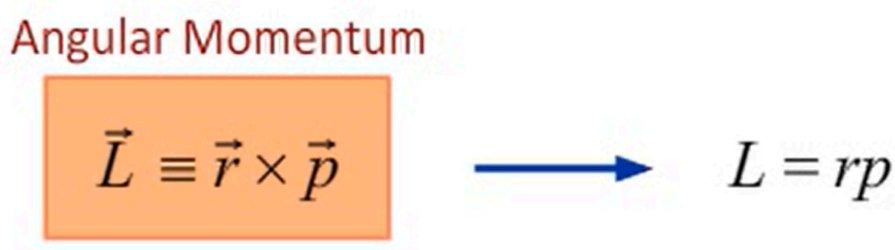
# L20_Angular Momentum

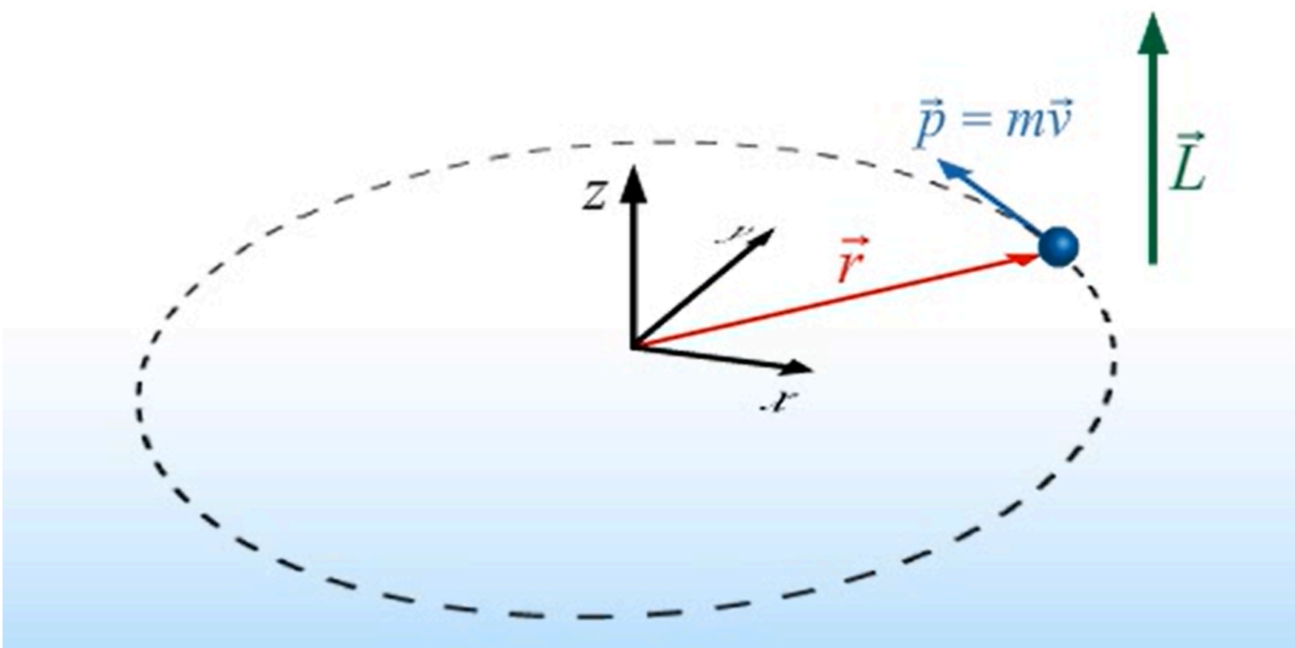
# Conservation of angular momentum

1. Analogous to the moment in translational movement, the angular momentum is conserved if the `torque of external force is zero`

# Formula

1. We use `L` to represent angular momentum, which equals to the cross product of the position vector `r` and the translational momentum `p`, and `p` is `mv`




