Faculty of Physics - 16 ottobre 2021

Optometry and wave lab course

Birefringence experiment .

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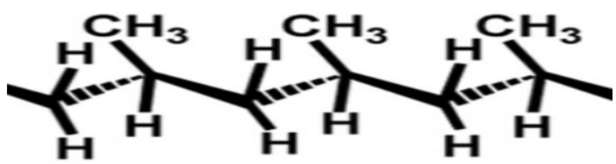
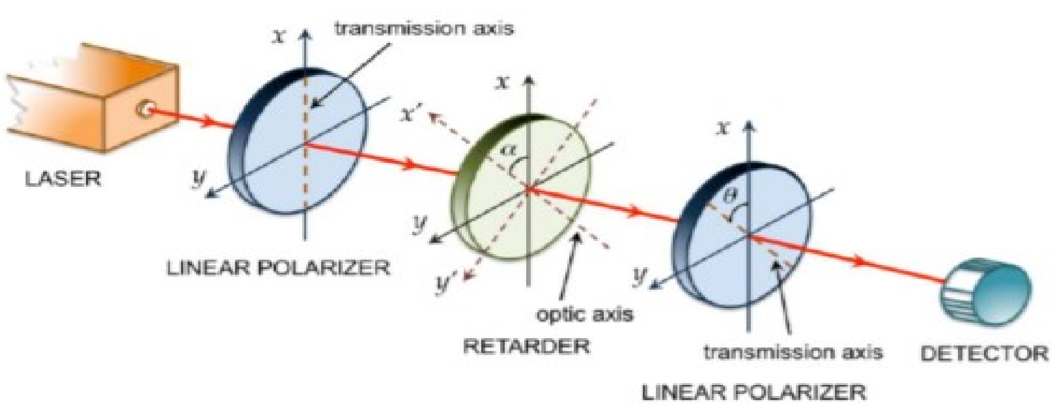
Indice

1 Introduction to Birefringence of duct tape.

In this experiment we will study the Birefringence phenomenon. Birefringence was discovered in 1669 by Erasmus Bartholinus (1625-1698) using calcite. By reflecting a 650 [nm] laser beam through 2 polarisation Plates, 4 layers of industrial duct tape, and a light detector we can measure the intensity which depends only on the angle of the duct tape to initial polarizer. Where θ is the angle between the first polarizer and the duct tape. In the second attempt we had 2 fixed angles: 45○ - 135○ to the two polarizers and the duct tape had a positive change in the amount of layers going from 4 to 10 layers. We conduct a simple experiment to study the effects of polarized light. A simple optical system composed of a polarizer, a retarder (cellotape) and an analyser is used to study the effect on the polarization state of the light which impinges on the setup. The optical system is characterized by means of a Jones matrix, and a simple procedure based on Jones vectors is used to obtain an expression for the intensity after the light passes through the optical system. The light intensity is measured by a photodetector and the expression obtained theoretically is experimentally validated. By fitting the experimental intensity data, the value of the retardation introduced by the retarder can also be obtained. the duct tape is made out of polymeric strings in the form of a thin layer. The light source passes through the strings of the polymer material that is used as a polarizer to.



The effect of the added glue on the duck tape to the measurement with the laser beam will be notes, still the result will be satisfying enough for the purpose of observing the Birefringence phenomena.
If instead we rotate the polarizers in relation to the Sellotape we will get the same result. The duct tape made from polymer material. In The Production of the duct tape it becomes uniaxial with a different index of refraction along the tape. For a general direction of the Electric field perpendicular to the k vector, one can decompose it to a sum in the E_o direction and the E_e direction. with that we can figure out the final intensity.



