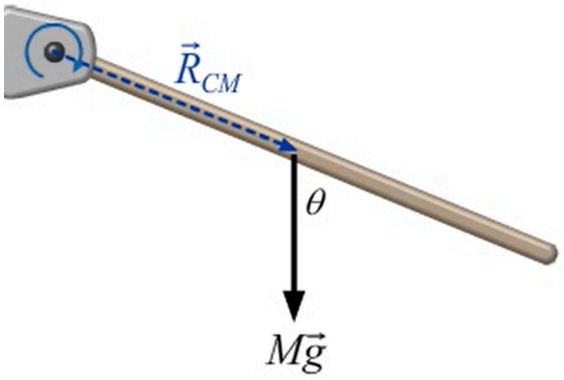
L18&19_Rotational Statics I & II

- [ ] SP_U_20

Torque by gravity

- Consider the torque due to gravity acts on the object as at the cm
  - $\tau = R_{cm}Mgsin\theta$



$$\vec{\tau}_{gravity} = \vec{R}_{CM} \times M\vec{g}$$ Torque by the Gravitational Force

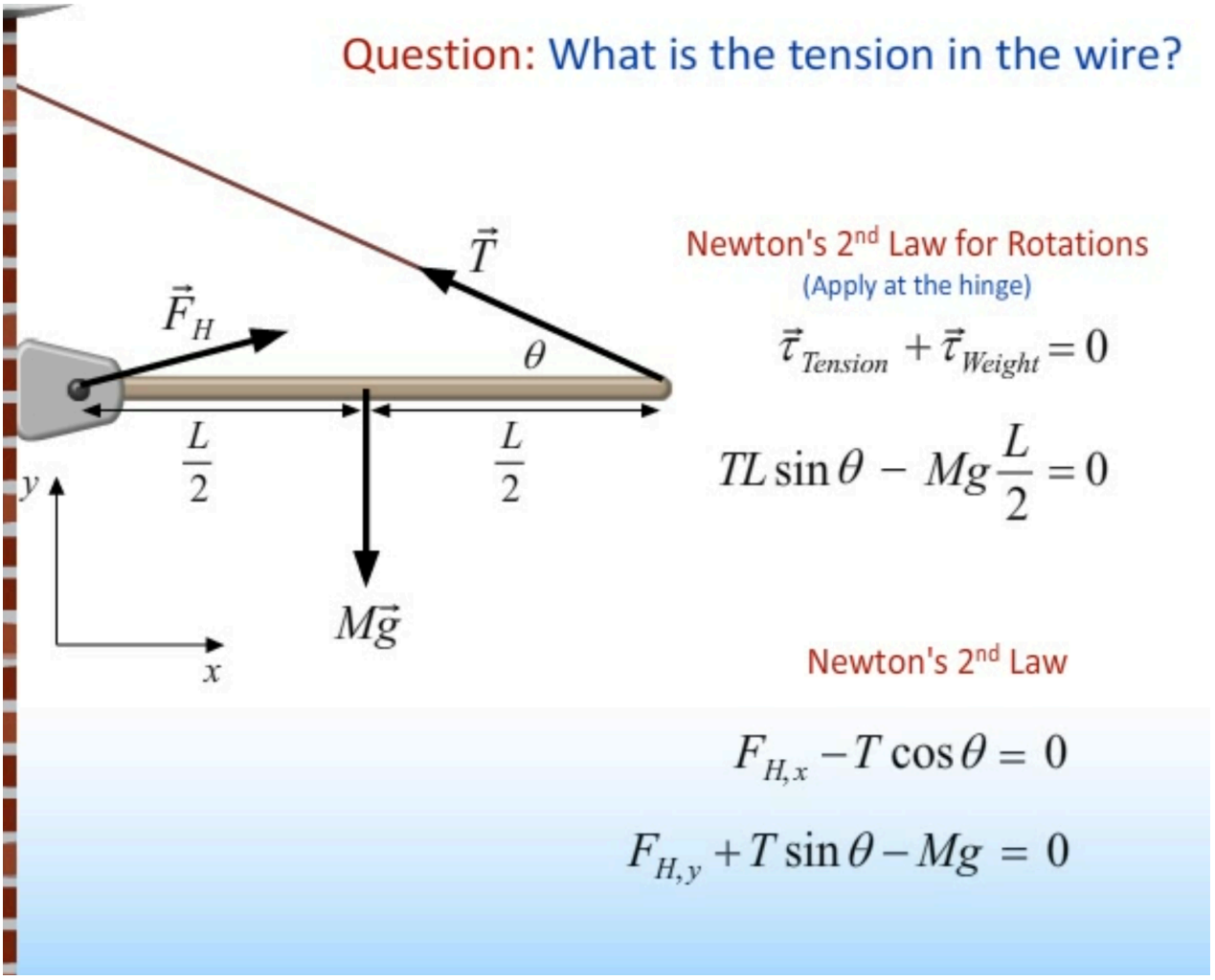
Note that here the multiplication is `cross multiplication`, which is expressed to $absin\theta$