2. Simplification

$$L = rp$$

$$L = r(mv)$$

$$L = r(m\omega r)$$

$$L = (mr^2)\omega$$

$$L = I\omega$$

1. Note that `I` is a scalar
   - For a single particle:

   $$L = I\omega$$

   - For a system of particles having the same angular velocity:

   $$L_{Total} = I_{Total}\omega$$

   Where $I_{Total} \equiv \sum m_i r_i^2$, the total moment of inertia.
2. The formula works for any solid objects
3. Angular Momentum and Kinetic Energy

   $$L = I\omega$$

   $$k = \frac{1}{2}I\omega^2$$

   $$\omega = \frac{L}{I}$$

   $$k = \frac{1}{2}I\frac{L^2}{I^2}$$

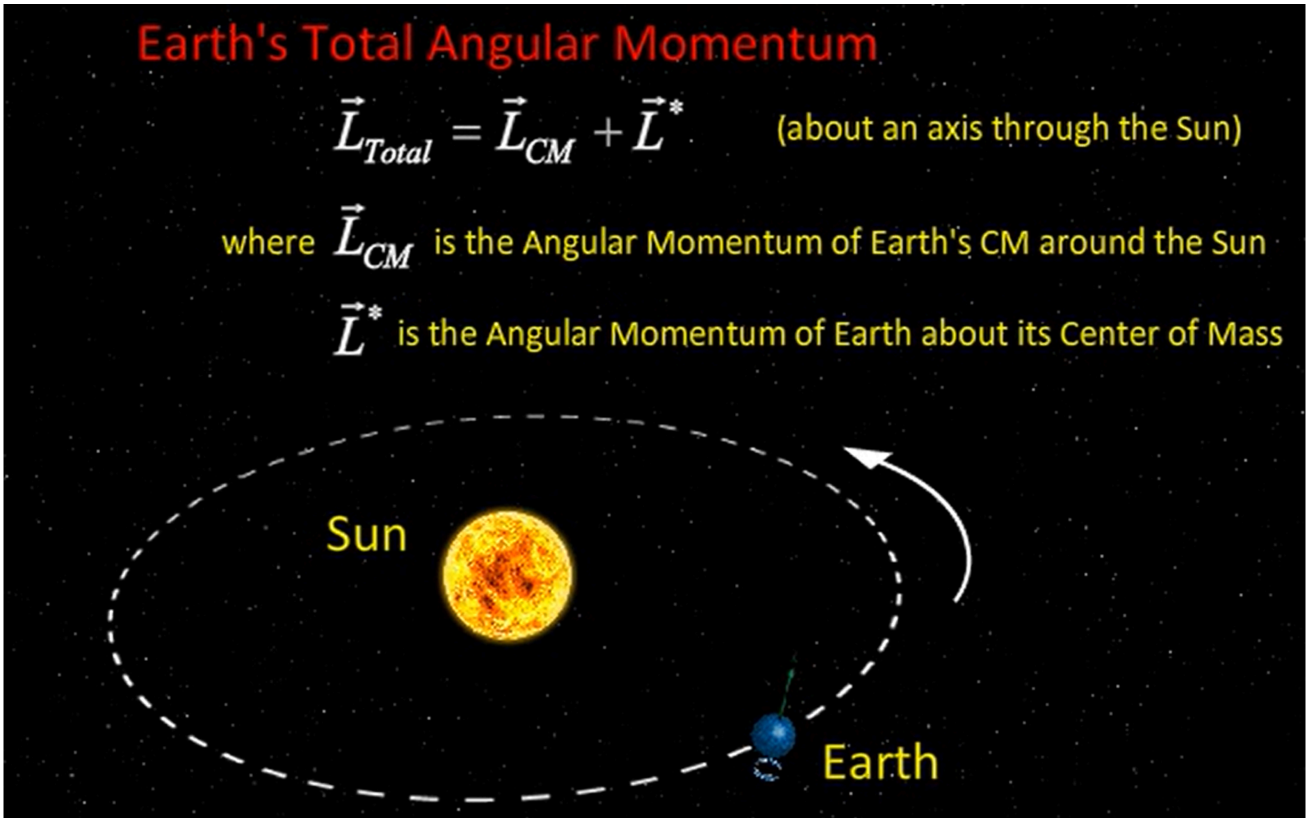
   $$k = \frac{1}{2}\frac{L^2}{I}$$

   $$k = \frac{L^2}{2I}$$

4. Direction: apply right hand rule: To tell the direction of the angular momentum $L$, same as the $\omega$ ,so curl the fingers as the direction of rotation, the direction of the thumb is the answer

