L17_Rotational Dynamics

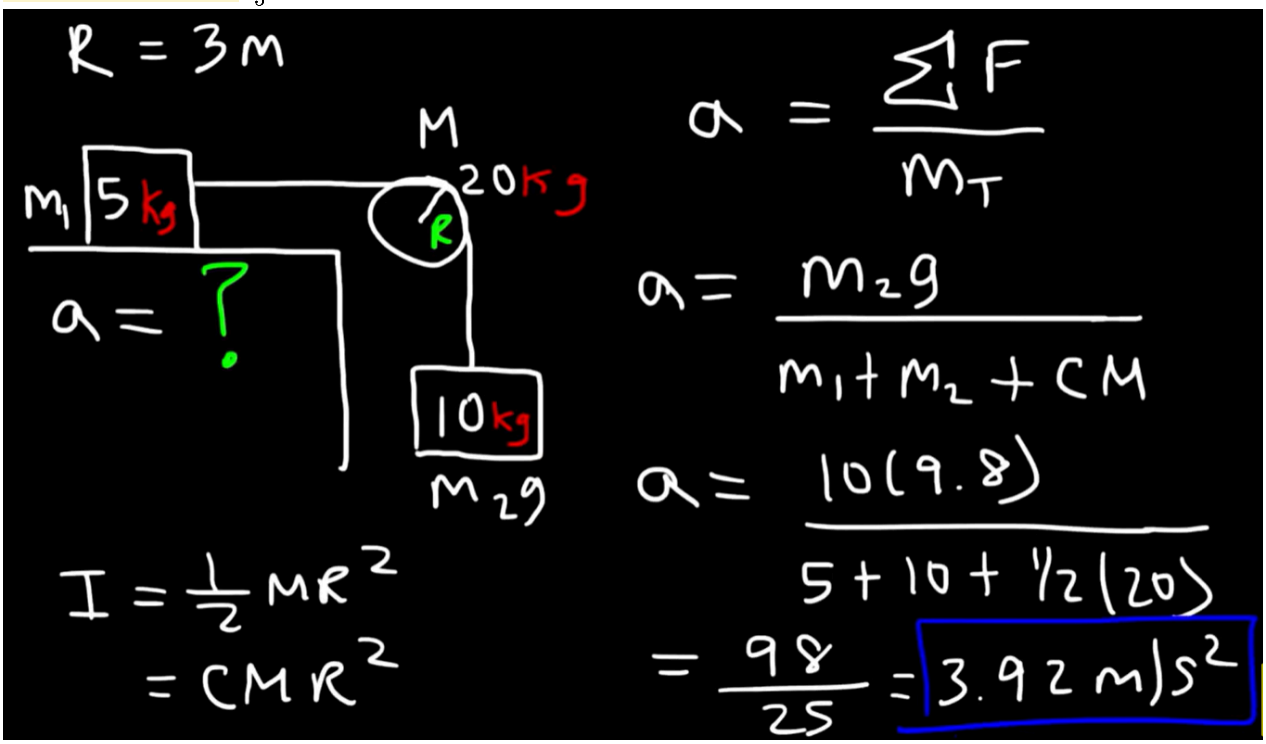
- [ ] Unit 18 optional exercise for Test preparation

Energy

- The work done by torque is the change in rotational energy
- The total kinemetic energy is the sum of linear kinematic energy and rotational kinematic energy

$$K_{Total} = \frac{1}{2} M_{Total} v_{CM}^2 + \frac{1}{2} I_{CM} \omega^2$$

Trick

- Calculating the acceleration of the system
  The core is $a = \frac{F}{m}$ and m for the pulley is CM where C is the coefficient in I, Note that if there is a sphere, need to consider its translational mass and rotational mass, so its effective mass is $\frac{7}{5}m$
  

- Note that the pulley makes the tension of both sides the string different.

Good Questions

1. Remember the process of a bowling ball

Initial Phase (Sliding Phase)

1. **Sliding State**: When the bowling ball is released, it has an initial velocity. At this point, the ball slides relative to the surface without rolling, and the angular velocity is zero.
2. **Effect of Sliding Friction**: The sliding friction force acts in the direction opposite to the motion, causing the linear velocity to decrease gradually.
3. **Torque and Angular Acceleration**: The friction force generates a torque leading to angular acceleration, causing the angular velocity to increase gradually.
4. **Change in Linear and Angular Velocities**:
   - Linear velocity as a function of time: $v(t) = v_0 - \mu_k g t$
   - Angular velocity as a function of time: $\omega(t) = \frac{\tau}{I} t$

Pure Rolling Phase

1. **Conditions for Pure Rolling**: When the linear velocity and angular velocity satisfy $v(t) = \omega(t)R$, the ball begins to roll without slipping.
2. **Calculation of Transition Time**: By solving the equation $v_0 - \mu_k g t = \left(\frac{\tau}{I} t\right) R$, the transition time can be found when the ball starts rolling without slipping.
3. **Disappearance of Sliding Friction**: In pure rolling, there is no relative sliding between the ball and the surface, so sliding friction disappears. At this point, there are no external forces in the horizontal direction, and the linear velocity remains constant according to Newton's first law.
4. **Effect of Static Friction**: In pure rolling, static friction is zero (assuming a horizontal surface and no other external forces), thus the linear velocity remains constant.

