Sequential Elimination Contests with All-pay Auctions

Fupeng Sun[1], Yanwei Sun[2], Chiwei Yan[3], Li Jin[4]

[1]Imperial College Business School, [2]Imperial College Business School, [3]University of California, Berkeley, [4] Shanghai Jiao Tong University

I. Motivation

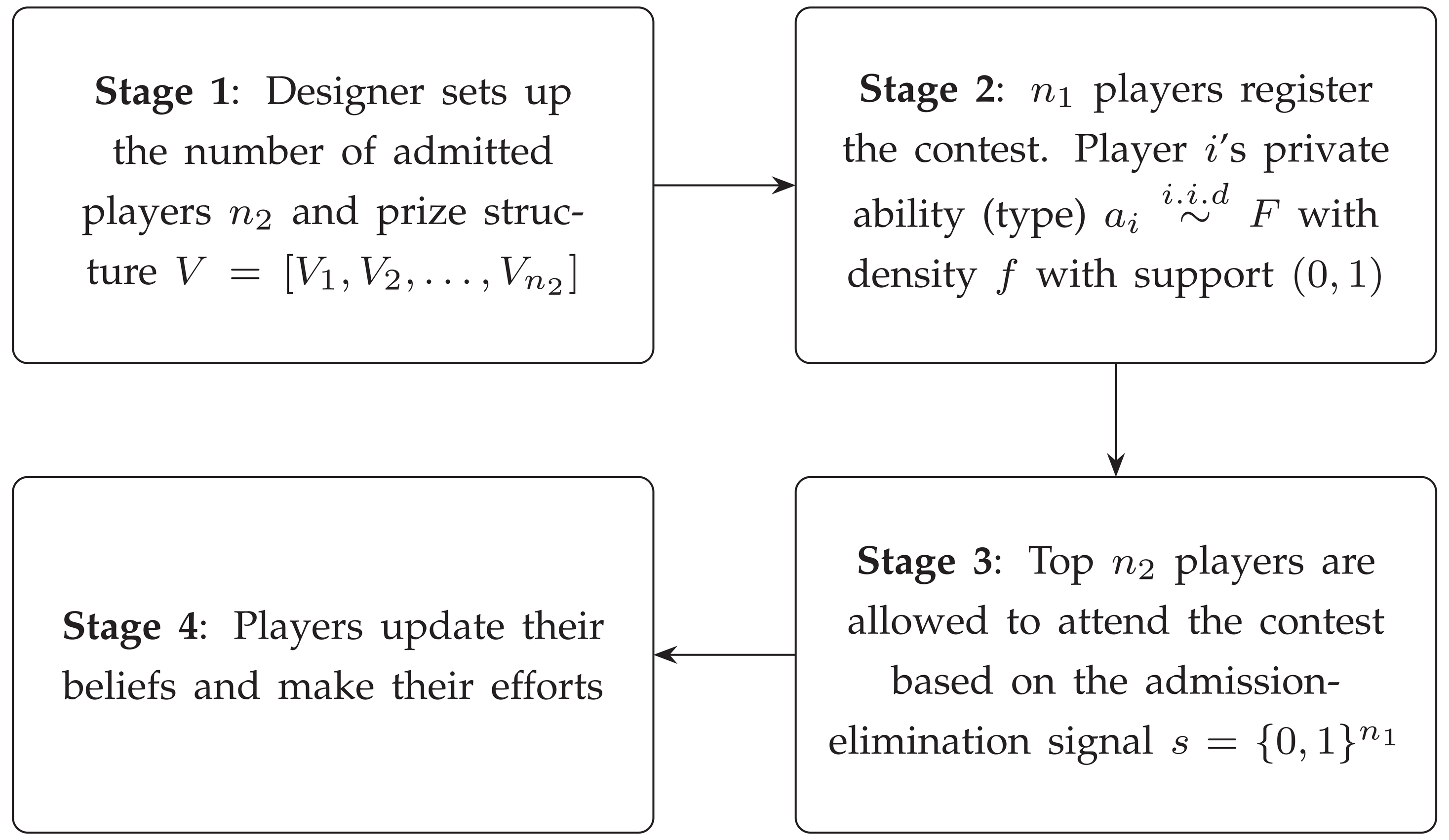
In many contests, the designer filters candidates prior to the round of competing for prizes:

- NSF Proposals: National Science Foundation in the US filters potential low-quality submissions without going through the costly formal review process.
- Start-up competitions: Y Combinator, a top incubator in the San Francisco Bay area, invite roughly top 7% of the applicants who fill out a short application form for on-site interviews to compete for a $500,000 funding.