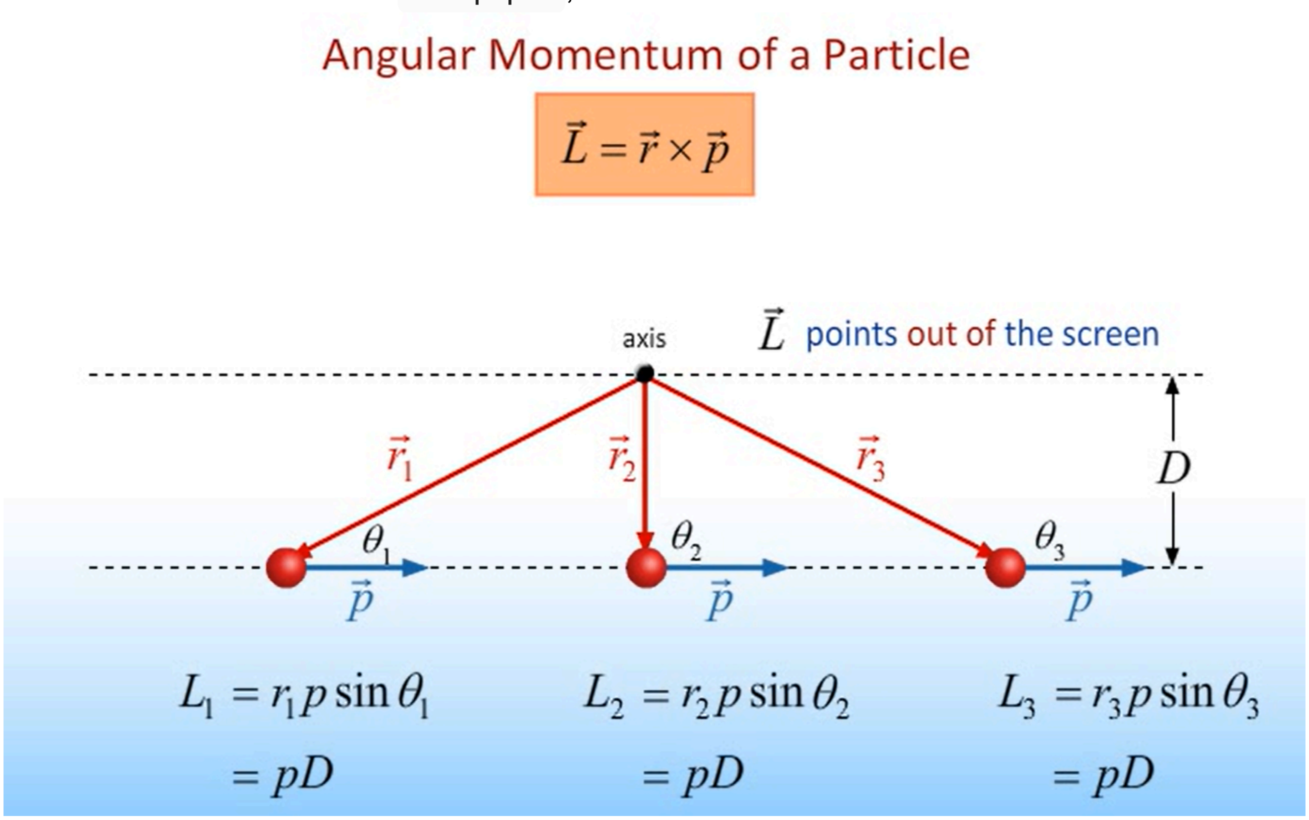
# Two spinning at the same time

1. If the object is spinning about its cm axis and another axis at the same time like the Earth and the Sun, the total `L` is just the sum

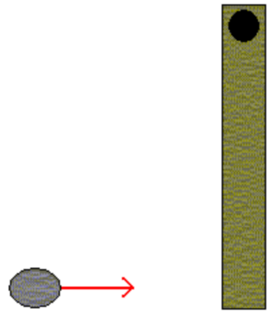



## Angular momentum of a particle

1. Actually, a particle does not have to be spinning to have `L`, in the image below, the `L` is conservative because no external force is exerted on the particle, which aligns with the conclusion with the formula `L=r×p=p*h`, h is the vertical distance



# Good Questions



A piece of putty of mass **m = 0.75 kg** and velocity **v = 2.5 m/s** moves on a horizontal frictionless surface. It collides with and sticks to a rod of mass **M = 2 kg** and length **L = 0.9 m** which pivots about a fixed vertical axis at the opposite end of the rod as shown. What fraction of the initial kinetic energy of the putty is lost in this collision?

