L15_Rotational Kinematics and Moment of Inertia

Rotational Kinematics

Formulas

1. Note the comparisons between linear motion (translation motion) and angular motion

Linear Motion	Angular Motion
$d = vt$	$\theta = \omega t$
$v_f = v_0 + at$	$\omega_f = \omega_0 + \alpha t$
$v_f^2 = v_0^2 + 2ad$	$\omega_f^2 = \omega_0^2 + 2\alpha\theta$
$d = v_0 t + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$d = \frac{1}{2}[v_0 + v_f]t$	$\theta = \frac{1}{2}[\omega_0 + \omega_f]t$

2. Related linear parameters to angular ones

- $d = \theta R$
- $v = \omega R$
- $a = \alpha R$

2. Direction:

- The object rotates in a clockwise direction, ω negative
- The object rotates in a counterclockwise direction, ω is positive

Rotational Energy

- $$E_{rot} = \frac{1}{2}mv^2 = \frac{1}{2}m(\omega r)^2 = \frac{1}{2}mr^2(\omega)^2 = \frac{1}{2}I\omega^2$$
- Note that the definition of I is different for different object

Moment of inertia

- **Thin/Slender Rod**

- Axis is the symmetric axis

$$I = \frac{1}{12}ML^2$$

- Axis is one end

$$I = \frac{1}{3}ML^2$$

- Here h is the distance from the axis to the left and L is the length of the rod

$$I = \frac{M}{3}(L^2 - 3Lh + 3h^2)$$

- **Disk/Solid Cylinder/Puck**

$$I = \frac{1}{2}MR^2$$

- **Hoop/Block**

$$I = mR^2$$

- **Sphere**

$$I = \frac{2}{5}mR^2$$

- **Shell**
  - Cylinderical Shell

$$I = mR^2$$

  - Spherical Shell

$$I = \frac{2}{3}mR^2$$

System

- Add every component's moment of inertia up