Assignment 05

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Due: 2022/12/13

In [1]:

```
import random
from math import *
import matplotlib.pyplot as plt
import matplotlib as mpl
import numpy as np
import pandas as pd
import datetime
import netCDF4
import xarray as xr
%matplotlib inline
import matplotlib.ticker as mticker
import cartopy.crs as ccrs
import cartopy.feature as cfeature
import cartopy.io.shapereader as shapereader
import scipy
from scipy.integrate import odeint
from scipy.optimize import curve_fit
plt.rcParams['font.sans-serif'] = ['SimHei']
```

```
import warnings
warnings.filterwarnings("ignore")
```

Ref: All the programming details were referred to the handout of course ESE5023 by professor Zhu (https://zhu-group.github.io/ese5023).

1. Modeling of carbon cycle

Data reference:
Global Fossil-Fuel CO2 Emissions;
Mauna Loa CO2 annual mean data;
Historical CO2 Records from the Law Dome DE08, DE08-2, and DSS Ice Cores.

Accordiing to the article "*Is a box model effective for understanding the carbon cycle?*", the two-bos model to compute the atmospheric carbon dioxide level was described as the equation (1) and (2):

$$\frac{dN_1}{dt} = -k_{12}N_1 + k_{21}N_2 + \gamma \tag{1}$$

$$\frac{dN_2}{dt} = k_{12}N_1 - k_{21}N_2 \tag{2}$$

1.1

Where N_1 and N_2 denote the concentration of carbon in the atmosphere and the surface of the ocean. And the constant flux from atmosphere to ocean is 105, while the constant flux from ocean to atmosphere is 102. Therefore, the transfer coefficinet $k_{12} = 105/N_1, K_{21} = 102/N_2$. The rate of production of CO_2 by fossil-fuel burnign is denoted by γ, whose data is referred to Global Fossil-Fuel CO2 Emissions. As the data is recored annually, $dt = 1$, so equation (1) and (2) can be reduced onto equation (1') and (2'):

$$ dN_{1LastYear} = -105 + 102 +\gamma_{ThisYear} \tag{1'}$$

$$ dN_{2LastYear} = 105 - 102 \tag{2'}$$

In [2]:

```
# 导入γ数据并做单位换算，换算后的单位为PgC
#1 million ton C = 1 * 10^12 g C = 1 * 10-3 PgC
df1 = pd.read_csv("gamma.csv")
df11 = df1.loc[(df1['Year'] >= 1986) & (df1['Year']<2005)]
df11['gamma'] = df11['Total']/1000

#基于dt=1year, 计算dN1
df11['delta_N1'] = df11['gamma'] - 3

#初始值1986年N1=740
df11['N1'] = 740
```

```python
#删去不需要的列
df11.drop(columns=['Total','Gas','Liquids','Cement Solids',
                   'Gas Production','Flaring','Capita'],inplace=True)

#重新排序号
df11.reset_index(drop=True, inplace=True)

#计算不考虑buffer效应的two-box模型N1结果:
for i in range(18):
    df11.iloc[i+1,3]=df11.iloc[i,3]+df11.iloc[i+1,2]

# 单位换算成ppm
df11['N1_ppm'] = df11['N1'] * 347 / 740
df11
```

Out[2]:

	Year	gamma	delta_N1	N1	N1_ppm
0	1986	5.609	2.609	740.000	347.000000
1	1987	5.755	2.755	742.755	348.291872
2	1988	5.968	2.968	745.723	349.683623
3	1989	6.088	3.088	748.811	351.131645
4	1990	6.151	3.151	751.962	352.609208
5	1991	6.239	3.239	755.201	354.128036
6	1992	6.178	3.178	758.379	355.618261
7	1993	6.172	3.172	761.551	357.105672
8	1994	6.284	3.284	764.835	358.645601
9	1995	6.422	3.422	768.257	360.250242
10	1996	6.550	3.550	771.807	361.914904
11	1997	6.663	3.663	775.470	363.632554
12	1998	6.638	3.638	779.108	365.338481
13	1999	6.584	3.584	782.692	367.019086
14	2000	6.750	3.750	786.442	368.777532
15	2001	6.916	3.916	790.358	370.613819

	Year	gamma	delta_N1	N1	N1_ppm
16	2002	6.981	3.981	794.339	372.480585
17	2003	7.397	4.397	798.736	374.542422
18	2004	7.782	4.782	803.518	376.784792

