

一. 函数.

function $\begin{cases} \text{domain} \\ \text{codomain} \\ \text{Range} \\ \text{rule} \end{cases}$ $\begin{cases} \text{function } (f) \\ \text{inverse-function } (f^{-1}) \end{cases}$ 有无 inverse 用 horizontal line test

f 与 f^{-1} 的图像关于 $y=x$ 对称。

对于有 f^{-1} 的 f , 则有 $f(f^{-1}(y)) = y$, $f^{-1}(f(x)) = x$. 对于 any y in the Range of f , 都有 $f^{-1}(y) = x$. f^{-1} 的 domain 为 f 的 range.

The inverse of the inverse function is the original function.

function composition: $h(x) = f(g(x)) = f \circ g$.

tangent: $\tan(x)$ reciprocals: $\frac{1}{x}$

$f(x)$ 和 $f(x+5)$ shifted 5 units right

> odd and even-function: $f(x) = f(-x)$ $f(x) = -f(-x)$

只有 $f(0)=0$ 这个函数是非奇非偶的. 对 odd 函数, $f(x) = -f(-x) \therefore 2f(0)=0$

mirror symmetry about the y -axis. even function

180° symmetry about the origin. odd function

product of two odd function: $f(x) \cdot g(x) = -f(-x) \cdot -g(-x) = f(-x) \cdot g(-x)$. is always a even function.

奇奇得偶, 奇偶得奇.

linear function: $f(x) = mx + b$

point-slope form of a linear function:

$$f(x) - f(x_0) = m(x - x_0)$$

common function and Graphs.

1. polynomials:

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$. a_n 是 coefficient of x^n . $a_{n_{\max}}$ 为 leading coefficient.

degree two polynomial: $y = ax^2 + bx + c$ $\Delta = b^2 - 4ac$ root: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

completing the square: $y = ax^2 + bx + c = y = a(x^2 + \frac{b}{a}x + \frac{c}{a}) = a(x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 + \frac{c}{a} - (\frac{b}{2a})^2) = a[(x + \frac{b}{2a})^2 + \frac{c}{a} - (\frac{b}{2a})^2]$

2. rational function:

function in the form of $\frac{p(x)}{q(x)}$, where p and q are polynomials. eg: $\frac{1}{x}$, $\frac{1}{x^2}$

3. Exponentials and logarithms:

$y = a^x$ $y = \log_a x$ 图像关于 $y=x$ 对称.

$$y = a^{-x} = (a^{-1})^x = (\frac{1}{a})^x$$

4. Trig function.

5. Functions involved with absolute value.

$f(x) = |x|$ the definition is:

$$f(x) = \begin{cases} x, & x > 0 \\ -x, & x \leq 0 \end{cases}$$