**$KE_{lost}/KE_{initial}$** =

## 分步解答：

**步骤1：计算油灰的初始动能**

$$KE_{\text{initial}} = \frac{1}{2}mv^2 = \frac{1}{2} \cdot 0.75 \cdot (2.5)^2 = 2.34375\,\text{J}$$

**步骤2：应用角动量守恒**

碰撞后系统（杆+油灰）的角动量守恒：

$$L_{\text{initial}} = mvL = 0.75 \cdot 2.5 \cdot 0.9 = 1.6875$$

**步骤3：计算总转动惯量**

杆绕一端的转动惯量：

$$I_{\text{rod}} = \frac{1}{3}ML^2 = \frac{1}{3} \cdot 2 \cdot (0.9)^2 = 0.54$$

油灰的转动惯量：

$$I_{\text{putty}} = mL^2 = 0.75 \cdot (0.9)^2 = 0.6075$$

总转动惯量：

$$I_{\text{total}} = I_{\text{rod}} + I_{\text{putty}} = 0.54 + 0.6075 = 1.1475$$

**步骤4：求碰撞后角速度**

$$\omega = \frac{L_{\text{initial}}}{I_{\text{total}}} = \frac{1.6875}{1.1475} \approx 1.4706\,\text{rad/s}$$

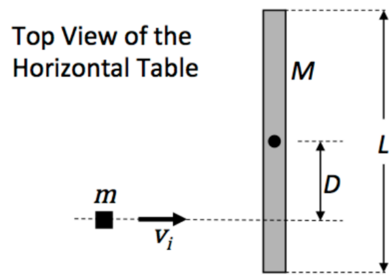
**步骤5：计算碰撞后的动能**

$$KE_{\text{final}} = \frac{1}{2}I_{\text{total}}\omega^2 = \frac{1}{2} \cdot 1.1475 \cdot (1.4706)^2 \approx 1.2403\,\text{J}$$

**步骤6：计算动能损失比例**

$$\frac{KE_{\text{loss}}}{KE_{\text{initial}}} = \frac{2.34375 - 1.2403}{2.34375} \approx 0.4706 \approx 47.1\%$$

Note that we can't calculate the $L$ of the rod after collision using the formula `rp` and note how we calculate kenematic energy using the angular motion arguments.




A block (which you should treat like a point particle) has mass m = 1.515 kg and slides with initial speed $v_i$ in the +x direction. It collides with a rod of length L = 1.3 m and mass M = 30.3 kg, which is initially at rest and oriented perpendicular to the path of the block. The block hits the rod a distance D = 0.39 from the center of the rod. After the collision, the block is at rest and the rod spins with angular velocity of $\omega_f$ = 28 rad/s. Everything is on top of a horizontal frictionless table, and the rod has a fixed frictionless pivot through its center that allows it to rotate freely but keeps its center from moving.

1) Which of the following statements best describes the collision?

- (●) The angular momentum about the pivot is conserved, but the linear momentum is not conserved.
- ( ) The angular momentum about the pivot and the linear momentum are both conserved.
- ( ) Neither the angular momentum about the pivot nor the linear momentum are conserved.

在碰撞过程中，分析线动量和角动量是否守恒：

1. **线动量守恒分析**：
   - 系统的初始线动量由滑块提供，大小为 $m \cdot v_{\text{initial}}$。
   - 碰撞后，滑块静止，杆子的中心被枢轴固定，因此杆子的质心速度为零，导致系统总线动量为零。
   - 由于枢轴对杆子施加了外力，阻止其中心移动，这个外力使得系统的线动量不守恒。
2. **角动量守恒分析**：
   - 碰撞前，滑块相对于枢轴的角动量为 $m \cdot v_{\text{initial}} \cdot D$。
   - 碰撞后，杆子绕枢轴旋转，其角动量为 $I \cdot \omega$，其中 $I$ 是杆子的转动惯量（对于均匀细杆绕中心转动惯量为 $\frac{1}{12}ML^2$）。
   - 枢轴对杆子的力作用在枢轴上，力矩为零，因此系统关于枢轴的总角动量守恒。