Mid3 15-21

L15_Rotational Kinematics and Moment of Inertia

L16_Parallel Axis Theorem and Torque

L17_Rotational Dynamics

L18&19_Rotational Statics I & II

L18&19_Rotational Statics I & II

L20_Angular Momentum

L21_Angular Momentum Vector and Precession

Formula check

Energy

- Rotational energy

$$E_{rot} = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

- Total energy

$$E_{total} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

Moment of inertia

- **Thin/Slender Rod**
  - $$
  I=\frac{1}{12}ML^2\ (axis\ is \ the \ symmetric \ axis)

$$-$$

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I=\frac{1}{3}ML^2\ (axis\ is \ one \ end )
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$$-$$

I=\frac{M}{3}(L^2-3Lh+3h^2)

$$-Herehisthedistancefromtheaxistotheleft andListhelengthoftherod$$

- **Disk/Solid Cylinder/Puck**

$$I = \frac{1}{2}MR^2$$

- **Hoop/Block**

$$I = mR^2$$

- **Sphere**

$$I = \frac{2}{5}mR^2$$

- **Shell**
  - Cylinderical Shell

$$I = mR^2$$

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- Spherical Shell
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$$I = \frac{2}{3}mR^2$$

Torque

$$T = Fr = r \times f = I\alpha$$

Accelaration

1. Tangential component: 切向分量 α is the rotational acceleration

$$a_t = r\alpha$$

2. Radial component: 径向分量

$$a_r = r\omega^2$$

Angular Momentum

- We use `L` to represent angular momentum, which equals to the cross product of the position vector `r` and the translational momentum `p`, and `p` is `mv`
- $$L = r \times p = I\omega$$

Precession

- $$\Omega = \frac{\tau_{external}}{L}$$

- $$Period = \frac{2\pi}{\Omega}$$

Right hand rule

1. To tell the direction of ω, curl the fingers as the direction of rotation, the direction of the thumb is the answer
2. To tell the direction of the angular accleration α, if the ω is increasing, then α is to the same direction of ω
3. To tell the direction of the angular momentum L, same as the ω ,so curl the fingers as the direction of rotation, the direction of the thumb is the answer
4. To tell the direction of the torque τ , point the fingers from the axis to the action point, curl the fingers in the direction of the force, the direction of the thumb is the answer
5. To tell the direction of the `precession` , tell by how the angular momentum changes, as shown in the image.

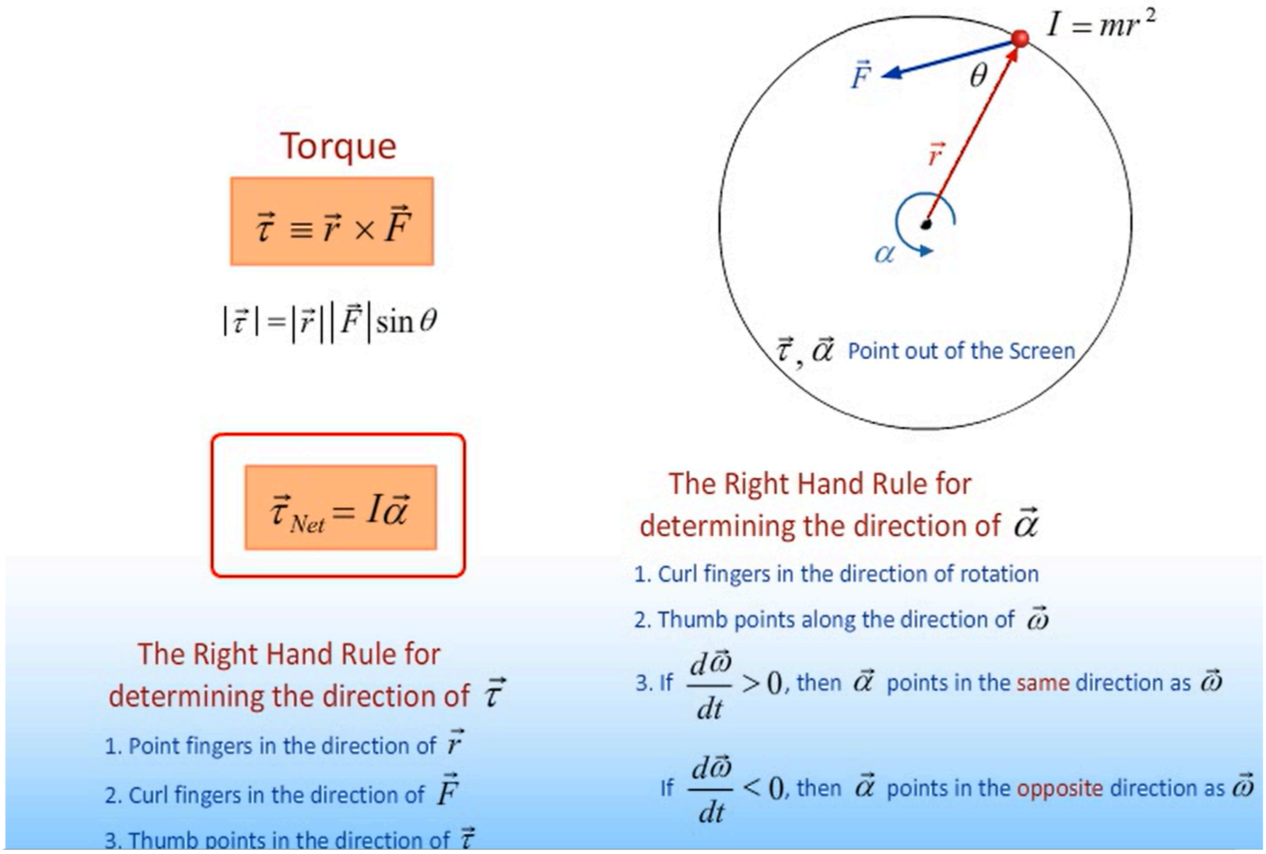

image-29.png

Conclusion Check

Parallel Axis Theorem

$$I_{new} = I_{cm} + Md^2$$

- Note that in a system, the I_{cm} is the respective I when rotating about their own center of mass

F=ma

- Calculating the acceleration of the system
  The core is $a = \frac{F}{m}$ and m for the pulley is CM where C is the coefficient in I, Note that if there is a sphere, need to consider its translational mass and rotational mass, so its effective mass is $\frac{7}{5}m$
- Note that the pulley makes the tension of both sides the string different.

Rolling without slipping

- $v = \omega R$ is true only when the object is rolling without slipping
- The equivalent mass of a sphere when it is rolling without slipping is $\frac{7}{5}m$