Model as solved with planned investment as unkown

1. Shocks: $Z = \{\epsilon^G, \epsilon^M\}$
2. Unkowns: $U = \{Y, w, r, I^p\}$
3. Targets: Goods market clearing, Wage residual, Fisher equation, Investment equation

The model can be summarized as the equation system

$$\boldsymbol{H}(\{Y_t, w_t, r_t, Z_t\}_{t=0}^{T}) = 0$$

$$\begin{bmatrix} \text{Good market clearing} \\ \text{Wage residual} \\ \text{Fisher residual} \\ \text{Investment equation} \end{bmatrix} = 0$$

$$\begin{bmatrix} C_t + G_t + I_t + I_t S\left(\frac{I_t}{I_{t-1}}\right) + \zeta L_{t-1} - Y_t \\ log\left(\frac{w_t}{w_{t-1}}\right) - (\pi_t^w - \pi_t) \\ 1 + i_t - (1+r_t)\mathbb{E}_t\left[\frac{P_{t+1}}{P_t}\right] \\ 1 + S\left(\frac{I_{t+1}}{I_t}\right) + \frac{I_{t+1}}{I_t} S'\left(\frac{I_{t+1}}{I_t}\right) - Q_t - \mathbb{E}\left[\frac{1}{1+r_{t+1}}\left(\frac{I_{t+2}}{I_{t+1}}\right)^2 S'\left(\frac{I_{t+2}}{I_{t+1}}\right)\right] \end{bmatrix} = 0$$

and implicitely

$$L_t = \int \ell(w, r^\ell, r^a)\mathcal{D}_t$$

$$A_t = \int a(r^a)\mathcal{D}_t$$

$$C_t = \int c(w, r^\ell, r^a)\mathcal{D}_t$$

(The household problem is specified below)

$$r_{t+1}^l = r_t - \xi$$

$$r_t^a = \Theta_p(\frac{d_t + p_t}{p_{t-1}}) + (1 - \Theta_p)(\frac{1 + \delta q_t}{q_{t-1}}) - 1$$

$$\Theta_p = \frac{p_{jt-1}}{A_{t-1}}$$

$$q_t = \frac{\mathbb{E}_t[1 + \delta q_{t+1}]}{1 + r_t}$$

$$p_{jt} = \frac{1}{1 + r_t}\mathbb{E}_t[D_{jt+1} + p_{jt+1}]$$

$$I_t = \begin{cases} I^{ss} & , t = 0 \\ I_{t-1}^p & , t > 0 \end{cases}$$

$$K_{t+1} = (1 - \delta)K_t + I_t$$

$$Q_t = \mathbb{E}_t\left[\frac{1}{1 + r_{t+1}}(r_{t+2}^K + (1 - \delta)Q_{t+1})\right]$$

$$D_t = \int D_{jt} dj + D_t^K = Y_t - w_t N_t - I_t \left(1 + S(\left(\frac{I_t}{I_{t-1}}\right)\right)$$

$$D_t^K = r_t^K K_t - I_t \left(1 + S\left(\frac{I_t}{I_{t-1}}\right)\right)$$

$$S\left(\frac{I_{t+1}}{I_t}\right) = \frac{\phi}{2}\left(\frac{I_{t+1}}{I_t} - 1\right)^2$$

$$s_t = \frac{w_t * N_t}{Y_t}\frac{1}{1-\alpha}$$

$$N_t = \left(\frac{Y}{\Theta K_t^\alpha}\right)^{\frac{1}{1-\alpha}}$$

$$G_t = G\left(1 + e_t^G\right)$$

$$G\epsilon_t^G = \left(\phi^G q_t B_t + (1-\phi^G)\tau_t \frac{W_t}{P_t} N_t\right)\epsilon_t^G$$

$$\tau_t = \underbrace{\frac{\left(1-\phi^G\right) G\epsilon_t^G}{\frac{W_t}{P_t} N_t}}_{\tau_t^{noShock}} + \phi q^{ss}\frac{\left(B_{t-1} - B^{ss}\right)}{Y^{ss}} + \tau^{ss}$$

$$B_t = \frac{\phi^G \epsilon_t^G G}{q_t} + \frac{G + (1+\delta q_t)B_{t-1} - \tau_t^{noShock}\frac{W_t}{P_t}N_t}{q_t}$$