2 Mathematical development

To understand Birefringence phenomena in a better way we will develop the equation. Starting with an incident ray we get:

$$E_0 \cdot \exp\big(-(\omega t + \vec{k}\cdot\vec{r})\big) = [E_0 exp(-in_0\vec{k}\cdot\vec{r}\bar{o}) + E_e exp((-in_e\vec{k}\cdot\vec{r}\bar{e})]exp(-i\omega t) \tag{1}$$

The input wave is polarized in the e1 direction and can be written as:

$$\vec{E_1} = E_1 cos(\omega t - \vec{k}\cdot\vec{z})\bar{e_1} \tag{2}$$

The detector is sensitive to the intensity of the light and this is linked to the field by

$$I_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}E_1^2 \tag{3}$$

As sellotape is birefringent this introduces a phase difference between the projections of E1 on to the axes p and o.at the entrance to the strip we have :

$$\vec{E_r} = E_1 cos(\omega t - \vec{k}\cdot\vec{z})(cos(\theta)\bar{o} + sin(\theta)\bar{p}) \tag{4}$$

$$\vec{E_r} = E_1[cos(\theta)cos(\omega t - n_0\vec{k}\cdot\vec{z})\bar{o} + sin(\theta)cos(\omega t - n_p\vec{k}\cdot\vec{z})\bar{p})] \tag{5}$$

If the thickness of the film is s we can define a Retardation factor as

$$R = (n_0 - n_p)s \tag{6}$$

Arranging the argument:

$$cos(\omega t - n_p\vec{k}\cdot\vec{z}) = cos(\omega t - (n_o\vec{k}\cdot\vec{z} - n_o\vec{k}\cdot\vec{z}) - n_p\vec{k}\cdot\vec{z}) = cos(\omega t - n_o\vec{k}\cdot\vec{z} + (n_o - n_p)\vec{k}\cdot\vec{z}) \tag{7}$$

Phase difference between the two directions is given by: $(n_o - n_p)\vec{k}\cdot\vec{z} = R_z k$
therefore:

$$\vec{E_r} = E_1[cos(\theta)cos(\omega t - \vec{k}\cdot\vec{z})\bar{o} + sin(\theta)cos(\omega t - \vec{k}\cdot\vec{z} + \frac{R}{\lambda})\bar{p})] \tag{8}$$

$$\vec{E_f} = \vec{E_r}\cdot\bar{e_2} = E_1 sin(\theta)cos(\theta)[cos(\omega t - \vec{k}\cdot\vec{z}) - cos(\omega t - \vec{k}\cdot\vec{z} + \frac{R}{\lambda})]\bar{e_2} \tag{9}$$

The final time averaged intensity due to the field in equation (9) will be given by

$$I_f = \frac{1}{4}I_0 sin^2(2\theta)\bigg(1 - cos(2\pi\frac{R}{\lambda})\bigg) \tag{10}$$

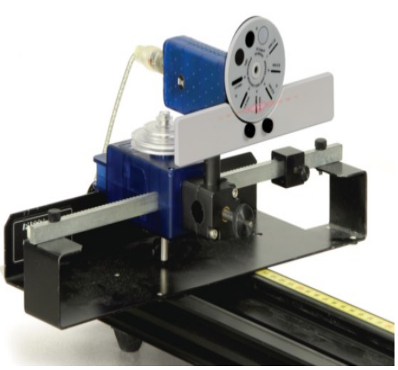
3 Experiment set up

using an Optical trail to place the polarizers, duct tape, laser beam, and the light detector in the same optical line we can measure the signal of light intensity. Moving the polarizers together, This way the reference frame of the polarizers remains orthogonal and the Sellotape is orientated in this frame by the wanted angle θ. Changing the angle of the first one from $0 - \frac{\pi}{2}$ and the second one from $\frac{\pi}{2} - \pi$, we can show the phase change of the beam. Measuring the light intensity I_f of the transmitted beam as a function of the first polarizer angle θ_1 we can see the Birefringence behavior and compare it to the theoretical development. In our experiment the width of the duct tape is $d = 50\ [\mu m] \pm 0.1\%$, $\lambda = 650\ [nm]$, $K = \lambda$ is the explicit wavelength - 650 nm and R is the Retardation factor. The final equation that will be used to the experiment is:

$$I_f = \frac{1}{4} I_0 sin^2(2\theta)\left(1 - cos(2\pi\frac{R}{\lambda})\right)$$

After testing the equation we got a linear fit with the transformed into a straight line y = mx + n (17) where m = 1 2 I0 cos (18) and n = 1 2 I0. (19) From equations (18) and (19), it is possible to obtain the value of cos from intensity measurements as follows: cos = m n Due to the restrictions

of equipment we cannot rotate the Sellotape strip, which will be mounted on one of the holders. If the polarizers are at 90○ between them then the measured intensity depends only on the angle of the Sellotape to initial polarizer.



