Continulity and Differentiability.

A function f is continuous at $X=a$ if $\lim_{x\to a} f(x) = f(a)$.

在 $[a,b]$ 上连续，(a,b) 每一点都连续 a上右连续，b上左连续.

连续的函数：polynomials。

连续函数的加，减，乘，嵌套依然连续。

中值定理：Intermediate Value Theorem.

If f is continous on $[a,b]$, and $f(a)<0$ and $f(b)>0$, then there is at least one number c in (a,b) such that $f(c)=0$.

eg: Any polynomial of odd degree has a least one root.

Max-Min Theorem: If f is continuous on $[a,b]$, then f has at least one maximum and one minmum on $[a,b]$.

Differentiablity.

Displacement and velocity. Instantaneous velocity at time $t = \lim_{u\to t} v_{t\leftrightarrow u}$.

$f(t)$ = position of car at time t.

$v_{t\leftrightarrow u} = \dfrac{\text{position at time } u - \text{position at } t}{u-t} = \dfrac{f(u)-f(t)}{u-t}$ 令 $h=u-t$，instaneous velocity at time $t = \lim_{h\to 0}\dfrac{f(t+h)-f(t)}{h}$

tangent lines:

$f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ 如果极限存在，则说 f 在 x 点可导 (differentiable).

$$f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$$

$\Delta x = x_{new} - x \quad \Delta y = y_{new} - y = f(x_{new}) - f(x) = f(x+\Delta x) - f(x)$.

Δx means "change in x". dx means "really really tiny change in x".

$\frac{dy}{dx}$ the derivative of y with respect to x.

Second and higher-order derivatives.

$f'(x) = \dfrac{dy}{dx}$, $f''(x) = \dfrac{d^2y}{dx^2}$, $f'''(x) = f^{(3)}(x) = \dfrac{d^3y}{dx^3} = \dfrac{d^3}{dx^3}(y)$

$y=|x|$ 导数不存在，但左导数或右导数存在.

Relationship between differentiablity and continuity.

If a function f is differentiable at x, then it's continuous at x.

可导一定连续，连续不一定可导。