CS 4810 Artificial Intelligence
CS 6810 Topics in Artificial Intelligence

Review: Tree search

- Initialize the **frontier** using the **starting state**
- While the frontier is not empty
  - Choose a frontier node to expand according to **search strategy** and take it off the frontier
  - If the node contains the **goal state**, return solution
  - Else **expand** the node and add its children to the frontier

- To handle repeated states:
  - Keep an **explored set**; add each node to the explored set every time you expand it
  - Every time you add a node to the frontier, check whether it already exists in the frontier with a higher path cost, and if yes, replace that node with the new one

Review: Uninformed search strategies

- Breadth-first search
- Depth-first search
- Iterative deepening search
- Uniform-cost search

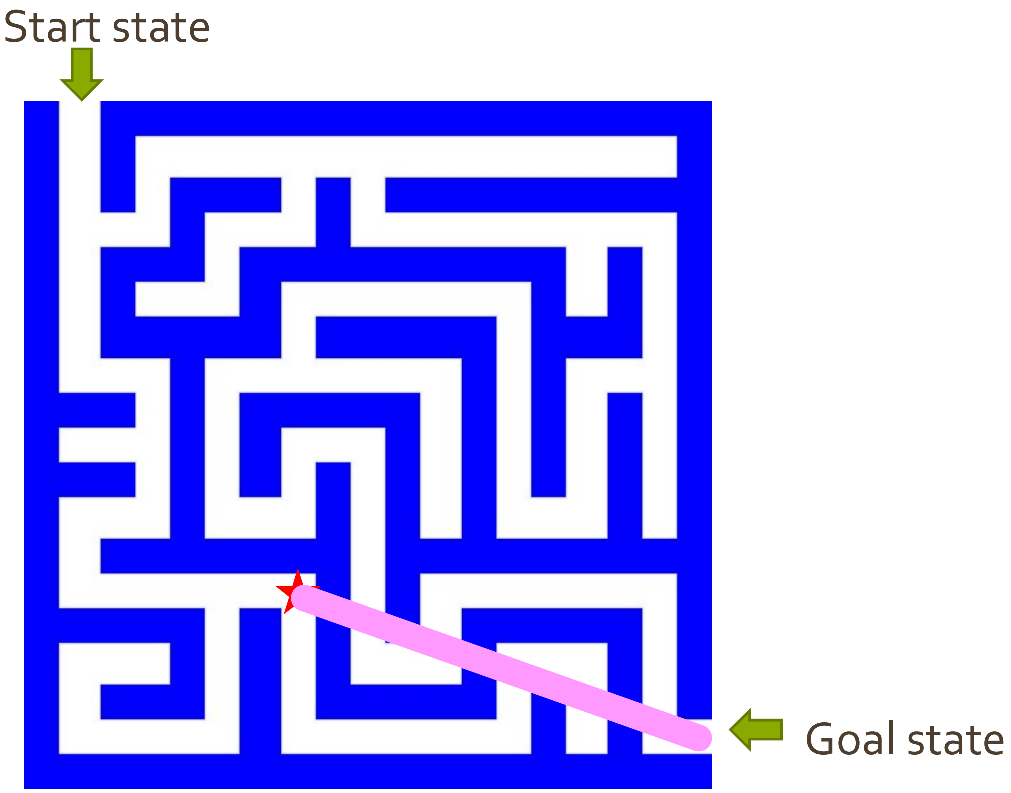
Informed search strategies (Sections 3.5-3.6)

- Idea: give the algorithm "hints" about the desirability of different states
  - Use an ***evaluation function*** to rank nodes and select the most promising one for expansion

