1.

x = 2 mod 297359071

x = 3 mod 837582957839

x = 4 mod 112889478

Ok so follow the chinese remainder theorem M=2*3*4=24

M1=837582957839*112889478=94554302892140718042

M2=297359071*112889478=33568710303754938

M3=249062890228437207569

y1*M1=1 mod 297359071

Using sage to do that find y1=165024624

y2*M2=1 mod 837582957839

Using sage y2=413637535290

y3*M3=1 mod 112889478

y3=107378873

Now our solution will be

2*165024624*94554302892140718042+3*413637535290*33568710303754938+4*107378873*249062890228437207569 mod(297359071*837582957839*112889478)

=111403041764946653500552235 32 (mod 28116579667059577118242058982)

So this will be the solution for this problem.

2. Let id be your student id number, p be the prime number

93935935937584760927320853927657, and q be the prime number 20395358947549853439147504976967820947509174847. Find an integer x such that x^37 = id (mod n ),

where n = p* q.

If you do not know the factorization of n, can you find x quickly?

Answer:

Using the chinese remainder theorem in order to solve the above equation, we just need to do the x^37=112889478 mod(p)

And x^37=112889478 mod (q)

So I need to find the inverse of 37 inside of z/(p-1)z:10155236317576730911061713938125

And find the inverse of 37 inside of z/(q-1)z: 16536777525040421707416895927271206173656087713

Now need to do exponential on both side of the equation x=112889478^10155236317576730911061713938125=11783559208677482719271449498166

(mod p))

x=112889478^7165936927517516073213988235150856008584304676

=6482461011087543415699077379576030823377770097

mod(q)

Ok, now we apply chinese remainder theorem,

M1=q

M2=p

M=p*q

y1*M1=1 mod (p)

y2*M2=1 mod(q)

y1=87745427609293285329933113785974

y2=1344082412796660241629822315953887863176717086

x=11783559208677482719271449498166*87745427609293285329933113785974*20395358947549853439147504976967820947509174847+6482461011087543415699077379576030823377770097*1344082412796660241629822315953887863176717086*93935935937584760927320853927657 mod(19158571315210891847847100831099236304685424909875913407370458411497031020434 79)=54520971385949737698613600763362301397190654353527082717485559807058596728484 1=1100134094274534720338365095230521398148842066655162744899298845948280904758835

So based on above
x=110013409427453472033836509523052139814884206665516274489929884594828 0904758835

Is my solution and I tested with sage that x^37 mod(p*q)=112889478!

If we don’t know the factorization of n, we can not find the solution quickly , since the chinese remainder theorem require that to find the factorization in order to do the next step, this is the who idea that why the block cypher is hard to be decryoted since we don’t know the factorization of a big number.

3. So we know that
phi(4)=4-2=2
phi(5)=4
By chinese remainder theorem we know that
phi(6)=phi(2)*phi(3)=1*2=2
phi(7)=6
phi(8)=phi(2^3)=8-4=4
phi(9)=phi(3^2)=9-3=6
phi(10)=phi(2)*phi(5)=1*4=4
phi(11)=10
phi(12)=phi(4)*phi(3)=2*2=4
After 12 is not possible since based on chinese remainder theorem, either it is is a prime it will have the order as p-1 will not be 4 any more, and 12=4*3 that is the biggest number that all its factors can still be as small as 4, so I think we are done,
M can be 5,8,10,12.

4.
Calculate 31^(30^45) mod 35
Need to calculate above first need to calculate 30^45
So z/35z has order phi(35)=phi(5)*phi(7)=4*6=24
So 6^45 ->6^21
6^21->(6^2)^10*6->(12^2)^5*6->0
31^0->1
So the final answer should be 1.