In [3]:

```
# 打印1987-2004的大气CO2浓度（单位：ppm）
print(df11.iloc[1:19,4])
```

```
1       348.291872
2       349.683623
3       351.131645
4       352.609208
5       354.128036
6       355.618261
7       357.105672
8       358.645601
9       360.250242
10      361.914904
11      363.632554
12      365.338481
13      367.019086
14      368.777532
15      370.613819
16      372.480585
17      374.542422
18      376.784792
Name: N1_ppm, dtype: float64
```

1.2

The two-box model with the buffer effect is defined as equation (3) and (4):

$$\frac{dN_1}{dt} = -k_{12}N_1 + k_{21}(N_2^0 + \xi(N_2 - N_2^0)) + \gamma \tag{3}$$

$$\frac{dN_2}{dt} = k_{12}N_1 - k_{21}(N_2^0 + \xi(N_2 - N_2^0)) \tag{4}$$

Where ξ is the buffer factor (equation (5) shows its defination expression), whihc can be deriviated as the function of atmospheric CO_2 concentration of ppm unit (equation (6), where z is the atmospheric CO_2 concentration of ppm unit).

$$\xi = [\frac{(P - P_0)/P_0}{(C - C_0)/C_0}]_{ConstantAlkalinity} \tag{5}$$

$$\xi(z) \approx 3.69 + 1.86 \times 10^{-2}z - 1.80 \times 10^{-6}z^2 \tag{6}$$

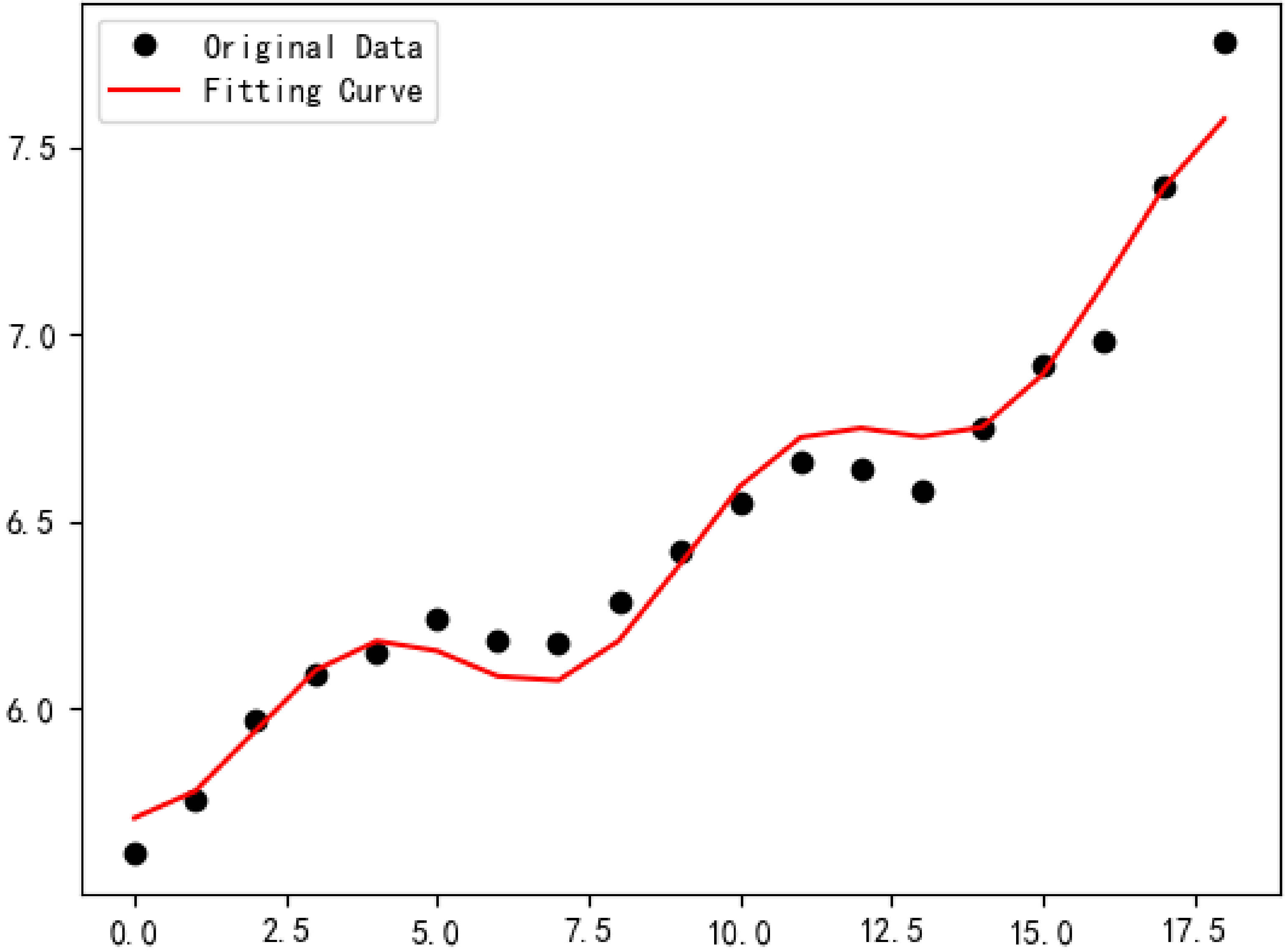
Covert the unit from ppm to PgC, the equation (6) turns into :