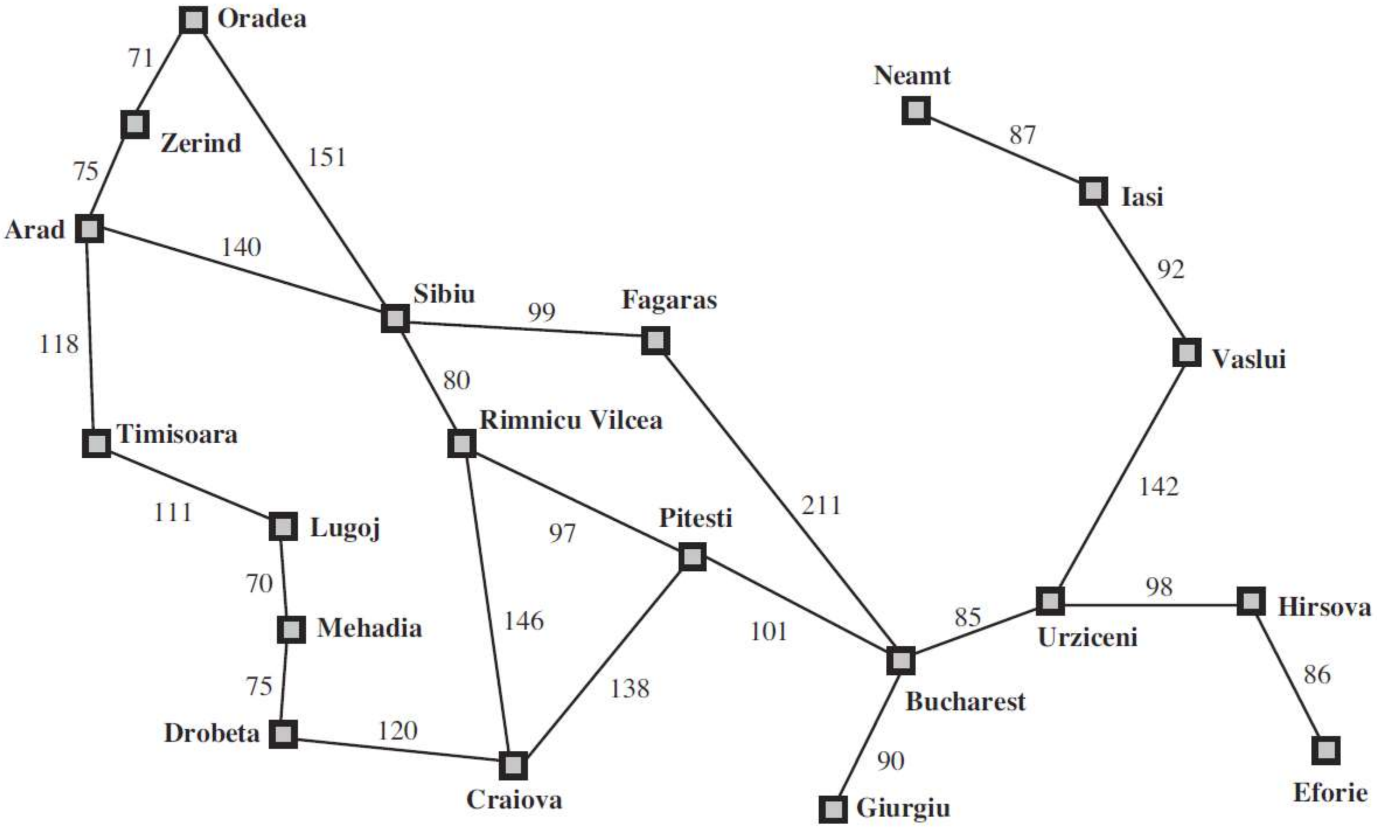
- Greedy best-first search
- A* search

Heuristic function

- **Heuristic function** *h*(*n*) estimates the cost of reaching goal from node *n*
- Example:



Heuristic for the Romania problem



Straight–line distance to Bucharest	
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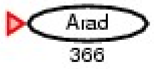
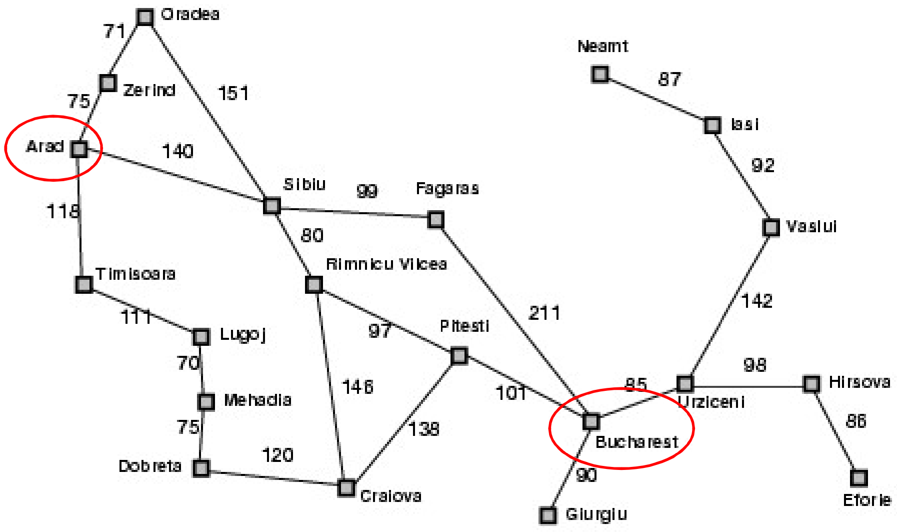
Greedy best-first search

- Expand the node that has the lowest value of the heuristic function *h*(*n*)