$$\pi_{w,t} - \pi_{t-1} = \frac{\left(1-\beta\xi_w\right)\left(1-\xi_w\right)}{\xi_w}\frac{\epsilon_w}{\epsilon_w + v_w - 1}\left(s_{w,t} - \frac{\epsilon_w - 1}{\epsilon_w}\right) + \beta\mathbb{E}_t\left[\pi_{w,t+1} - \pi_t\right]$$

$$s_{w,t} = \frac{\int v'(n_{it})di}{(1-\tau_t)w_t \int \epsilon_{it} u'(c_{it})di}$$

$$\pi_t - \pi_{t-1} = \frac{\left(1-\xi_p\right)\left(1 - \frac{\xi_p}{1+r}\right)}{\xi_p}\frac{\epsilon_p}{v_p + \epsilon_p - 1}\left(s_t - \frac{\epsilon_p - 1}{\epsilon_p}\right) + \left(\frac{1}{1+r}\right)\mathbb{E}_t\left(\pi_{t+1} - \pi_t\right)$$

$$1 + i_t = (1 + r^{ss})^{1-\rho^m}(1 + i_{t-1})^{\rho^m}\left(\frac{P_t}{P_{t-1}}\right)^{(1-\rho^m)\phi}\left(1 + \epsilon_t^m\right)$$

Other relevant equations:

Household Block:

Households receive after-tax labor income is given as

$$V_t(l,a,s) = \max_{c,l} u(c) - v(N_t) + \beta\mathbb{E}_t\left[V_{t+1}(l',a',s')|s\right]$$

s.t.

$$c + l' = (1 + r_t^l)l + Z_t e(s) + d_t(a)$$
$$a' = (1 + r_t^a)a - d_t(a)$$
$$l' \geq 0$$

$$z_t = Z_t e(s)$$

$$Z_t = (1-\tau_t) w_t N_t$$

$$d_t(a) = (1+r_t^a)a + \chi((1+r_t^a)a - (1+r^{a,ss})\bar{a})$$

$\bar{a}$ is the target value for illiquid assets.

Other equations

Equations that are implicitely used to derive the variables in the block structure:

- Flow of funds constraint for the intermediary
  - beginning of period

$$(1+r_t^a)A_{t-1} + (1+r_t^l)L_{t-1} = (1+\delta q_t)B_{t-1} + \int (p_{jt} + D_{jt})v_{jt-1}dj - \xi L_{t-1}$$

  - end of period

$$\int p_{jt} v_{jt} dj + q_t B_t = A_t + L_t$$

- Asset pricing equations

$$\mathbb{E}_t[1+r_{t+1}^a] = \frac{\mathbb{E}_t[1+\delta q_{t+1}]}{q_t} = \frac{\mathbb{E}_t[p_{jt+1} + D_{jt+1}]}{p_{jt}} = 1 + r_{t+1}^l + \xi \equiv 1 + r_t$$

- Final good production

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{e_p}{e_p - 1}} dj \right)^{\frac{e_p - 1}{e_p}}$$

- Intermediate good production

$$Y_{jt} = \Theta K_{jt}^{\alpha} N_{jt}^{1-\alpha}$$

- Investment equations for deciding on optimal capital

$$1 + S\left(\frac{I_{t+1}}{I_t}\right) + \frac{I_{t+1}}{I_t} S'\left(\frac{I_{t+1}}{I_t}\right) = Q_t + \mathbb{E}\left[\frac{1}{1+r_{t+1}} \left(\frac{I_{t+2}}{I_{t+1}}\right)^2 S'\left(\frac{I_{t+2}}{I_{t+1}}\right)\right]$$

- Government budget constraint

$$q_t B_t \left(1 + \phi^G e_t^G\right) + \tau_t \frac{W_t}{P_t} N_t \left(1 + \left(1 - \phi^G\right) e_t^G\right) = G\left(1 + e_t^G\right) + (1+\delta q_t) B_{t-1}$$

$$\tau_t \frac{W_t}{P_t} N_t \mathbb{E}[e(s)] = \tau_t \frac{W_t}{P_t} N_t$$