We can keep simplifying this expression by changing $R_{CM}sin\theta$ into the horizontal length between the cm and the rotating axis. Furthermore, the length does not have to be horizontal if we are not considering gravity.

Solution strategy to a static system

1. Use Newton's 2_{nd} Law
2. Use the $\tau = 0$ to find other equations, note that here you should use the right hand rule to determine the direction of torques

3. Example:



- If the wire broke: the only torque is caused by the gravity, so that we can get an expression for α, which is not a constant $\tau = I\alpha$, express τ and I to get α

Torque of a box on the truck

Question: What is the maximum acceleration the truck can have before the box tips over?

$f = Ma$

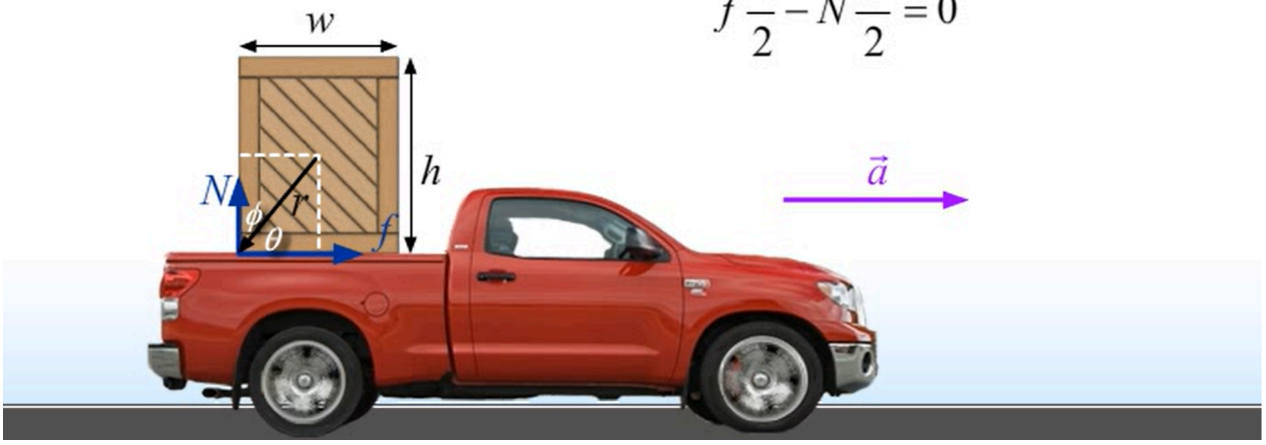
$N = Mg$

Newton's 2nd Law for Rotations

$\vec{\tau}_{friction} + \vec{\tau}_{normal} = 0$ (about an axis through the Center of Mass)

$f(r\sin\theta) - N(r\sin\phi) = 0$

$f\frac{h}{2} - N\frac{w}{2} = 0$

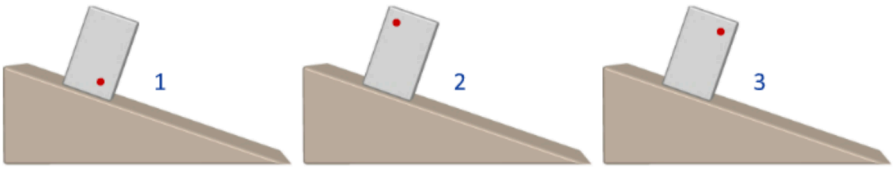


1. Note how we analyze at the tip in this problem

Stability of an object

1. If the cm is inside the footprint of an object, it stays stable
2. If the cm is outside the footprint of an object, it falls down so that it can lower its cm
3. In other words, if the object can fall down without an increament in cm, then it falls

Three boxes are placed on identical ramps as shown below. Friction prevents them from sliding. The center of mass of each box is indicated by a red dot. In which case(s) does the box tip over?