Greedy best-first search example





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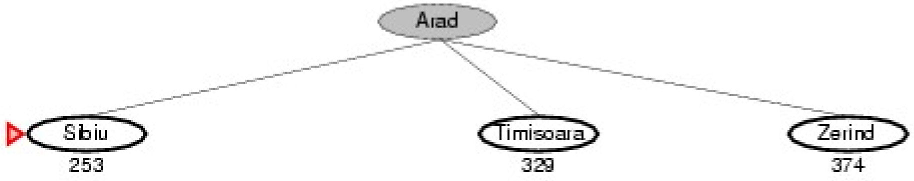

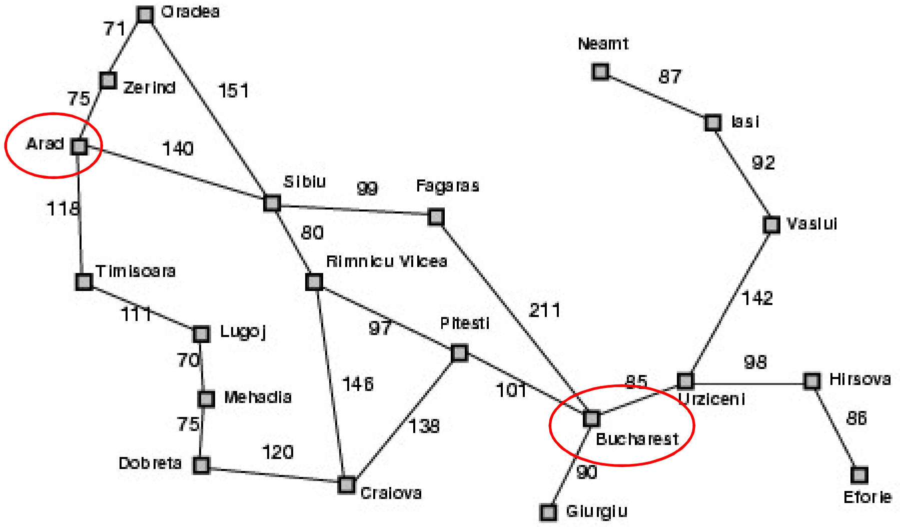

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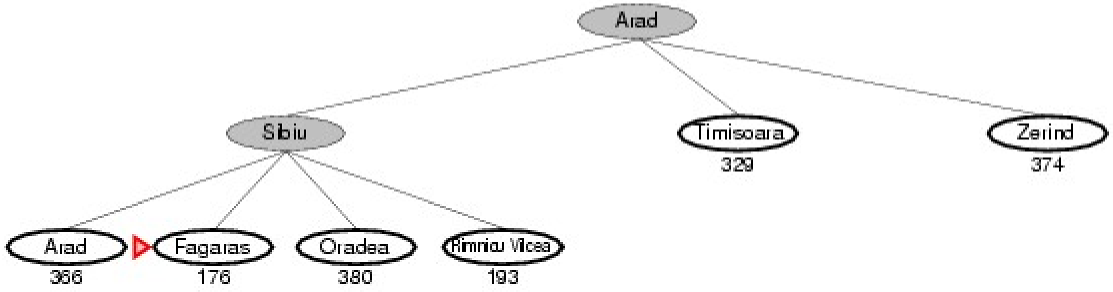
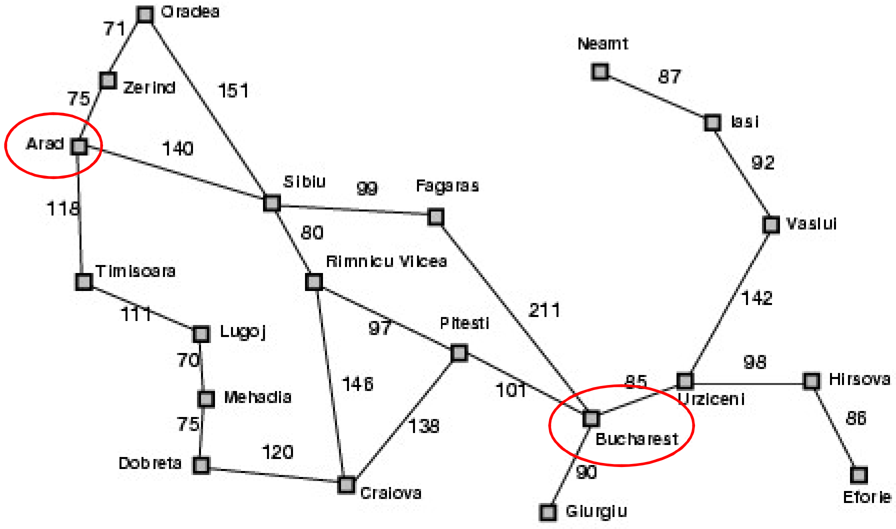
Greedy best-first search example





