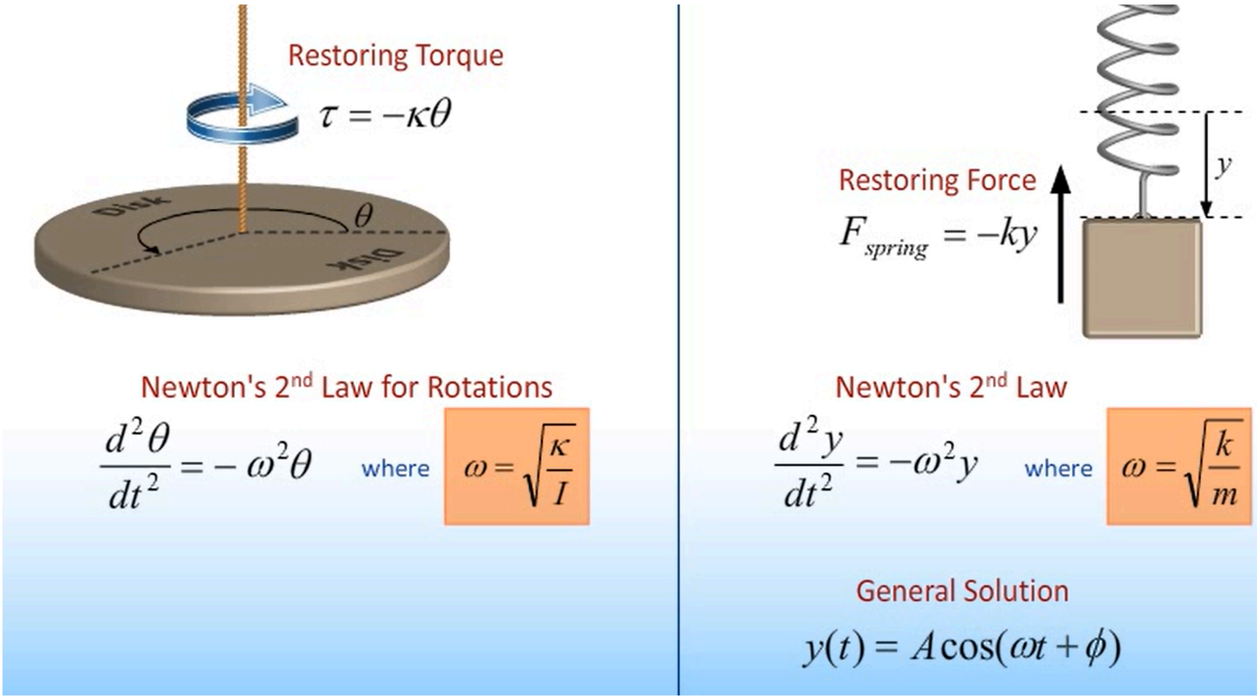
L23_Simple and Physical Pendula

Torsion Pendulum

1. Formula



2. Here κ is the torsion constant $T = 2\pi\sqrt{\frac{I}{\kappa}}$, we can calculate the κ using two methods
    1. When we know the balance angle, we can use the balance of torsion, where the torsion of exerted force equals to the reverse torsion, so $FR = \kappa\theta$
    2. When the system is dynamic, we apply $T = 2\pi\sqrt{\frac{I}{\kappa}}$,

Pendula

- Formula

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\omega = \sqrt{\frac{g}{L}}$$

$$\theta(t) = \theta_{\max}\cos(\omega t + \phi)$$

Physical Pendula

1. 知识点与公式

- **核心特征**：任何一个有质量有形状的刚体，绕一个非质心的转轴摆动。你的作业里是一个杆。

- **角频率 (Angular Frequency, ω)**：
- **周期 (Period, T)**：

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

- **公式解读**：
    - `m`：刚体的总质量。
    - $d(R_{cm})$：**转轴 (pivot)** 到 **质心 (center of mass)** 的距离。
    - `I`：刚体绕 **转轴 (pivot)** 的 **转动惯量 (Moment of Inertia)**。

2. 应试解题步骤（最关键的一步是求 `I` 和 `d`）

1. **确定质心位置，计算** `d`：
    - 对于均匀杆 (uniform rod)，质心在杆的几何中心 (L/2 处)。
    - `d` 就是转轴到这个几何中心的距离。
2. **计算转动惯量** `I`：用 **平行轴定理 (Parallel Axis Theorem)**。

$$I = I_{cm} + md^2$$

    - **第一步：查表得到** `I_cm`，即绕质心的转动惯量。
    - **第二步：利用平行轴定理**，把转轴从质心“平移”到实际的转轴位置。把上一步算出的 `m` 和 `d` 代入 `I = I_cm + md²`，算出最终的 `I`。
3. **计算** `ω` **或** `T`：把 `m, g, d, I` 代入公式

$$\omega = \sqrt{\frac{mgd}{I}}$$

Physical pendulum

- Formula for angular frequency

$$\omega = \sqrt{\frac{MgR_{CM}}{I}}$$

- Angular Velocity and Angular Frequency

Feature	Angular Velocity	Angular Frequency
Description of Object	Object's rotation (Rotation)	System's oscillation or periodic motion (Oscillation/Periodic Motion)
Physical Meaning	"How fast an object is currently spinning"	"The inherent 'quickness' of a system's full cycle"
Variable	Can change (can accelerate or decelerate)	For a certain system, it's a constant

Feature	Angular Velocity	Angular Frequency
Typical Example	Rotating Earth, car wheels, accelerating fan	Oscillating pendulum, spring, physical pendulum
Core Formula	$\omega = \frac{\Delta\theta}{\Delta t}$	$\omega = 2\pi f = \frac{2\pi}{T}$

In the special case of uniform circular motion, the magnitude of angular velocity is equal to angular frequency:

$$|\omega_{velocity}| = \omega_{frequency}$$

Due to this perfect correspondence, physics uses the symbol ω to represent both.

Period for any Physical Pendulum

The period (T) of a physical pendulum is given by:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MgR_{CM}}}$$

Since (I \propto M), it follows that (T) is independent of (M).

- Formula for T

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MgR_{CM}}}$$

Since $I \propto M$, T is independent of M.

Reminder

1. All the formulas above are using radian measure, not degree measure, but sometimes the initial angle would be given in degree measure, so make sure set the calculator to the right mode