Faculty of Physics - 24 giugno 2021

modern physics lab course

Millikan experiment. Finding the change of an electron.

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Indice

1 Theoretical introduction

The purpose of this experiment is to measure the charge of the electron . The experiment is called "Millikan's experiment" . Millikan was the first physicist to measure the charge of the electron . A drop of oil is entered into an apparatus , there a radioactive source produces emission of alpha particles that meats the oil drop. A single droplet captured an ion and changed its charge. When a neutral substance emits particles of positive charge it gets ionized in negative charge . Modern physics states that this net total charge is an integer multiple of the charge of the electron $e = 1.6 \cdot 10^{-19}[C]$. In order to find out if this is so , one needs to be able to measure the amount of electric charge on an oil drop that its mass is unknown.

The first step is to calculate the velocity of the oil drop when it is free falling and the force of gravity is equal to the drag force that opposes its fall . The second step is to construct the expression for the forces acting on the ionized drop under the effect of a known electric field with a specific magnitude $E\ [\frac{N}{m}]$. Having these two equations we can express the charge of the oil drop without any dependency on the drag coefficient $\kappa\ [\frac{kg}{s}]$

$$mg = \kappa v_f$$

Equation for free falling un-ionized drop. Where v_f is the velocity of fall

$$Eq = mg + kv_u$$

Equation of motion for the ionized drop. Where v_u is the velocity of movement in the direction of the applied E field . Our apparatus contains a lens that allows us to inspect the motion of the oil drop in both cases, There are ticks on the viewing screen and both velocities can be measured with a stop watch. If we acquire the velocites via measurement we have an analytical expression for the charge .

$$q = \frac{mg \cdot (v_u + v_f)}{E \cdot v_f}$$

The expression for the mass of the oil drop is needed in order to find out the charge. The mass is very hard to measure and therefore we will approximate that the density of mass ρ is constant ,stating that the fluid uncompressible.

$$m = V \cdot \rho = \frac{4}{3} \cdot \pi \cdot a_r^3 \cdot \rho$$

We have used the expression of a symmetrical sphere for volume , where a_r is the radius of the sphere . Stokes's law gives an expression to calculate the radius of a sphere.

$$a = \sqrt{\frac{9 \cdot \eta \cdot v_f}{2 \cdot \rho \cdot g}} \quad [mm]$$

In our measurement the range of velocities was between 0.1 $[\frac{mm}{s}]$ to 0.5$[\frac{mm}{s}]$. Since this range of velocities is under 1 $[\frac{mm}{s}]$ there is an additional expression that needs to be added to Stoke's theorem in order for us to be accuarate

$$\eta \longrightarrow \eta \cdot \frac{1}{1 + \frac{b}{p \cdot}} [\frac{kg}{m \cdot s}]$$

$$b = const = 8.2 * 10^{-3} \frac{Pa}{m^2}$$

The final expression for the charge q is :

$$q = \frac{4\pi}{3} \cdot [\sqrt{(\frac{b}{2p})^2 + \frac{9\eta v_f}{2\rho g}} - \frac{b}{2p}]^3 \cdot \frac{\rho g d(v_f + v_r)}{V v_f}$$