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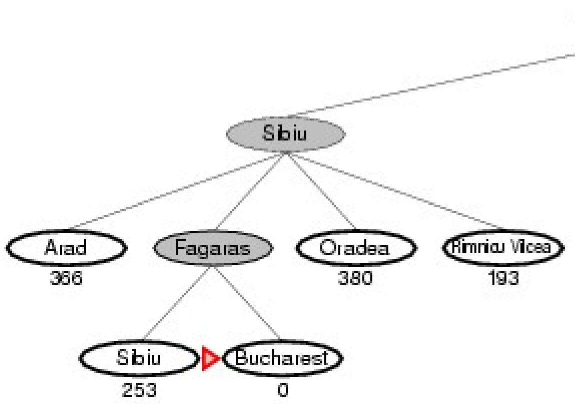
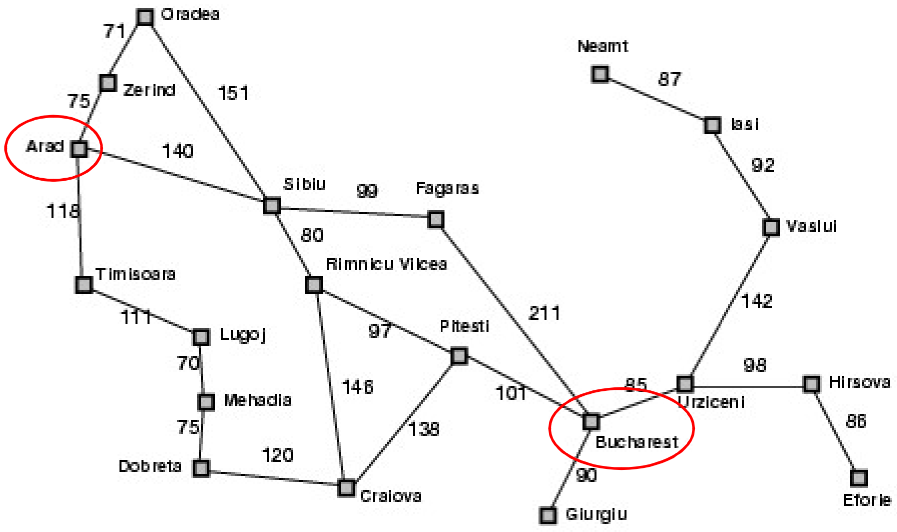
Greedy best-first search example





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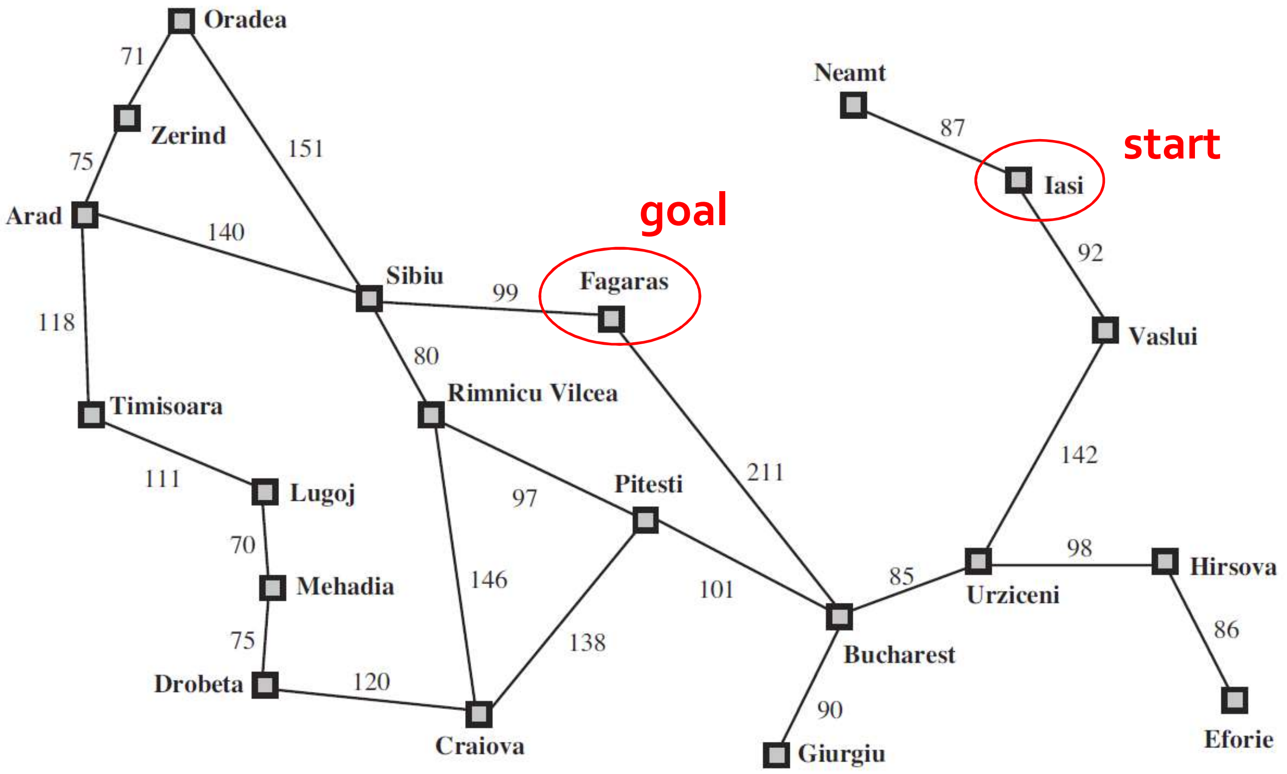
Properties of greedy best-first search