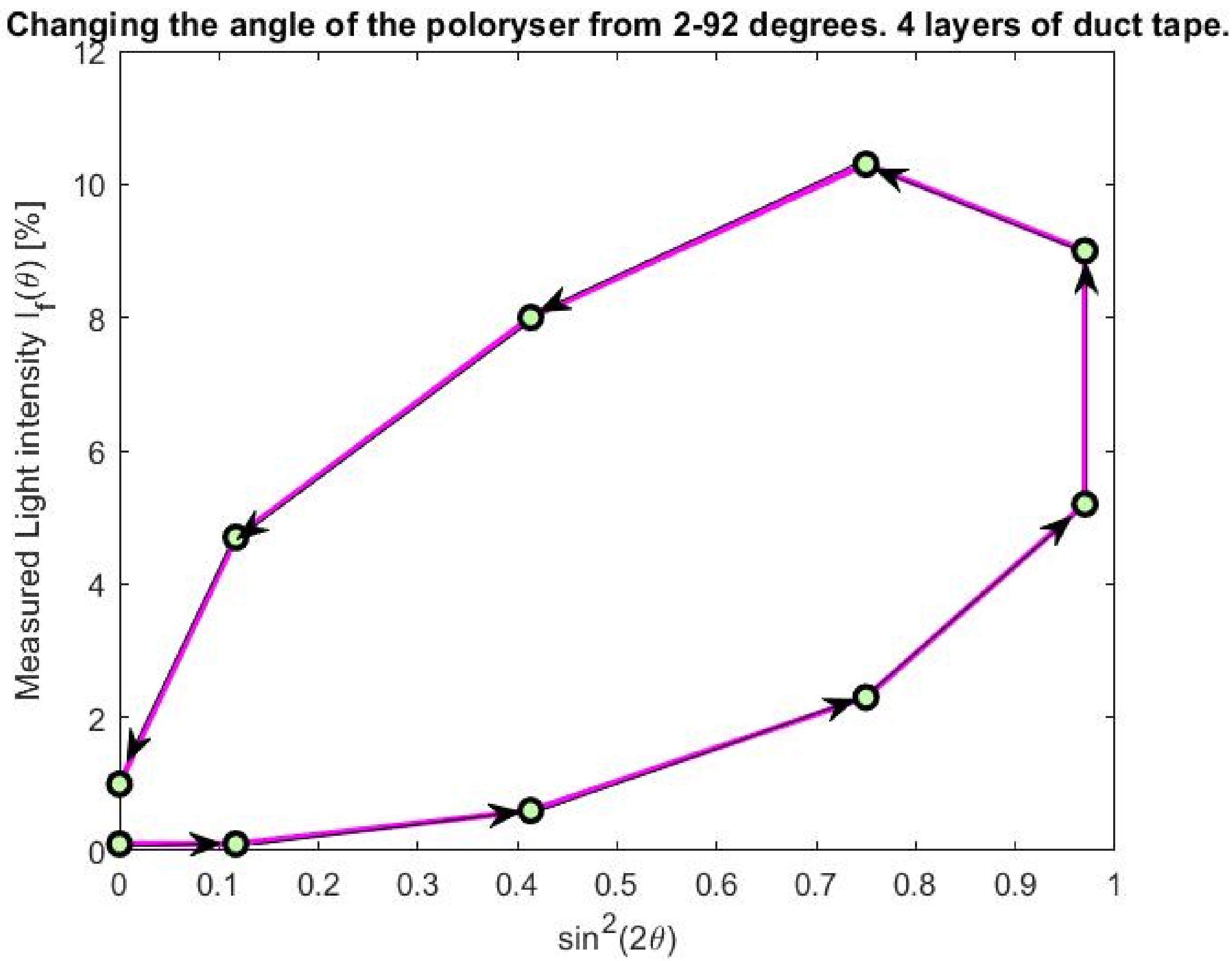
Natural sun light and most of the artificial illuminations transmits light waves. The electric field vectors vibrate in all perpendicular planes with respect to the direction of light. When the electric field vectors are restricted to a single plane, then the light is polarized with respect to the direction of the light source. all waves vibrate in the same plane.