2 Introduction to the system of the experiment.

Parts Diagram

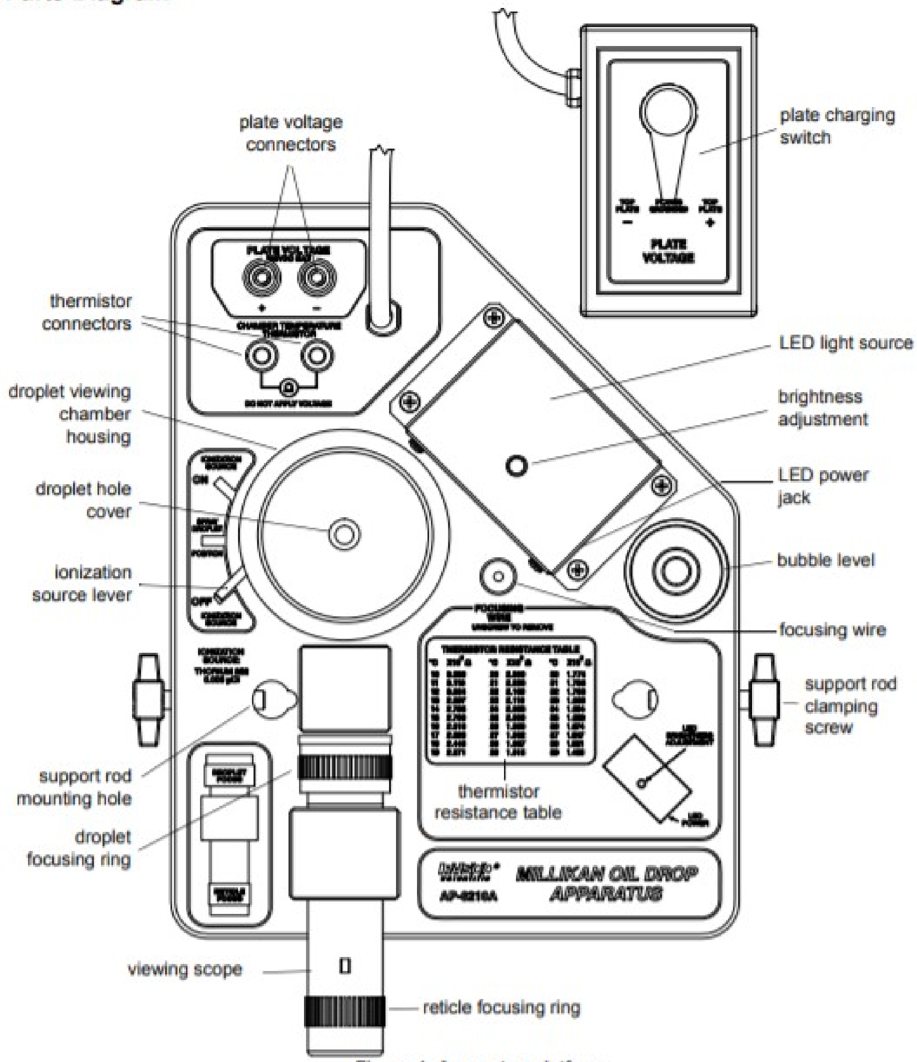
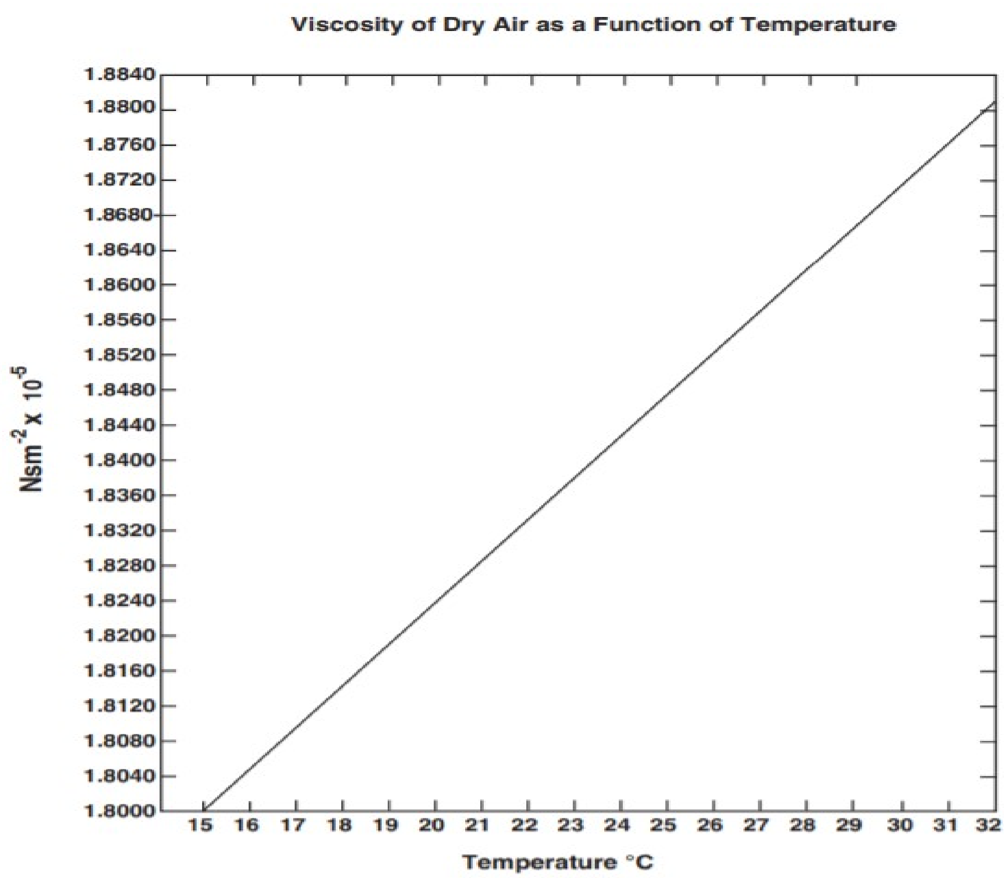


Figure 4: Apparatus platform

To create the experiment system we used several devises as shown in figure 4.

