have

$$l = \frac{1}{2} \tag{2.0.14}$$

The plot of the hyperbola is given below

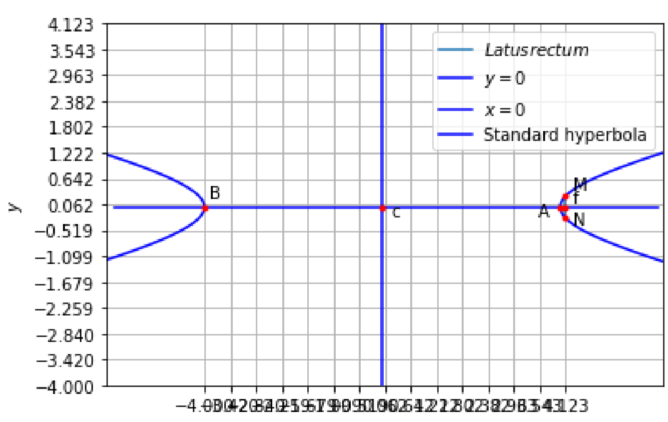

Fig. 2.1: Plot of standard hyperbola