The incident light electric field vectors vibrate perpendicular to the direction of the light source in an equal distribution of all planes before encountering the first polarizer. The duct tape is behaving like a polarizer. This specific materials are actually filters containing long-chain polymer molecules that are oriented in a single direction. Only the incident light that is vibrating in the same plane as the oriented polymer molecules is absorbed.

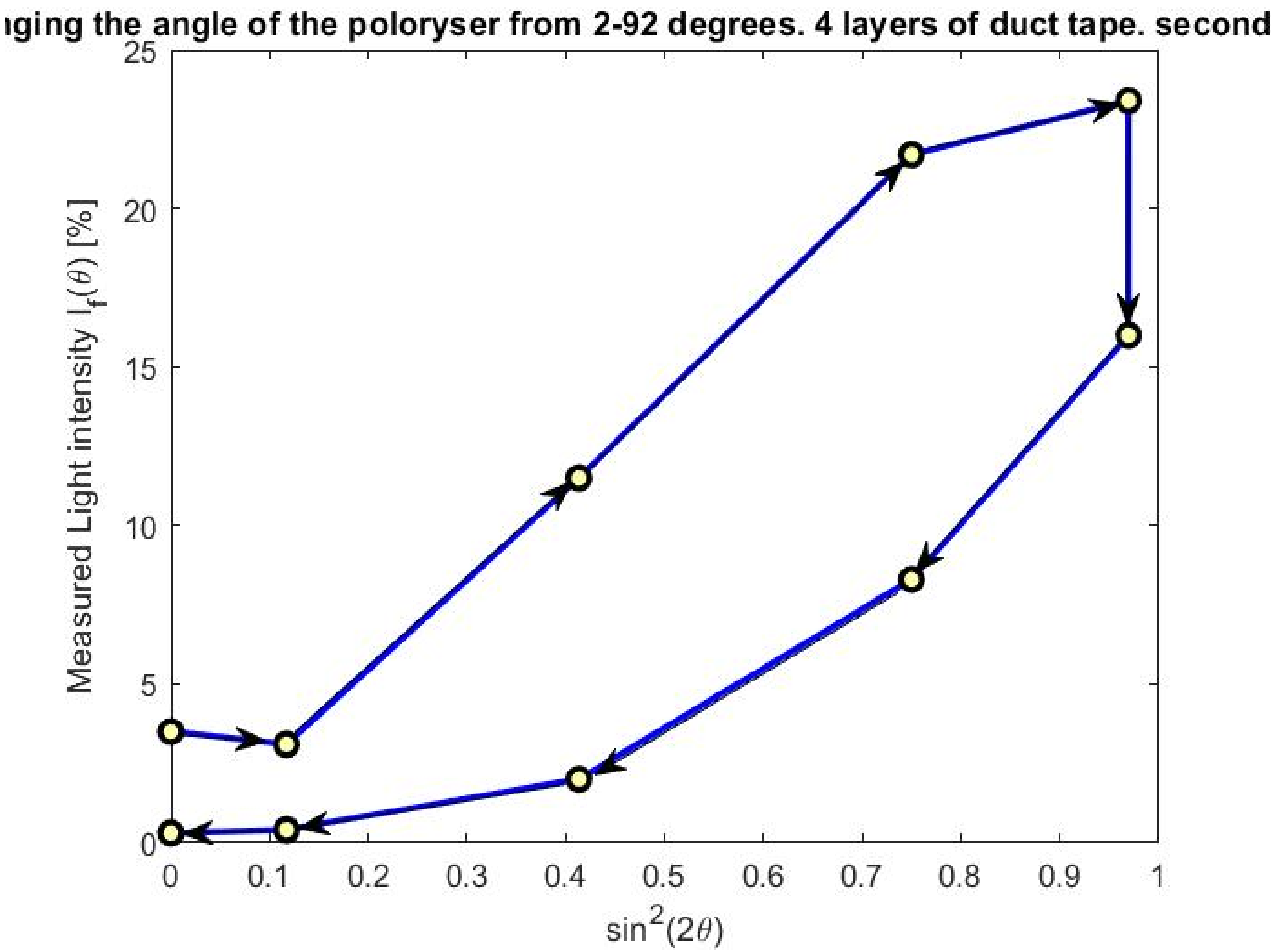
4 Measure Course experiment number one

lighting the 650 [nm] on the optical trail with rotating polarizers left and right to the 4 layers duct tape