Summary

- **Sliding Phase**: The ball slides without rolling, subject to sliding friction, with decreasing linear velocity and increasing angular velocity.
- **Transition Phase**: When the linear velocity and angular velocity satisfy $v(t) = \omega(t)R$, the ball begins pure rolling.
- **Pure Rolling Phase**: There is no relative sliding between the ball and the surface, sliding friction disappears, and there are no external forces in the horizontal direction, so the linear velocity remains constant.

Once the ball starts pure rolling, there are no external forces in the horizontal direction, thus the linear velocity remains constant. At this point, the linear velocity is related to the angular velocity by the equation

$$\mathbf{v} = \omega \mathbf{R}$$

2. When to use $a = \mu g$ and when to use $a = \alpha R$

1. **系统与单点**
   - **系统**：当我们说 $\mu_k g$ 是针对系统时，指的是这个加速度是作用于整个保龄球这个物体的整体线性加速度，是物体整体在滑动过程中质心的加速度。它描述的是整个球体在水平方向上的运动变化情况，是作用于整个系统的力（滑动摩擦力）导致的结果。
   - **单点**：而 αr 是针对旋转体上某一点的切向加速度。角加速度 α 描述的是物体绕轴旋转的角速度变化快慢，乘以半径 r 就可以得到该点的切向加速度，表示物体上某一点由于旋转而产生的线速度变化率。
2. **适用场景**
   - **线性加速度（$a = \mu_k g$）**：适用于分析保龄球整体的平动运动，即球心处的线速度变化。这是由于滑动摩擦力导致整个球体在水平方向上运动状态的改变，是物体整体运动的一个参数。
   - **切向加速度（$a_{tan} = \alpha r$）**：适用于分析保龄球上某一点的线速度变化。在旋转运动中，每个点都有自己的切向加速度，这个加速度只与旋转有关，是旋转运动的局部参数。

- Example: A spherical bowling ball with mass $m = 4.3\,\mathrm{kg}$ and radius $R = 0.103\,\mathrm{m}$ is thrown down the lane with an initial speed of $v = 8.4\,\mathrm{m/s}$. The coefficient of kinetic friction between the sliding ball and the ground is $\mu = 0.31$. Once the ball begins to roll without slipping it moves with a constant velocity down the lane.
  What is the magnitude of the linear acceleration of the bowling ball as it slides down the lane?
  Answer: $\mu g = 3.0411\ \mathrm{m/s^2}$
- In other words, $a = \alpha R$ works only for rotation without slipping while $a = \mu g$ works for the system, and has the same effect as $F = ma$

3. How to calculate to tension that makes a sphere rolling without slipping

- Use the equivalent mass of sphere

$$T = \frac{7}{5}ma$$

4. A sphere sliding down a ramp

- What is the speed at the bottom of the ramp

$$\Delta\left(\frac{1}{2}Mv_{CM}^2\right) + \Delta\left(\frac{1}{2}I_{CM}\omega^2\right) = MgH$$

The speed at the bottom of the ramp for a sphere rolling without slipping is:

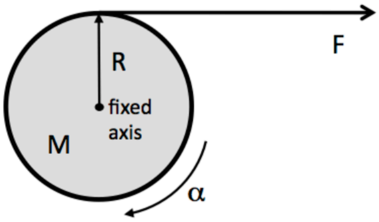
$$v_{CM} = \sqrt{\frac{10}{7}gH}$$

Which is less than the speed of a block sliding without friction:

$$v_{CM} = \sqrt{2gH}$$

5. Whether the force or torque is zero

A solid cylinder of radius R = 0.5 m starts from rest and can rotate without friction about an axis through its center of mass as shown. A string is wrapped around the circumference of the cylinder and is pulled with a constant force F=4 N. The rotation axis is fixed so that the center of mass of the cylinder does not move as the cylinder rotates. The magnitude of the angular acceleration of the cylinder is α = 13 rad/s^2.



1) Which of the following statements is true?

- ○ There is a non-zero net force on the cylinder, but the net torque on the cylinder is zero.
- ◉ There is a non-zero net torque on the cylinder, but the net force on the cylinder is zero.
- ○ There is both a non-zero net torque and a non-zero net force on the cylinder.

题目中给出一个半径为R=0.5米的实心圆柱体，绕通过质心的固定轴无摩擦转动。绳子绕在圆柱体表面并用恒力F=4N拉动，质心保持不动。需要判断此时圆柱体的受力和扭矩情况。

1. **质心运动分析**：由于质心保持不动，说明质心的加速度为零，根据牛顿第二定律，合外力必须为零。拉力F被转轴提供的反作用力平衡，因此合外力为零。
2. **扭矩分析**：拉力F作用在圆柱体的边缘，距离转轴的半径为R，因此产生的扭矩为τ = F * R。转轴的反作用力作用在转轴上（半径为0），因此不会产生扭矩。净扭矩为F * R，不为零，导致角加速度。
3. **结论**：虽然合外力为零（转轴反作用力平衡了拉力F），但拉力F产生的扭矩未被平衡，因此存在非零的净扭矩，导致角加速度。