$$\xi(z) \approx 3.69 + \frac{3441}{86750} N_1 - 3.84 \times 10^{-6} (N_1)^2 \tag{6'}$$

In [4]:

```
# 对ξ做散点拟合（1986-2004年间），得到ξ(t)
x = np.arange(0, 19, 1)
y = df11['gamma']
popt,pcov=curve_fit(lambda t,a,b,c,d,e,f: a+b*t+c*t**2+d*np.sin(e*t+f),x,y)
print('popt: ',popt)
plt.figure()
a,b,c,d,e,f=popt
y_pre = a+b*x+c*x**2+d*np.sin(e*x+f)
plt.plot(x, y, 'ko', label="Original Data")
plt.plot(x, y_pre, 'r-', label="Fitting Curve")
plt.legend()
plt.show()
```

```
popt:  [ 5.85850616e+00  3.02738354e-02  3.15089632e-03 -1.54492232e-01
  8.86989926e-01  1.41978441e+00]
```



The result of curve fitting for $\gamma(t)$ is:

$$\gamma(t) = 5.86 + 3.03 \times 10^{-2} t + 3.15 \times 10^{-3} t^2 - 1.54 \times 10^{-1} sin(8.87 \times 10^{-1} t + 1.42)$$

In [5]:

```
#（7）（6’）式代入（3）（4）式后解常微分方程组
def f(t,u):
    #x1,x2...xn = u
    x,y = u
    #dx1dt=...,dx2dt=...,dxndt=...
    dxdt = -105 + 102/y*(821+(3.69+3441/86750*x-0.00000384*x**2)*(y-821))+(5.86+0.(
    dydt = 105 - 102/y*(821+(3.69+3441/86750*x-0.00000384*x**2)*(y-821))
    #dudt = [dx1dt,dx2dt,...dxndt]
    dudt = [dxdt,dydt]
    return dudt
#x1(0)=...,x2(0)=...,xn(0)=...
u0=[740,900]
t = np.linspace(0,18,19)
sol = odeint(f,u0,t,tfirst=True)
```

In [6]:

```
#单位换算为ppm,并打印1987-2004年的N1
res_buffer = []
for i in range(19):
    res_buffer.append(sol[i][0]/740*347)
res_buffer[1:19]
```

Out[6]:
```
[385.6206105245448,
 389.1217264712141,
 391.9626696925123,
 394.8494323488009,
 397.7468394323386,
 400.618628846124,
 403.46853359849166,
 406.3399318489883,
 409.2860521926387,
 412.33296678618683,
 415.4625413448939,
 418.6280035441636,
 421.79054223497957,
 424.9500662339734,
 428.1474725163775,
 431.43668109339546,
 434.84710630031134,
 438.36415269246237]
```

Ref: The thought of the curve fitting of $\gamma(t)$ was inspired by *Jiang Xiating*, and the corresponding code was referred to *Together_CZ* (https://blog.csdn.net/together_cz/article/details/100538951). The method to handle the system of the ODE was referred to *whr804* (https://blog.csdn.net/weixin_61268973/article/details/122462611).