stripe. Starting from 2○ up to 92○ while the rotation of the polarizers is synchronized. Measuring the light intensity as a function of $sin^2(2\theta)$ and we get a close loop.

5 Results experiment number one



In the second attempt:



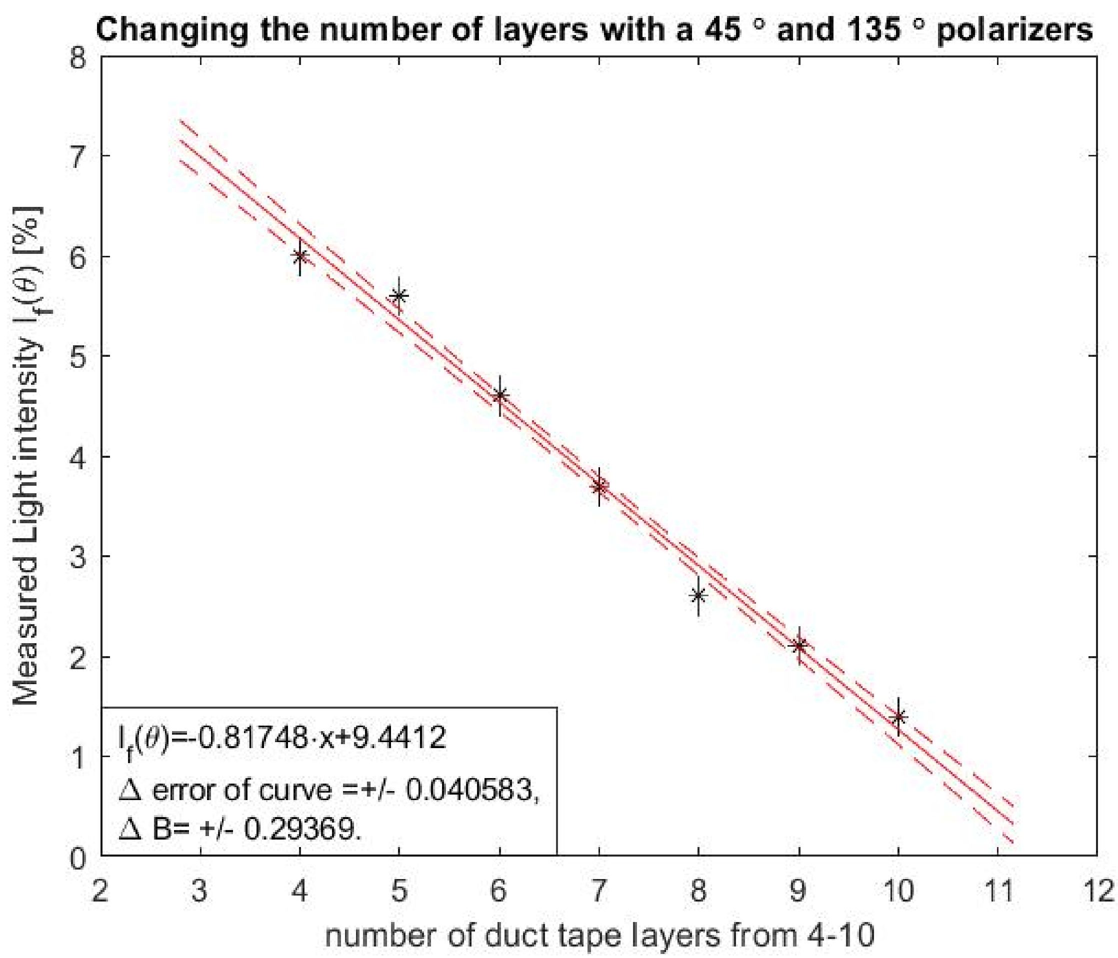
6 Measure Course experiment number two

lighting the 650 [nm] on the optical trail with fixed polarizers at angles of 45○ and 135 left and right to the 4 layers duct tape stripe. Starting from 4 layers up to 10 layers of duct tape. Measuring the light intensity as a function of the number of strips and we get the linear equation.We can calculate R the Retardation factor.