- **Complete?**

  No – can get stuck in loops



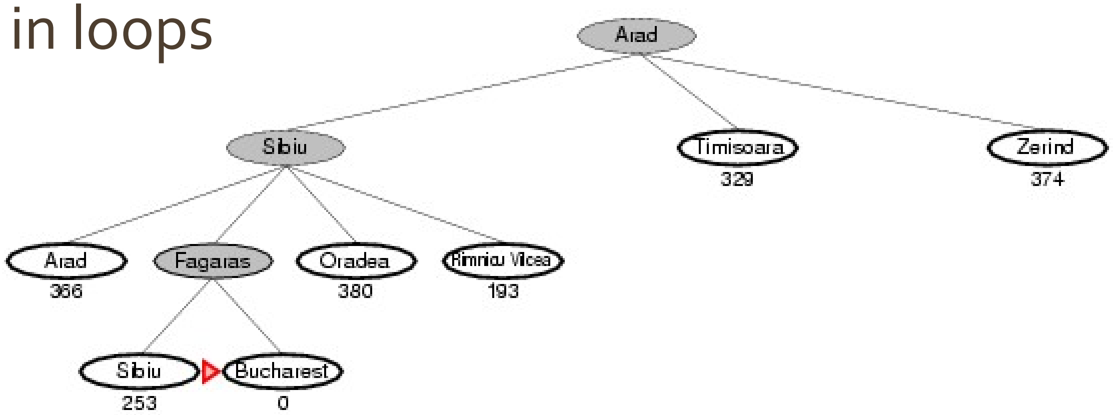
Properties of greedy best-first search

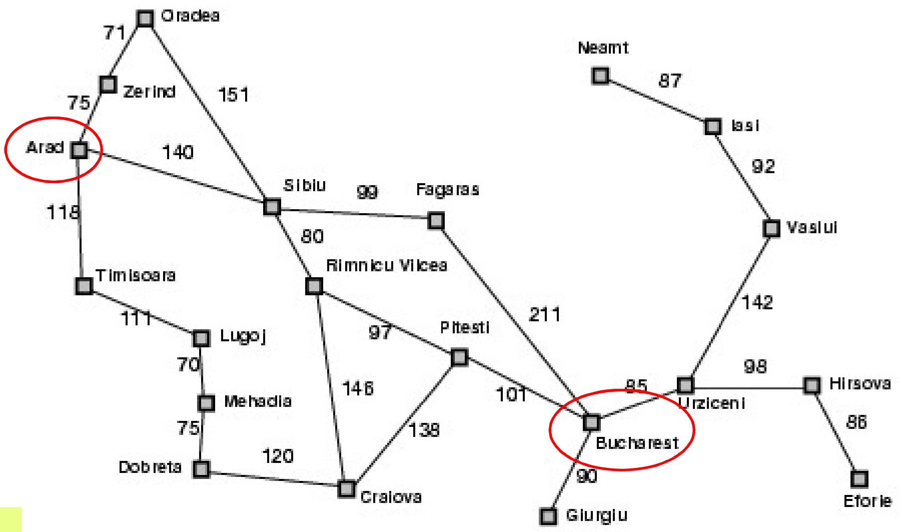
- **Complete?**

  No – can get stuck in loops

- **Optimal?**

  No





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Properties of greedy best-first search