The system includes:

1.) Digital Multi meter to measure voltage and resistance. Taking the resistance from the Multi meter and calculating the temperature using the thermistor table and then taking the viscosity constant with an estimate error of $\Delta\eta = 0.004 \cdot 10^{-5}$ $[\frac{kg}{m \cdot s}]$ using the graph below.
2.) Digital Stopwatch to calculate the time it takes for a drop of oil to cross a constant distance. With an estimated error of $\Delta t = 0.1$ $[sec]$ and $\Delta d = 0.02\ m$ $[m]$
3.) power supply, high voltage, well-regulated, 500 VDC, 10 mA (minimum). With an estimated error of $\Delta V = \pm 1$ $[v]$
4.) Viewing scope (30X, bright-field, erect image).
5.) Reticle focusing ring, and droplet focusing ring.
6.) Nonvolatilemineraloil,120mL(4ounces)
7.) AC Adapter,100240VACto12V DC,1.0A



3 Experiment course

At first we opened the light source to be able to see the plaid paper inside the Droplet Viewing chamber. After focusing the Viewing Scope we can use the net of the plaid paper to measure the movement of the oil drops between the lines. We switched on the power supply to 500V and calculated the existing resistance, by knowing the resistance we can evaluate the exact temperature of the system. In this experiment the goal is to measure the charge of the electron, to do so we built a system of oil drops that are ionized and caring a constant charge. In the mathematical development we saw that all we actually need to measure is the speed of the drops. At first we sprayed the oil to the chamber and measured there free fall speed, after that we measured the speed of the drops while a constant electric field is applied inside the chamber, via the capacitor that is charged with a constant potential of $V = 400\ [v]$. After measuring the time it took the drop to Cross a constant distance of $d = 0.1\ m\ [m]$ we took the average time and and used the simple kinematic equation:

$$V_f = \frac{d}{t_{average}}$$

$$V_r = \frac{d}{t_{average}}$$

After calculating the speed we entered the data to the formula and calculated the charge of the drop:

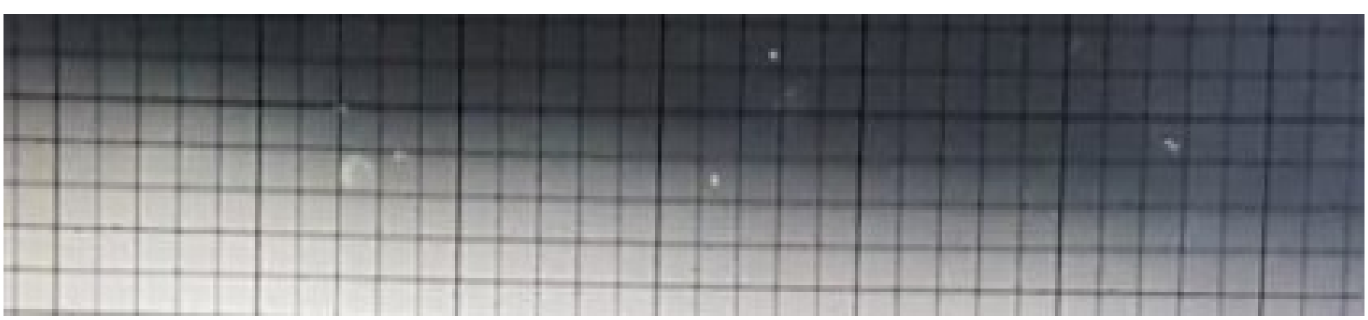
$$q = \frac{4\pi}{3} \cdot [\sqrt{(\frac{b}{2p})^2 + \frac{9\eta v_f}{2\rho g}} - \frac{b}{2p}]^3 \cdot \frac{\rho g d(v_f + v_r)}{V v_f}$$

Dividing the charge of the drop with the esstimated charge of the electron

$$\frac{q}{e} = k\ , \quad e = 1.6 \cdot 10^{-16}$$

$$k \in Z$$

We got a Integer means that the drop is charged with a finite numbers of electrons.
In the picture below we can see inside the chamber and have a glans of the actual drops.



4 Results

In this experiment we took 5 oil drops and calculated the number of electrons that make up the drop charge. The results of the experiment are shown in the table below:

drop number 1	drop number 2	drop number 3	drop number 4	drop number 5
$e_1 = 17.341$	$e_2 = 9.801$	$e_3 = 11.032$	$e_4 = 13.873$	$e_5 = 16.992$

The measurements of the total deviation of the number of electrons that make up the drop charge are taken for the fifth oil drop because it is the closest result to an integer we received. In the experiment we expected to get small numbers of electrons on the drop and instead we got numbers that are between 9-17. A reasonable explanation for why we didn't get a small number like 1-3 as we expected is because we measured only the big and fast drops. The big drops where easy to measure because we saw them clearly and where able to control them with the electric field, which implies That the big drops where also ionized with a total charge that is bigger then 1-3 electrons hence the the fast speed accord due to the force of the electric field implied on them.The bigger drops got more electrons this is the reason why we got those numbers.Let us calculate the deviation of the fifth drop.

