ASSIGNMENT-9

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1 Question No-2.2(Vectors)

Let $\mathbf{A} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ be the vertices of $\triangle ABC$.

a. The median from $\mathbf{A}$ meets BC at $\mathbf{D}$. Find the coordinates of the point $\mathbf{D}$.
b. Find the coordinates of the point $\mathbf{P}$ on AD such that $AP : PD = 2 : 1$.
c. Find the coordinates of the points $\mathbf{Q}$ and $\mathbf{R}$ on medians BE and CF respectively such that $BQ : QE = 2 : 1$ and $CR : RF = 2 : 1$.

2 Solution

a. Given $\triangle ABC$ with vertices

$$\mathbf{A} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{2.0.1}$$

Given that the median from $\mathbf{A}$ meets BC at $\mathbf{D}$, now the coordinate of $\mathbf{D}$ is given as,

$$\mathbf{D} = \frac{\mathbf{B} + \mathbf{C}}{2} = \frac{\begin{pmatrix} 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix}}{2} \tag{2.0.2}$$

$$\implies \mathbf{D} = \begin{pmatrix} \frac{7}{2} \\ \frac{9}{2} \end{pmatrix} \tag{2.0.3}$$

b. Result :The coordinates of point $\mathbf{C}$ dividing the line AB in the ratio $m : n$ is given by

$$\frac{n\mathbf{A} + m\mathbf{B}}{m + n} \tag{2.0.4}$$

Given that the point $\mathbf{P}$ divides AD in the ratio $2 : 1$, now to find $\mathbf{P}$ we use (2.0.4),

$$\mathbf{P} = \frac{1\begin{pmatrix} 4 \\ 2 \end{pmatrix} + 2\begin{pmatrix} \frac{7}{2} \\ \frac{9}{2} \end{pmatrix}}{3} = \begin{pmatrix} \frac{11}{3} \\ \frac{11}{3} \end{pmatrix} \tag{2.0.5}$$

c. Given that the point $\mathbf{Q}$ on the median BE divides it in the ratio $2 : 1$, first we find $\mathbf{E}$,

$$\mathbf{E} = \frac{\mathbf{A} + \mathbf{C}}{2} = \frac{\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix}}{2} \tag{2.0.6}$$

$$\implies \mathbf{E} = \begin{pmatrix} \frac{5}{2} \\ 3 \end{pmatrix}. \tag{2.0.7}$$

Now we find $\mathbf{Q}$ using (2.0.4)

$$\mathbf{Q} = \frac{1\begin{pmatrix} 6 \\ 5 \end{pmatrix} + 2\begin{pmatrix} \frac{5}{2} \\ 3 \end{pmatrix}}{3} = \begin{pmatrix} \frac{11}{3} \\ \frac{11}{3} \end{pmatrix} \tag{2.0.8}$$

Similarly,Given that the point $\mathbf{R}$ on the median CF divides it in the ratio $2 : 1$, first we find $\mathbf{F}$,

$$\mathbf{F} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 5 \end{pmatrix}}{2} \tag{2.0.9}$$

$$\implies \mathbf{F} = \begin{pmatrix} 5 \\ \frac{7}{2} \end{pmatrix}. \tag{2.0.10}$$

Now we find $\mathbf{R}$ using (2.0.4)

$$\mathbf{R} = \frac{1\begin{pmatrix} 1 \\ 4 \end{pmatrix} + 2\begin{pmatrix} 5 \\ \frac{7}{2} \end{pmatrix}}{3} = \begin{pmatrix} \frac{11}{3} \\ \frac{11}{3} \end{pmatrix} \tag{2.0.11}$$

The plot of the $\triangle ABC$ is given below

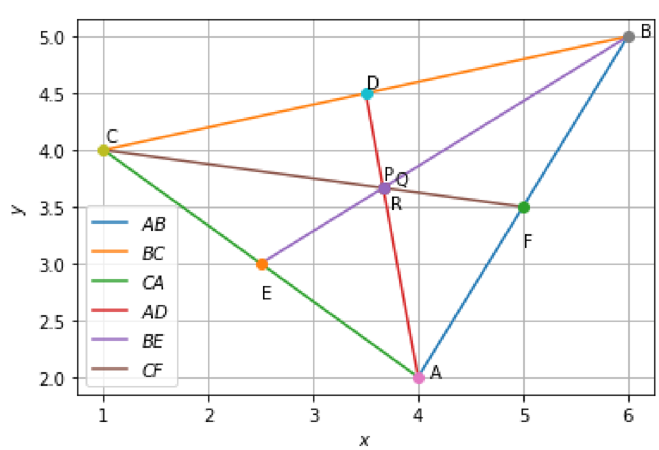

Fig. 2.1: Plot of $\triangle ABC$