- **Complete?**

  No – can get stuck in loops

- **Optimal?**

  No

- **Time?**

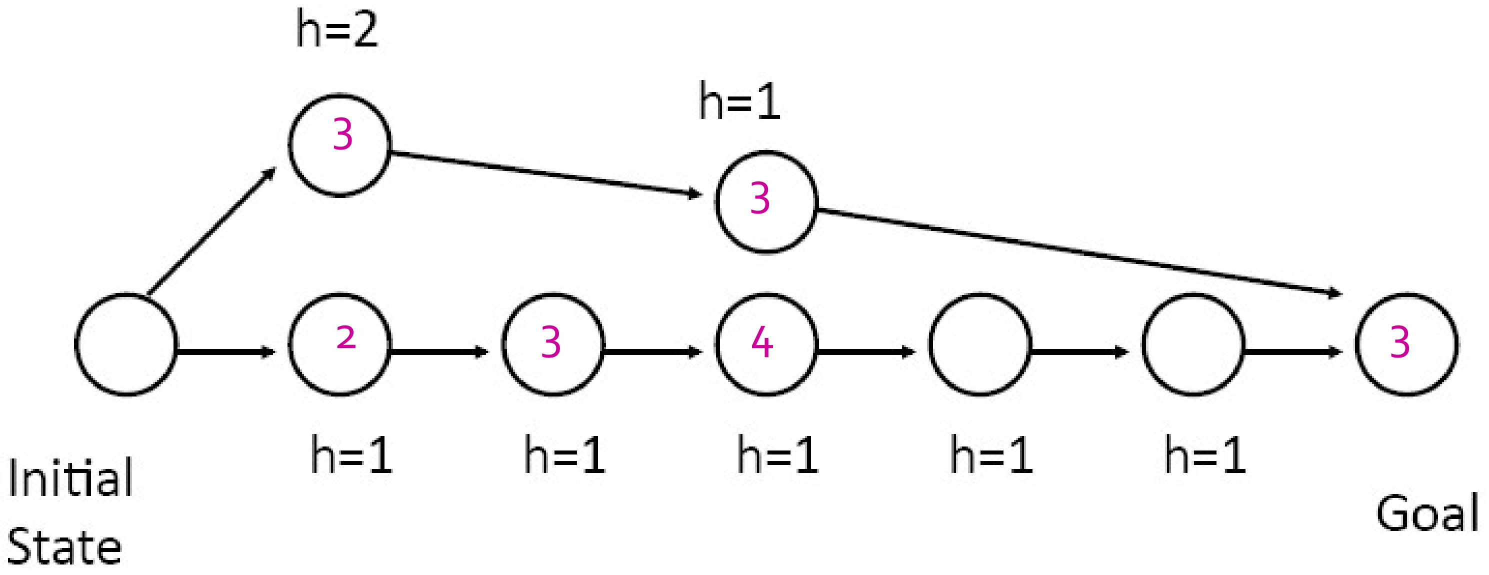
  Worst case: $O(b^m)$

  Can be much better with a good heuristic

- **Space?**

  Worst case: $O(b^m)$

How can we fix the greedy problem?



- How about keeping track of the distance already traveled in addition to the distance remaining?

A* search

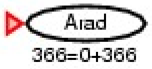
- Idea: avoid expanding paths that are already expensive
- The **evaluation function** $f(n)$ is the estimated total cost of the path through node *n* to the goal:

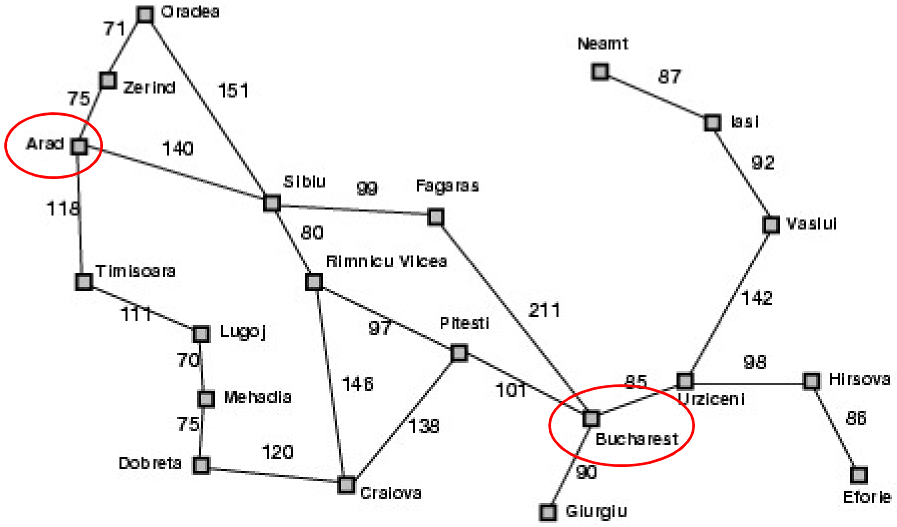
$$f(n) = g(n) + h(n)$$

$g(n)$: cost so far to reach *n* (path cost)

$h(n)$: estimated cost from *n* to goal (heuristic)

A* search example





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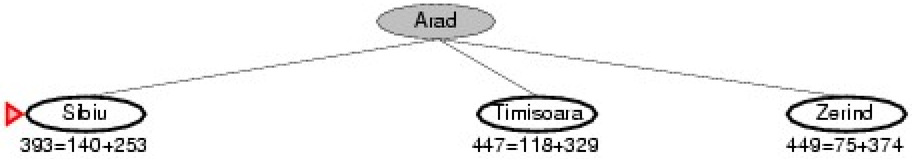

