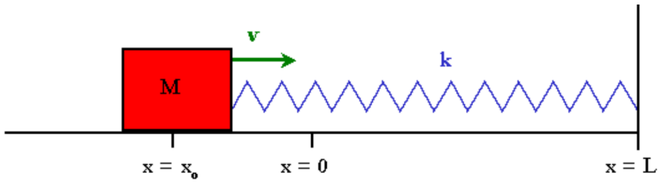
L22_Simple Harmonic Motion

1. The potential energy function for a mass on a vertical spring is identical to that for a mass on a horizontal spring as long as we measure displacements from the equilibrium length of the spring with the mass attached, which means we should choose the equilibrium position with the mass attached in the case of vertical spring, and choose the natural equilibrium when horizontal, so that we can apply $E = \frac{1}{2}kx^2$ in both cases
2. Formulas
   1. $\omega = \sqrt{\frac{k}{m}}$
   2. Special case where $t = 0$ $x_0 = A$ $v_0 = 0$, starts from rest at the maximum amplitude
   $$x(t) = A\cos(\omega t)$$
   $$v(t) \equiv \frac{dx}{dt} = -\omega A\sin(\omega t)$$
   $$a(t) \equiv \frac{dv}{dt} = -\omega^2 A\cos(\omega t)$$
   3. Special case where $t = 0$ $x_0 = 0$, starts at the equilibrium
   $$x(t) = A\sin(\omega t)$$
   $$v(t) \equiv \frac{dx}{dt} = \omega A\cos(\omega t) = v_0\cos(\omega t)$$
   $$a(t) \equiv \frac{dv}{dt} = -\omega^2 A\sin(\omega t)$$
   $$v_0(v_{max}) = A\omega, \text{ so we can calculate A using } A = \frac{v_0}{\omega}$$
   4. General case: we need to find the parameters A and ϕ
   $$x(t) = A\cos(\omega t + \phi)$$
   $$v(t) = -A\omega\sin(\omega t + \phi)$$
   5. Formula for calculating amplitude
   $$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$$
   This can be generated by consevation of energy
   $$E_{\text{total}} = \frac{1}{2}kA^2 = (\text{initial kinetic energy}) + (\text{initial potential energy}) = \frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2$$
3. One important property is that the frequency $f = \frac{1}{T}$, $T = \frac{2\pi}{\omega}$,
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

is only dependent on k and m, irrevelant with A

4. For vertical system, if we take the equilibrium position with the mass attached, we do not need to consider about gravity, just treat it as if it is on a horizontal frictionless floor, the highest point and the lowest point have same distance to the equilibrium position.
   However, the lowest point has larger elastic potential energy
   Example:



At **t = 0** a block with mass **M = 5 kg** moves with a velocity **v = 2 m/s** at position **x_o = -.33 m** from the equilibrium position of the spring. The block is attached to a massless spring of spring constant **k = 61.2 N/m** and slides on a frictionless surface. At what time will the block next pass **x = 0**, the place where the spring is unstretched?

t_1 =

0.15 seconds

要确定物块下一次经过平衡位置 $x = 0$ 的时间 t_1，我们遵循以下步骤：

1. **计算角频率** ω

   弹簧振子的角频率公式为：

$$\omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{61.2}{5}} \approx 3.498\,\mathrm{rad/s}.$$

2. **确定振幅** A

   利用初始条件 $x_0 = -0.33\,\mathrm{m}$ 和 $v_0 = 2\,\mathrm{m/s}$，振幅为：

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} = \sqrt{(-0.33)^2 + \left(\frac{2}{3.498}\right)^2} \approx 0.66\,\mathrm{m}.$$

3. **计算初始相位** ϕ

   由 $x_0 = A\cos\phi$ 和 $v_0 = -A\omega\sin\phi$，得：

$$\cos\phi = \frac{x_0}{A} = -0.5, \quad \sin\phi = -\frac{v_0}{A\omega} \approx -0.866.$$

   因此，$\phi = \frac{4\pi}{3}$。

4. **求解时间** t_1

   物块首次经过 $x = 0$ 时，相位需满足：

$$\omega t_1 + \phi = \frac{3\pi}{2} \quad \Rightarrow \quad t_1 = \frac{\frac{3\pi}{2} - \phi}{\omega}.$$

   代入 $\phi = \frac{4\pi}{3}$ 和 $\omega \approx 3.498\,\mathrm{rad/s}$，得：

$$t_1 = \frac{\frac{\pi}{6}}{3.498} \approx 0.150\,\mathrm{s}.$$