7 Results experiment number two

With this graph we could able extract the equation using our fitting tools.

In this graph we can see the correlation between the number of layers and light intensity. the slop of the fitted equation represents



Extracting the linear equation from the results we find it to be:

$$I_f(\theta) = -0.81748 \cdot x + 9.4412$$

Compering the equation to the expected intensity:

$$I_f = \frac{1}{4} I_0 sin^2(2\theta)\left(1 - cos(2\pi \frac{R}{\lambda})\right)$$

As we discussed before, if the two polorizers are at $\frac{\pi}{2}$ between them so the light intensity will be depended on the angle of the first polorizer. In our experiment the angle was $\frac{\pi}{4}$ for convenience so that $sin^2(2\theta) = 1$, the equation becomes:

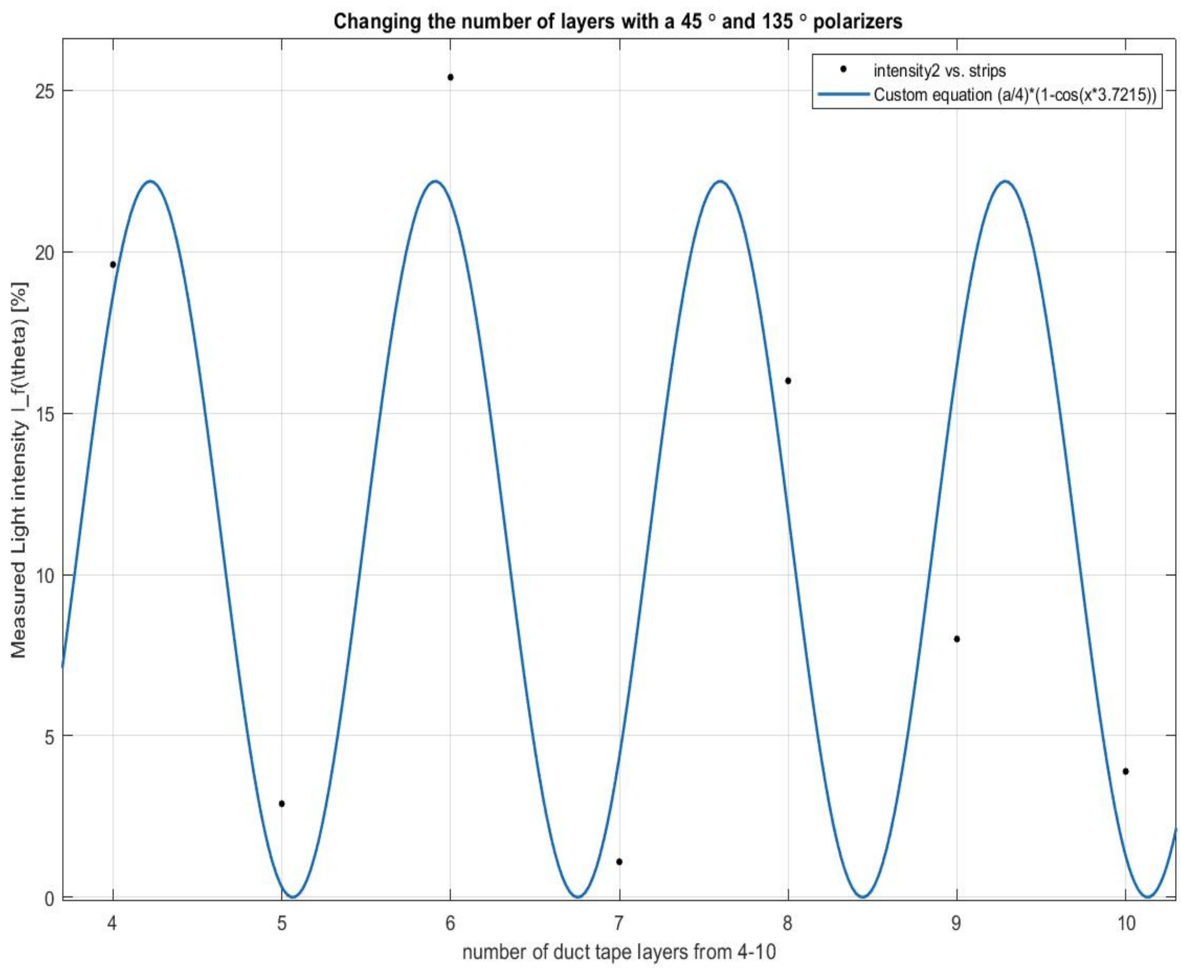
$$I_f = \frac{1}{4} I_0 \left(1 - cos(2\pi \frac{R}{\lambda})\right)$$