Questions: How should the designer eliminate candidates to maximize efforts?

II. Model Setup

Timeline of the contest:



Research Question: What is the optimal (n_2, V) in terms of i) expected *highest* equilibrium efforts? ii) expected *total* equilibrium efforts?

Bayesian Nash Equilibrium (BNE)

A BNE is a tuple of posterior beliefs (densities) $[\beta_i(a_{-i} \mid s, a_i) : i \in \mathcal{I}]$ and strategies $[b_i(\cdot) : i \in \mathcal{I}]$ that satisfies the following conditions:

1. (*Bayesian Updating*) For every admitted player $i \in \mathcal{I}$, which is the set of admitted n_2 players, Bayes' rule is used to update her posterior belief $\beta_i(a_{-i} \mid s, a_i)$.
2. (*Sequential Rationality*) For every admitted player $i \in \mathcal{I}$,

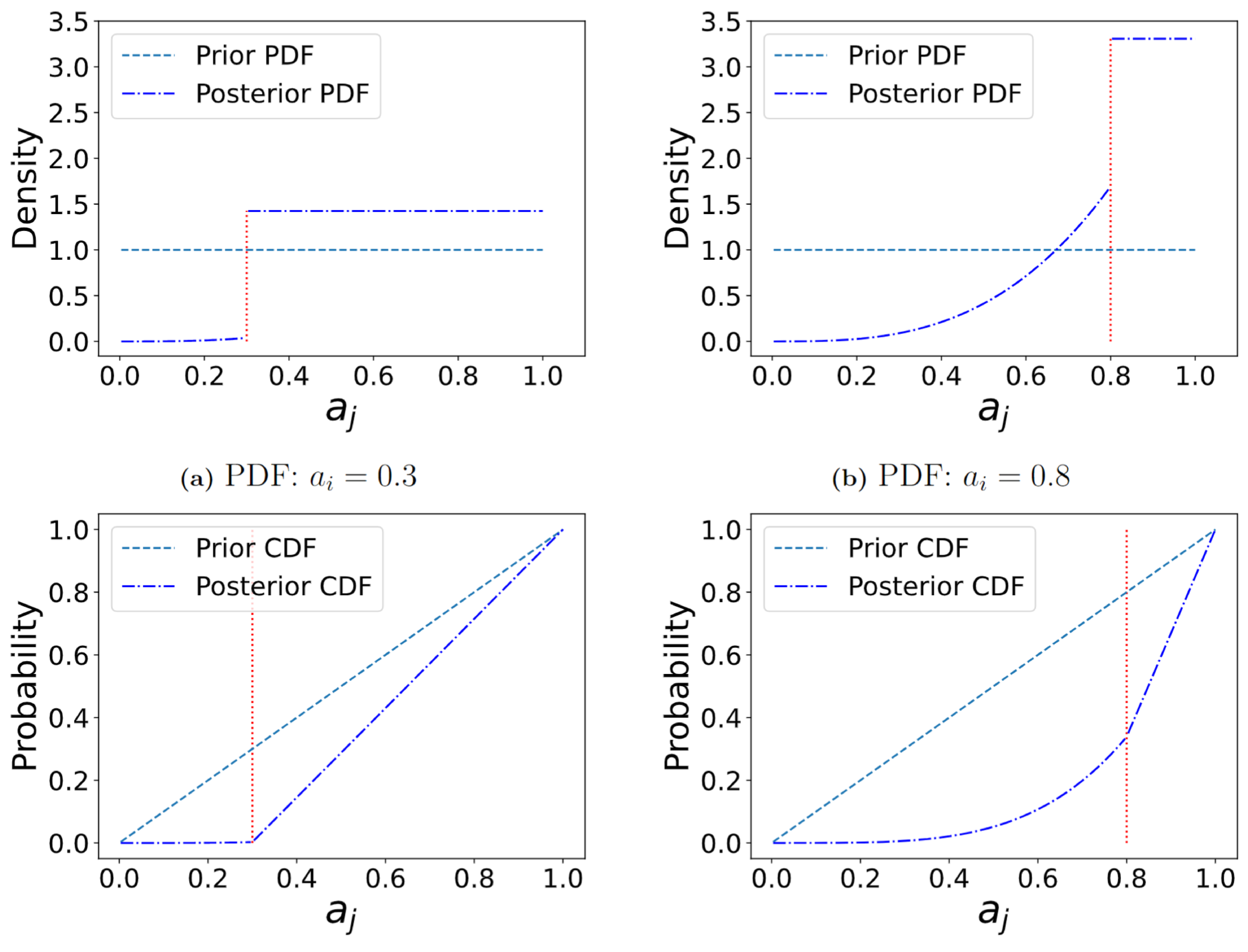
$$b_i(a_i) \in \arg\max_{e_i} \sum_{\ell=1}^{n_2} V_\ell P_{i\ell} - \frac{g(e_i)}{a_i},$$