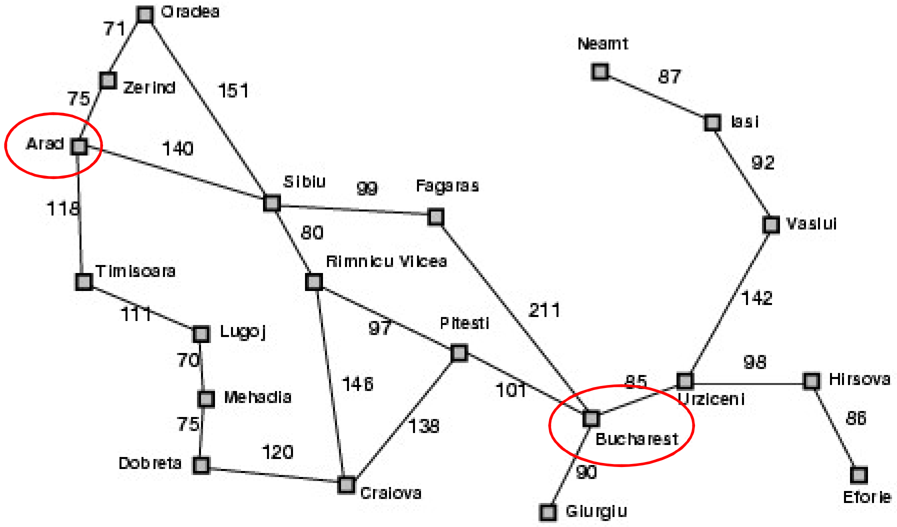

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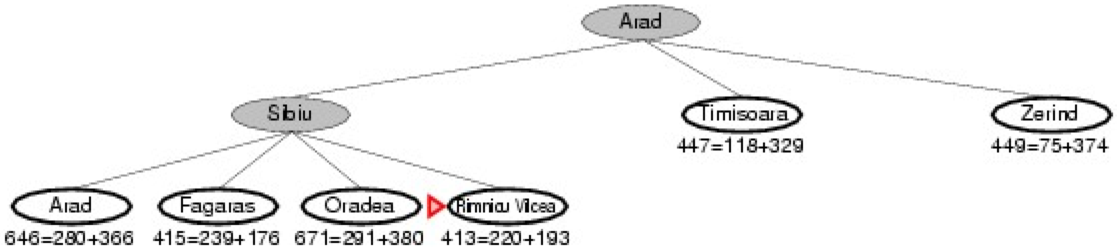

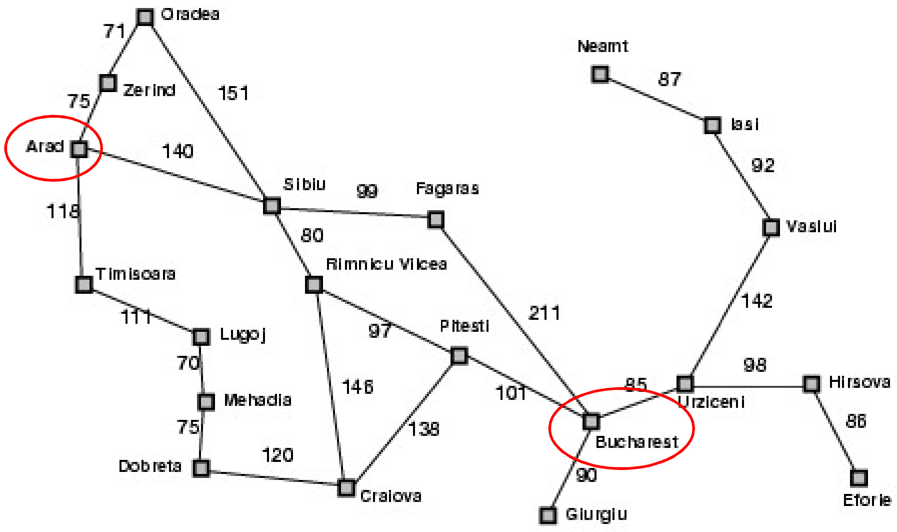

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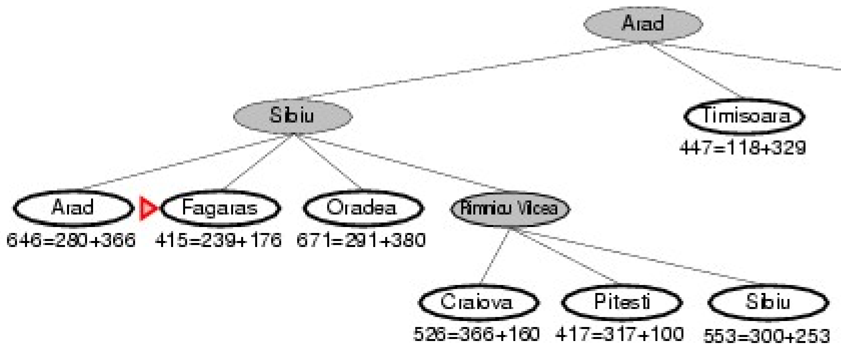

