Q3.

Given

$\alpha_p = 3dB$ $\omega_p = 2\times\pi\times1000 = 2000\pi$ rad/sec

$\alpha_s = 10 dB$ $\omega_s = 2\times\pi\times350 = 700\pi$ rad/sec

$f_s = 5000 Hz$

$T = \frac{1}{f_s} = 2\times10^{-4}$ sec

$\therefore$ Prewraping digital frequencies.

$$\Omega_p = \frac{2}{T} \tan\frac{\omega_p T}{2}$$

$$= \frac{2}{2\times10^{-4}} \tan\left(\frac{2000\pi\times2\times10^{-4}}{2}\right) = 7265 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T} \tan\frac{\omega_s T}{2}$$

$$= 2235 \text{ rad/sec}$$

The order of filter

$$N = \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\frac{\Omega_s}{\Omega_p}} = 0.932$$

Let $N=1$

the 1^{st} order butterworth filter for $\Omega_p = 1$ rad/sec is $H(s) = \frac{1}{1+s}$

Put $s \rightarrow \frac{\Omega_p}{s} = \frac{7265}{s}$

Tranfer function of high pass filter

$$H(s) = \frac{1}{s+1}\bigg|_{s=\frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

using bilinear transformation,

$$H(z) = H(s) \Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265} \Big|_{s = \frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$= \frac{0.5792\,(1-z^{-1})}{1 - 0.1584 z^{-1}}$$

$$H(z) = \frac{0.5792\,(1-z^{-1})}{1 - 0.1584 z^{-1}}$$