$$\text{The derivative with respect to } V_f \text{ shows :}$$

$$\frac{\partial}{\partial\ x}\left(\frac{4\pi}{3}\cdot\left(\sqrt{b+a\ \cdot(x\cdot\ \ y)}-c\right)^3\cdot\frac{d\,(x+e)}{x}\right)=$$

$$\frac{2\pi d\left(3axy\left(-c+\sqrt{axy+b}\right)^2(x+e)-2e\left(-c+\sqrt{axy+b}\right)^3\sqrt{axy+b}\right)}{3x^2\sqrt{axy+b}}$$

$$b=(\frac{b}{2p})^2=(c^2)=(4\cdot 10^-8)^2=1.6\cdot 10^{-15}[m]$$

$$a=5.18\cdot 10^-4[\frac{m^2\cdot s^2}{kg}],y=18.39\cdot 16^{-6}[\frac{kg}{m\cdot s}]$$

$$d=0.17\frac{kg}{ms^2V},e=v_r=1.85\cdot 10^{-4}[\frac{m}{s}]$$

Where the average value of $x = V_f$ for the fourth particle is calculated from Matlab and its deviation is shown :

$$\Delta V_f=\sqrt{(\frac{\partial V_f}{\partial d}\cdot\Delta d)^2+(\frac{\partial V_f}{\partial t_{ave}}\cdot\Delta t_{ave})^2}=9.827\cdot 10^{-6}[\frac{m}{s}]$$

and $\Delta V_f \cdot \frac{\partial q}{\partial V_f} = \cdot[c]$
The derivative with respect to η shows:

$$\frac{\partial}{\partial\ n}\left(\frac{4\pi}{3}\cdot\left(\sqrt{b+a\ \cdot(x\cdot\ \ n\ )}\ -c\right)^3\cdot\frac{d\,(x+e)}{x}\right)\frac{2\pi ad\,(x+e)\left(-c+\sqrt{anx+b}\right)^2}{\sqrt{anx+b}}$$

Where $x = v_f$ and $n = \eta$
The derivative with respect to v_r where r is v_r shows :

$$\frac{\partial}{\partial\ r}\left(\frac{4\pi}{3}\cdot\left(\sqrt{b+a\ \cdot(x\cdot\ \ n\ )}\ -c\right)^3\cdot\frac{d\,(x+r)}{x}\right)=\frac{4\pi d\left(\sqrt{b+anx}-c\right)^3}{3x}$$

$$\Delta V_f \cdot \frac{\partial q}{\partial V_f} = 1.55 \cdot 10^{-21} = a_1[c]$$

$$\Delta\eta \cdot \frac{\partial q}{\partial \eta} = 6.6 \cdot 10^{-21} = a_2[c]$$

$$\Delta V_r \cdot \frac{\partial q}{\partial V_r} = 5.6 \cdot 10^{-}20 = a_3[c]$$

$$\Delta q = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2} = e/3[c]$$

It should be noted that the deviation for η was approximated using the graph in the second figure . And we approximated the same deviation for V_r and V_f because the elements that cause deviation are the same for both of these parameters . The final calculation gives that the deviation of the charge we measured is:

$$\Delta q/q = \frac{e/3}{16.992 \cdot e} \approx 2\%$$

5 Discussion

The deviation of approximately 2 % is the deviation of the charge which means that the total deviation of the number of electrons found in the drop is the same 2 %.

The result implies that it is possible that we have measured an oil drop that has been ionized with a total charge of 17e . But the result does not show that the charge on the oil drop has to be a quantified amount of discrete charge, the only way to show this is to make sure that the oil drop is ionized with exactly one charge and perform the experiment many times until a certain expectation value is reached which should be the electron charge , this way it would prove that the lowest charge possible c for sure.

6 conclusion

In this experiment we saw the Millikan oil drop platform that allowed us to measure the number of electrons in a drop of ionized oil. The conclusion in this experiment is that the lowest charge possible from the ionization process is the charge of electron e. Also we found out that in the ionization process of the oil drops the charge q of the oil drop will always be an integer number of electrons. In the process of measurements we found out after measuring only the big and fast drops that The bigger drops got more electrons then the smaller ones due to the ionization. By measuring the velocity that the drop made at a "free fall" state and " Active force" state and taking the time with a stop watch we got to a results that where satisfying. The total estimation of our deviation is 2 % which is good iven all the changing factors of the experiment. To conclude we saw in this experiment how to calculate the charge of the electron using a complex model of ionized oil droplets.