1.3

In [7]:

```python
#将1.2结果存到df11中:
df11['bufferN1_ppm'] = 347
for i in range(19):
    df11.iloc[i,5]=res_buffer[i]

#将1.1和1.2在1986年的对应值赋值为空值,方便画图（图上没有1986年的）
df11.iloc[0,4:6]=np.nan
df11.head(3)
```

Out[7]:

	Year	gamma	delta_N1	N1	N1_ppm	bufferN1_ppm
0	1986	5.609	2.609	740.000	NaN	NaN
1	1987	5.755	2.755	742.755	348.291872	385.620611
2	1988	5.968	2.968	745.723	349.683623	389.121726

In [8]:

```python
#导入观测值，并与df11合并，再生成一个datetime列
from datetime import datetime, timedelta
from pandas import DataFrame
df2 = pd.read_csv("Loa.csv")
df22 = df2.loc[(df2['Year'] >= 1986) & (df2['Year']<2005)]
df_merge = df11.merge(df22,on='Year').rename(columns={'mean':'Observed'})
df_merge['Year']=pd.to_datetime(df_merge['Year'], format="%Y")
df_merge
```

Out[8]:

	Year	gamma	delta_N1	N1	N1_ppm	bufferN1_ppm	Observed	unc
0	1986-01-01	5.609	2.609	740.000	NaN	NaN	347.61	0.12
1	1987-01-01	5.755	2.755	742.755	348.291872	385.620611	349.31	0.12
2	1988-01-01	5.968	2.968	745.723	349.683623	389.121726	351.69	0.12
3	1989-01-01	6.088	3.088	748.811	351.131645	391.962670	353.20	0.12
4	1990-01-01	6.151	3.151	751.962	352.609208	394.849432	354.45	0.12
5	1991-01-01	6.239	3.239	755.201	354.128036	397.746839	355.70	0.12
6	1992-01-01	6.178	3.178	758.379	355.618261	400.618629	356.54	0.12
7	1993-01-01	6.172	3.172	761.551	357.105672	403.468534	357.21	0.12
8	1994-01-01	6.284	3.284	764.835	358.645601	406.339932	358.96	0.12
9	1995-01-01	6.422	3.422	768.257	360.250242	409.286052	360.97	0.12
10	1996-01-01	6.550	3.550	771.807	361.914904	412.332967	362.74	0.12
11	1997-01-01	6.663	3.663	775.470	363.632554	415.462541	363.88	0.12
12	1998-01-01	6.638	3.638	779.108	365.338481	418.628004	366.84	0.12
13	1999-01-01	6.584	3.584	782.692	367.019086	421.790542	368.54	0.12
14	2000-01-01	6.750	3.750	786.442	368.777532	424.950066	369.71	0.12
15	2001-01-01	6.916	3.916	790.358	370.613819	428.147473	371.32	0.12

	Year	gamma	delta_N1	N1	N1_ppm	bufferN1_ppm	Observed	unc
16	2002-01-01	6.981	3.981	794.339	372.480585	431.436681	373.45	0.12
17	2003-01-01	7.397	4.397	798.736	374.542422	434.847106	375.98	0.12
18	2004-01-01	7.782	4.782	803.518	376.784792	438.364153	377.70	0.12

