

1. Need to find the inverse of 122 mod 343 use x euclidean algo

$$343=2*122+99$$

$$122=1*99+23$$

$$99=4*23+9$$

$$23=3*7+2$$

$$7=3*2+1$$

Then reverse this process backward

$$1=7-3*2$$

$$=7-3*(23-3*7)$$

$$=-3*23+10*7$$

$$=-3*23+10*(99-4*23)$$

$$=-43*23+10*99$$

$$=-43*(122-1*99)+10*99$$

$$=-43*122+53*99$$

$$=-43*122+53*(343-2*122)$$

$$=-149*122+53*343$$

The we find the inverse is -149 we multiply -149 on both side then we get  $x=3*-149+2*343=239$ .

2. My id number is 112889478, so it is not invertible since  $\gcd(id, 2^{64}) \neq 1$

The next number that is invertible under  $2^{64}$  will be 1128894779

$a=112889479$  , and we want to find  $a*x=2018 \mod 2^{64}$

The same idea we need to find the inverse of a with mod  $2^{64}$ .

The answer from sage is 10282407867830009742

Adn u need the following code  $d,u,v=\text{xgcd}(112889479, 2^{64})$

Then  $x=\text{mod}(u*2018, 2^{64})$

Then  $x=10282407867830009742$

3. The unit group of  $\mathbb{Z}/16\mathbb{Z}$  should be all the numbers that is relative to 16.

$\{1,3,5,7,9,11,13,15\}$

And zero divisors will be

$\{2,4,6,8,10,12,14\}$

4. The unit group of  $\mathbb{Z}/15\mathbb{Z}$  is  $\{1,2,4,7,8,11,13,14\}$

And the zero divisor for this ring will be  $\{3,5,6,9,10,12\}$