# Gyroscope Operating Modes - Overview

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## 1 Gyroscope dynamics

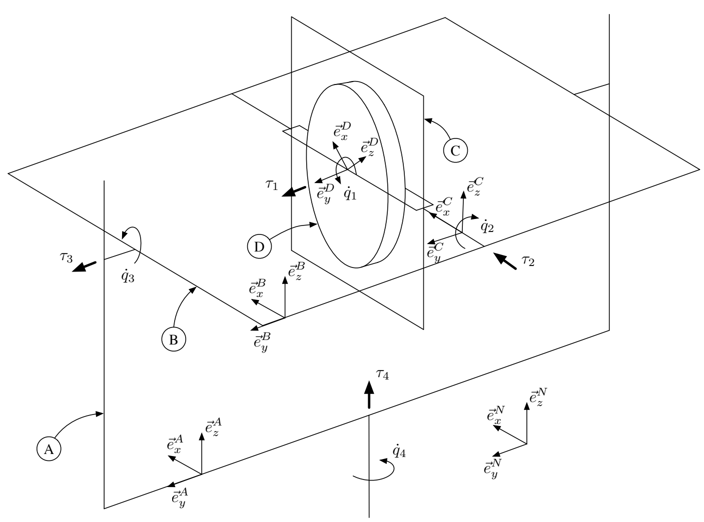


Figure 1: Schematic of the gyroscope

The dynamics of the gyroscope can be written in terms of an equation of motion, see [1], in the form

$$M(q(t))\ddot{q}(t) + \left(C\left(q(t), \dot{q}(t)\right) + F_\mathrm{v}\right)\dot{q}(t) = K_\mathrm{m} i(t) \tag{1}$$

where $q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^\top$ (in rad) is the generalized angular position vector (containing the angle of each gimbal), $\dot{q} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 \end{bmatrix}$ (in rad/s) the generalized angular velocity vector (containing the angular velocity of each gimbal), $i = \begin{bmatrix} i_1 & i_2 & i_3 & i_4 \end{bmatrix}$ (in A) is the motor currents vector, $M(q)$ is the inertia matrix, $C(q, \dot{q})$ is the Coriolis matrix, $F_\mathrm{v} = \mathrm{diag}(f_{\mathrm{v},1},\ f_{\mathrm{v},2},\ f_{\mathrm{v},3},\ f_{\mathrm{v},4})$

is the viscous friction vector, $K_\mathrm{m} = \mathrm{diag}(k_{\mathrm{m},1},\ k_{\mathrm{m},2},\ k_{\mathrm{m},3},\ k_{\mathrm{m},4}])$ is the motor constant vector. The equation (1) can be written in NL state-space form as

$$\underbrace{\begin{bmatrix}\dot{q}\\ \ddot{q}\end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix}I & 0\\ 0 & M(q)\end{bmatrix}^{-1}\begin{bmatrix}0 & I\\ 0 & -C(q,\dot{q}) - F_\mathrm{v}\end{bmatrix}}_{A(x)}\underbrace{\begin{bmatrix}q\\ \dot{q}\end{bmatrix}}_{x} + \underbrace{\begin{bmatrix}I & 0\\ 0 & M(q)\end{bmatrix}^{-1}\begin{bmatrix}0\\ K_\mathrm{m}\end{bmatrix}}_{B(x)}\underbrace{i}_{u}. \tag{2}$$

## 2 Operating Modes

Several operating modes of the gyroscope exists (by locking various gimbals), however, only three of them are interesting for our considered problem. For these three operating modes either one or none of the gimbals are locked. Locking more than one gimbal results in the dynamics becoming linear as the coupling due to the gyroscopic effect will not be there.

### 2.1 OM-1

The first operating mode, which will be referred to as OM-1, has the outer (silver gimbal) locked. This means that $q_4 = 0$, $\dot{q}_4 = 0$ and $i_4 = 0$. This means the states of the resulting NL-system are $x = \begin{bmatrix}q_2 & q_3 & \dot{q}_1 & \dot{q}_2 & \dot{q}_3\end{bmatrix}^\top$. For control purposes tracking of $q_2$, $q_3$ and $\dot{q}_1$ is of interest using the inputs $u = \begin{bmatrix}i_1 & i_2 & i_3\end{bmatrix}^\top$ (regulating the magnitude of $\dot{q}_2$ and $\dot{q}_3$ can also be added as objective). The resulting matrix $A(x)$ is in this case dependent on $q_2$, $\dot{q}_1$, $\dot{q}_2$ and $\dot{q}_3$ and the matrix $B(x)$ is dependent on $q_2$.

### 2.2 OM-2

The second operating mode, which will be referred to as OM-2, has the red gimbal locked. This means that $q_3 = 0$, $\dot{q}_3 = 0$ and $i_3 = 0$. This means the states of the resulting NL-system are $x = \begin{bmatrix}q_2 & q_4 & \dot{q}_1 & \dot{q}_2 & \dot{q}_4\end{bmatrix}^\top$. For control purposes tracking of $q_4$ and $\dot{q}_1$ is of interest using the inputs $u = \begin{bmatrix}i_1 & i_2\end{bmatrix}^\top$ (regulating the magnitude of $q_2$, $\dot{q}_2$ and $\dot{q}_4$ can also be added as objective). The resulting matrix $A(x)$ is in this case dependent on $q_2$, $\dot{q}_1$, $\dot{q}_2$ and $\dot{q}_4$ and the matrix $B(x)$ is dependent on $q_2$.

### 2.3 OM-3

The third and final operating mode, which will be referred to as OM-3, has all gimbals unlocked. his means that $q_3 = 0$, $\dot{q}_3 = 0$ and $i_3 = 0$. This means the states of the resulting NL-system are $x = \begin{bmatrix} q_2 & q_3 & q_4 & \dot{q}_1 & \dot{q}_3 & \dot{q}_2 & \dot{q}_4 \end{bmatrix}^\top$. For control purposes tracking of $q_3$, $q_4$, and $\dot{q}_1$ is of interest using the inputs $u = \begin{bmatrix} i_1 & i_2 \end{bmatrix}^\top$ (regulating the magnitude of $q_2$, $\dot{q}_2$, $\dot{q}_3$ and $\dot{q}_4$ can also be added as objective). The resulting matrix $A(x)$ is in this case dependent on $q_2$, $q_3$, $\dot{q}_1$, $\dot{q}_2$, $\dot{q}_3$ and $\dot{q}_4$ and the matrix $B(x)$ is dependent on $q_2$ and $q_3$.

### 2.4 Complexity overview

OM-1 and OM-2 are more or less similar in complexity, both in terms of complexity of the equations and (can be) in terms of the control objective. OM-3 is the most difficult operating mode as it considers the full gyroscope model.

## References

[1] T. Bloemers and R. Tóth, "Equations of motion of a control moment gyroscope," tech. rep., Eindhoven University of Technology, 2019.