where $P_{i\ell}$ is the probability that e_i ranks ℓ^{th} highest in $\{b_j(a_j) : j \in \mathcal{I}_{-i}\} \cup \{e_i\}$.

III. Equilibrium

Example of posterior beliefs: two admitted players. When $n_2 = 2$, for any player $i \neq j \in \mathcal{I}$, player i's posterior belief (PDF) about player j's ability is

$$\beta_i(a_j \mid s, a_i) = \begin{cases} \dfrac{f(a_j)}{\frac{F^{n_1-1}(a_i)}{n_1-1} + 1 - F(a_i)}, & a_i < a_j < 1, \\[2ex] \dfrac{f(a_j)F^{n_1-2}(a_j)}{\frac{F^{n_1-1}(a_i)}{n_1-1} + 1 - F(a_i)}, & 0 < a_j < a_i. \end{cases}$$



(a) PDF: $a_i = 0.3$ (b) PDF: $a_i = 0.8$ (c) CDF: $a_i = 0.3$ (d) CDF: $a_i = 0.8$

Comparison between prior and posterior beliefs ($n_1 = 5, n_2 = 2$ with uniform priors).

Remark 1: Player i's belief depends on her own private ability, thus *player i's belief is also private*, which is the key difference from standard Bayesian games.

Remark 2: One player's marginal posterior beliefs about different players' abilities can be correlated. Formally, unless $n_2 = n_1$, $\beta_i(a_{-i} \mid s, a_i) \neq \prod_{j \in \mathcal{I}_{-i}} \beta_i(a_j \mid s, a_i)$ in general.

Remark 3: The admitted player i's marginal posterior belief about another admitted player first-order stochastically dominates prior belief.