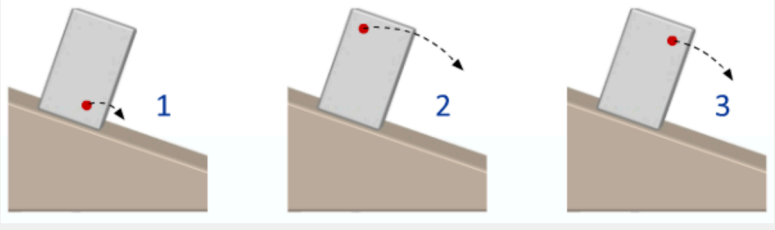
- none
- both 2 and 3
- only 3

Submit

Your submissions: C ✓

Submitted: Monday, April 21 at 11:20 AM

Feedback: The box will tip over if it can lower its center of mass by doing so. In both cases 1 and 2 the CM will have to move up a bit before moving down as the box tips, so these are both stable. In case 3 the CM will move down as soon as the box starts to tip, so it will be unstable.



其实这个内容讲的是物体的稳定性，和物体的重心位置有关。简单来说，如果物体的重心在它的支撑面（接触地面的部分）内，物体就稳定；如果重心超出支撑面，物体会倾倒。

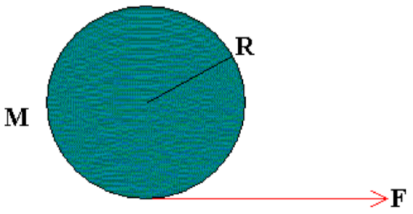
从图片上看，三个盒子放在斜坡上：

- **情况1和2**：盒子的重心位置在支撑面内，即使盒子开始倾斜，重心会先稍微升高再降低，所以它是稳定的，不会倾倒。
- **情况3**：盒子的重心位置在支撑面外，只要盒子开始倾斜，重心就会直接下降，所以它是不稳定的，会倾倒。

你可以想象一下推倒一个杯子：如果杯子的重心在底座范围内，你推它的时候它会摇晃但不会倒；如果重心超出底座范围，它就会倒下。

总结一下：只有当物体的重心超出支撑面时，物体才会倾倒。

Good questions



A puck rests on a horizontal frictionless plane. A string is wound around the puck and pulled on with constant force. What fraction of the disk's total kinetic energy is due to the rotation?

KE_{rot}/KE_{tot}

1. 2/3

在无摩擦平面上，冰球受拉力F作用，同时产生平动和转动。关键点在于无需假设线加速度a与角加速度α之间存在关系（如a=αR），因为无滑动条件不适用。正确推导如下：

1. **平动方程**：$F = ma \Rightarrow a = \frac{F}{m}$
2. **转动方程**：转矩$\tau = FR = I\alpha \Rightarrow \alpha = \frac{FR}{I}$
3. **动能计算**：
   - 平动动能：$K_{\text{trans}} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{F}{m}t\right)^2 = \frac{F^2t^2}{2m}$
   - 转动动能：$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{FR}{I}t\right)^2 = \frac{F^2R^2t^2}{2I}$
4. **能量比例**：

$$\frac{K_{\text{rot}}}{K_{\text{total}}} = \frac{\frac{F^2R^2t^2}{2I}}{\frac{F^2t^2}{2m} + \frac{F^2R^2t^2}{2I}} = \frac{R^2/I}{1/m + R^2/I}$$

5. **代入正确转动惯量**（冰球视为圆盘，$I = \frac{1}{2}mR^2$）：

$$\frac{R^2/(\frac{1}{2}mR^2)}{1/m + R^2/(\frac{1}{2}mR^2)} = \frac{2/m}{3/m} = \frac{2}{3}$$

错误根源：用户误用$a = \alpha R$导致错误转动惯量$I = mR^2$。实际上，无摩擦条件下该关系不成立，应分别由牛顿定律和转动定律独立求解。

答案：旋转动能占总动能的$\boxed{\frac{2}{3}}$。