One can notice that a linear equation and the equation above can not correlate with each other. only by testing the part of the equation $cos(2\pi \frac{R}{\lambda})$ with different values of $R = \Delta n \cdot d$ we can understand that the equation dose not act like a linear one. d is the thickness of the material, adding more layers multiplies d with the amount of layers, the result of a theoretical graph is nothing like a linear one. Looking at the graph we got we can infer that we had lots of problems in the measurement. We took the experiment again to be sure of the results, the graph we got was nothing like a linear one. Calculating the Retardation factor with results from another experiment found online and compering this to the second attempt. The Retardation factor is : $R = \Delta n \cdot d$ the thickness of the film is $d = 50 \pm 0.1$ $[\mu m]$ The refractive index of cello-tape Δn in wave lengths of 650 [nm] is approximately 0.0077. It is important to understand that the refractive index was taken from a similar experiment. In our experiment we cant find the refractive index because we had to do other measurements to obtain it. to find the Retardation factor we only need to input those numbers together and get

$$R = \Delta n \cdot d = 0.0077 \cdot 50 \cdot 10^{-6} = 0.385[\mu m]$$

We found the Retardation factor of 1 layer of duct tape, Multiplying R_1 by the number of duct tape layers we can get the Retardation factor for every d(x).

In the second attempt we used 10 layers of duct tape again. The angles are fixed with angle of 45○ and 135○ respectively. In this measurement at first we thought that we had it wrong but looking at the fit we created with the refractive index 0.0077, we found it close to the truth.



8 Discussion

In this experiment we propose and demonstrate a very simple method to fabricate interference multilayer birefringent filters. We employ the birefringence properties of common cellotape. Cellotape tape layers can be very easily superimposed with different orientations in order to generate different spectral responses.

9 conclusion

A simple system consisting of a strip of cellotape placed between two polarizers was used to analyze the behaviour of this cellotape layer as a birefringent retarder. The system was firstly theoretically analyzed using Jones matrices, which allows us to easily obtain an expression for the intensity transmitted by the whole system as a function of the angles between the optic axis of the retarder and the transmission axis of the analyzer, respectively, and the transmission axis of the polarizer.

Looking at the two measurements we did we infer that this experiment is not enough to predict the Retardation factor and to obtain the duct tape optical behavior.

Riferimenti bibliografici

[1] [Birefringence of cellotape: Jones representation and experimental analysis by Augusto Belendez, Elena Fernandez, Jorge Frances and Cristian Neipp https://physlab.lums.edu.pk/images/3/30/Cellophane2.pdf

[2] Using cellophane tape to experience interference birefringent filters D. Puerto1, P. Velásquez1,2, M. M. Sánchez-López2,3, I. Moreno1 and F. Mateos1 1. Dept. Ciencia y Tecnología de Materiales, Univ. Miguel Hernández. 03202 Elche, Spain.
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https://spie.org/etop/ETOP2005_088.pdf

[3] Birefringence of cellotape: Jones representation and experimental analysis. https://physlab.lums.edu.pk/images/3/30/Cellophane2.pdf