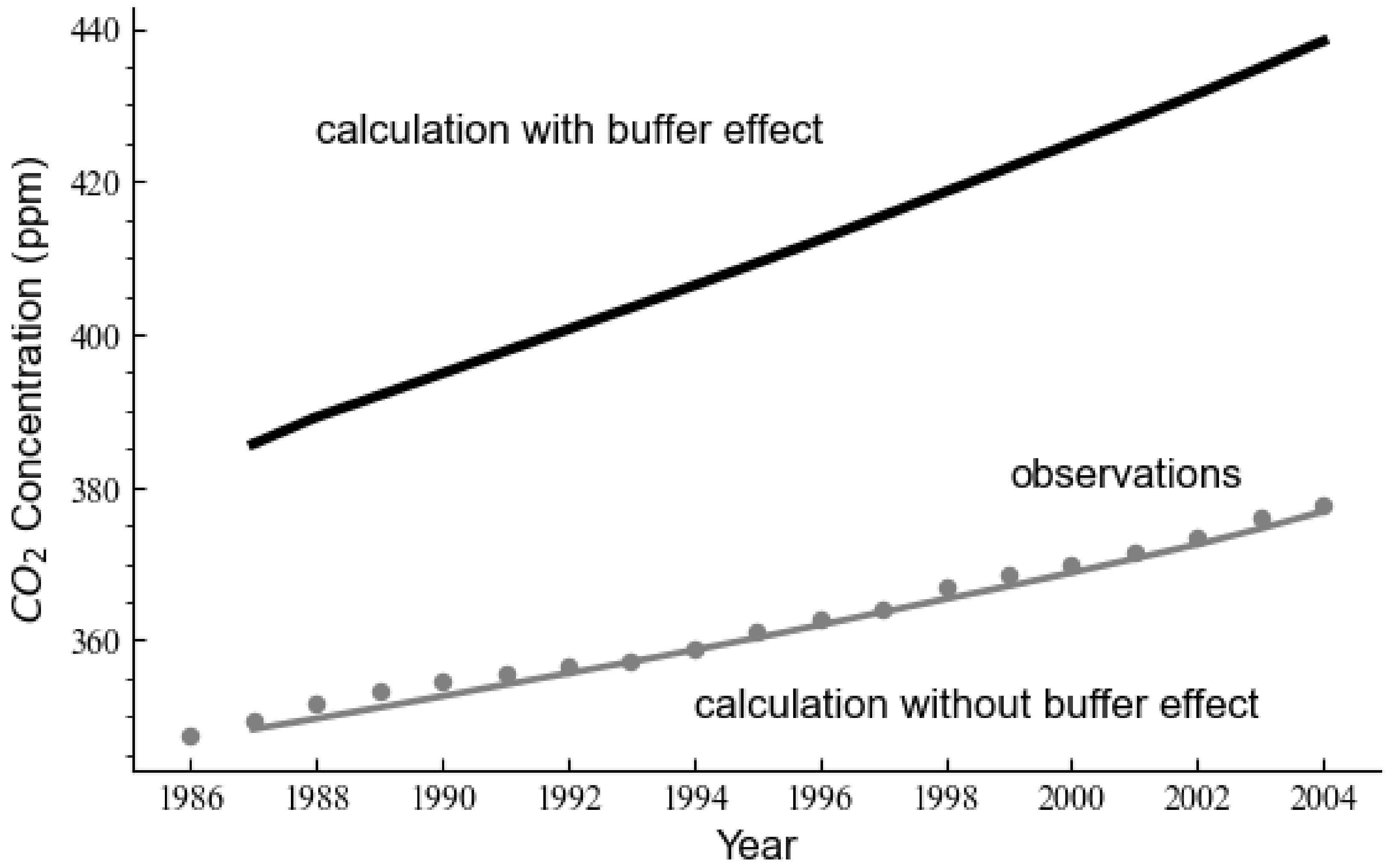
In [9]:
```
#画图
fig, ax = plt.subplots(figsize=(6.5,4))
plt.plot(df_merge['Year'],df_merge['N1_ppm'],lw=2,color='grey')
plt.plot(df_merge['Year'],df_merge['bufferN1_ppm'],lw=3,color='k')
plt.scatter(df_merge['Year'],df_merge['Observed'],marker='o',s=19,color='grey')
plt.xlabel("Year",fontsize=12,fontproperties="Arial")
plt.ylabel("$CO_2$ Concentration (ppm)",fontsize=12,fontproperties="Arial")
ax.spines['right'].set_visible(False)
ax.spines['top'].set_visible(False)


#刻度朝里、刻度字体新罗马
plt.tick_params(labelsize=10,direction='in')
x1_label = ax.get_xticklabels()
[x1_label_temp.set_fontname('Times New Roman') for x1_label_temp in x1_label];
y1_label = ax.get_yticklabels()
[y1_label_temp.set_fontname('Times New Roman') for y1_label_temp in y1_label];

#y轴坐标刻度：主+次
```

```
from matplotlib.ticker import MultipleLocator, FormatStrFormatter
yminorLocator   = MultipleLocator(5) #将y轴次刻度标签设置为n的倍数
ax.yaxis.set_minor_locator(yminorLocator)

#加上批注text
plt.text(df_merge.iloc[8,0],350,"calculation without buffer effect",fontsize=12,fontp
plt.text(df_merge.iloc[13,0],380,"observations",fontsize=12,fontproperties="Arial");
plt.text(df_merge.iloc[2,0],425,"calculation with buffer effect",fontsize=12,fontpro
```



Ref: The details about beautifing the plot was referred to *RS&Hydrology* (https://blog.csdn.net/qq_32649321/article/details/120324133).