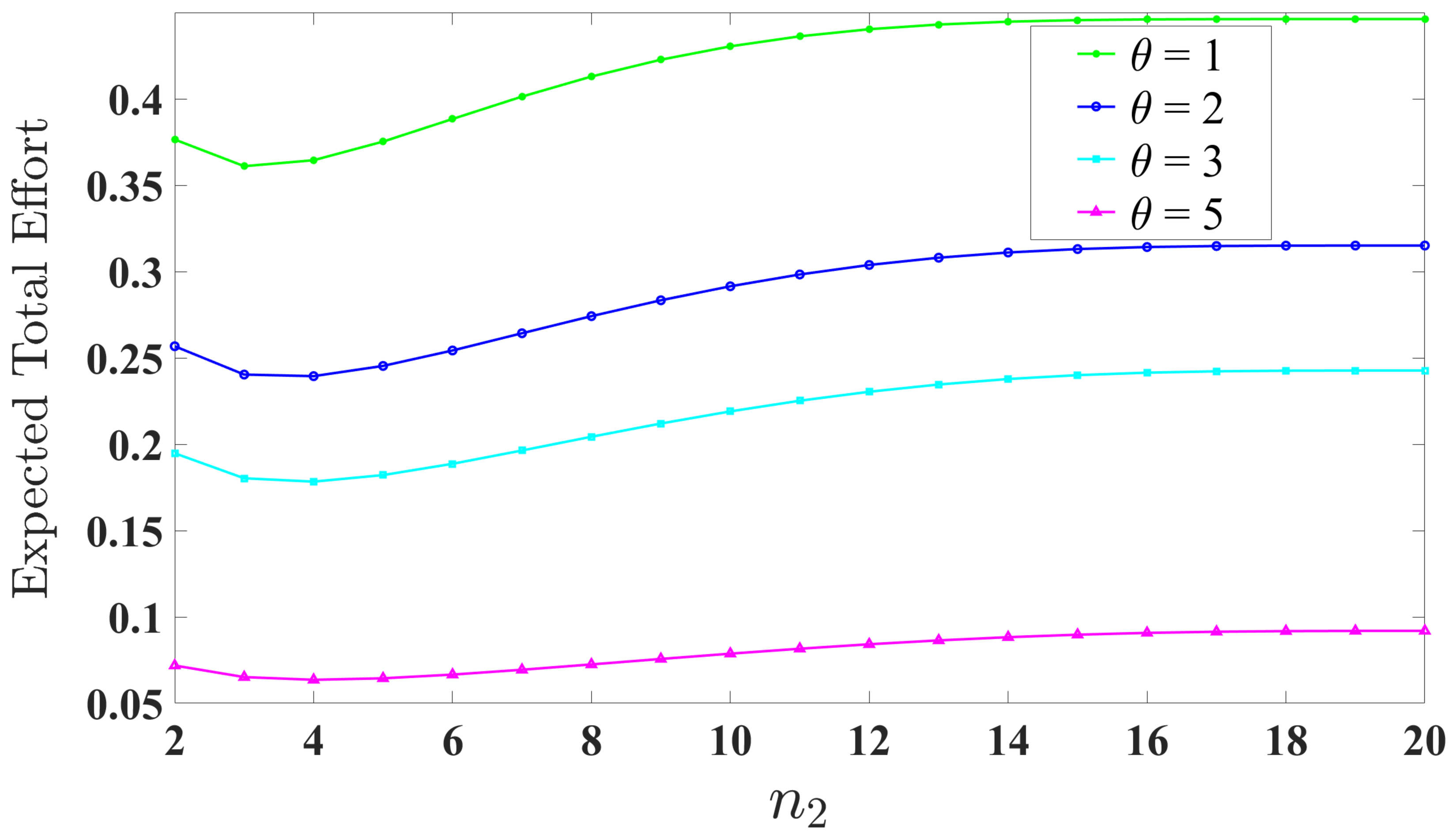
Proposition.
There exists a *unique symmetric* Bayesian Nash equilibrium strategy.

IV. Optimal Mechanism

Proposition. The equilibrium effort of player i with ability a_i is increasing in n_2 when $n_2 \in [\lfloor (n_1+1)/2 \rfloor + 1, n_1]$, and is in general *not monotone* in n_2 when $n_2 \in [2, \lfloor (n_1+1)/2 \rfloor + 1]$.

Theorem [*Equilibrium Dominance*]. Fix any prize structure V, all players exert weakly lower efforts compared with those under a regular one-round contest that admits all players.

Example: expected total efforts under winner-take-all reward structure, convex cost function $g(x) = x^5$, $n_1 = 20$, and prior ability distribution $F(x) = x^\theta$.



Remark 1: Given $n_2 = n_1$, optimal prize structure is winner-take-all when the cost function g is linear (Moldovanu and Sela 2001, Chawla et al. 2019).

Remark 2: When the designer cannot admit all registered n_1 players, numerical results show that the optimal n_2 lies at the corner points — either admitting 2 players or the maximum allowed.

V. Two-stage Sequential Elimination Contests

All n_1 players attend the first-stage contest, but only the players with top n_2 first-stage efforts can attend the second-stage contest.

Proposition. There does *not* exist a symmetric and monotone Perfect Bayesian Equilibrium (PBE) in any two-stage sequential elimination contest.