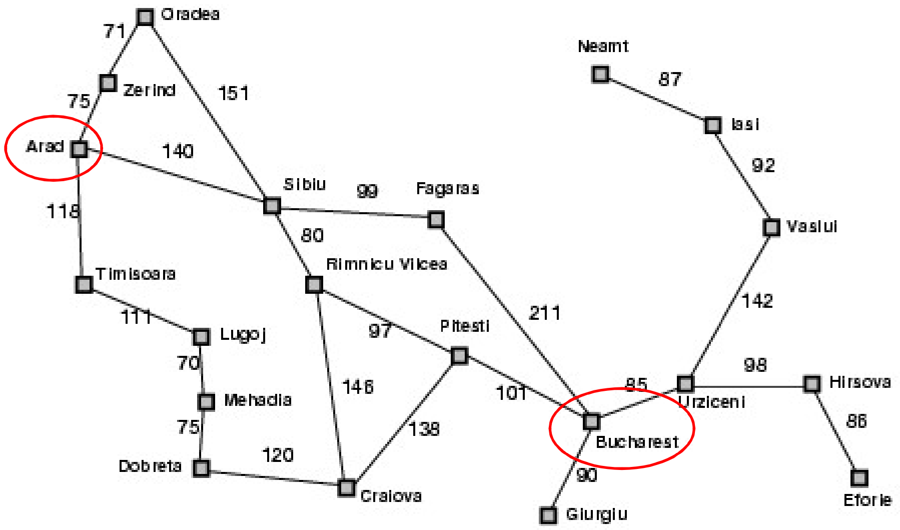

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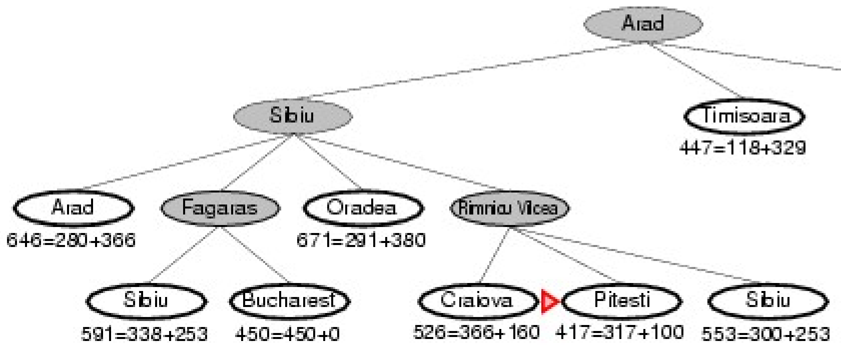

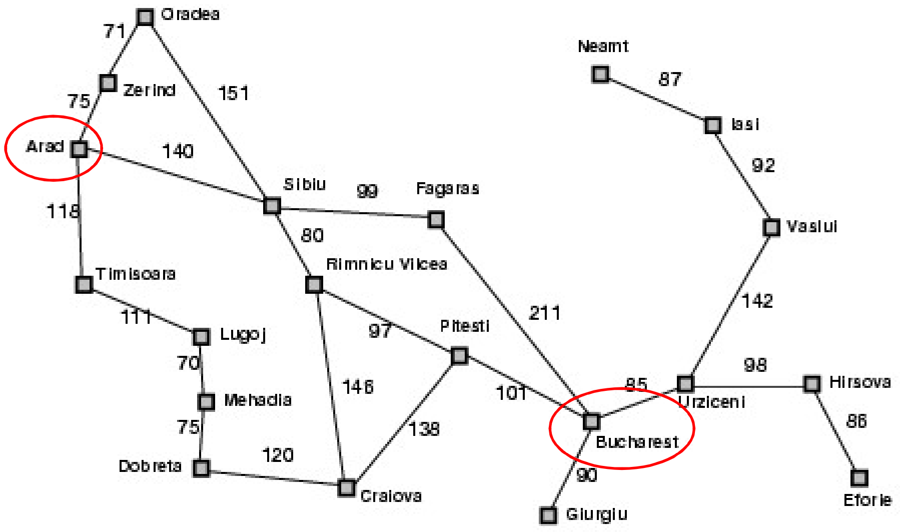

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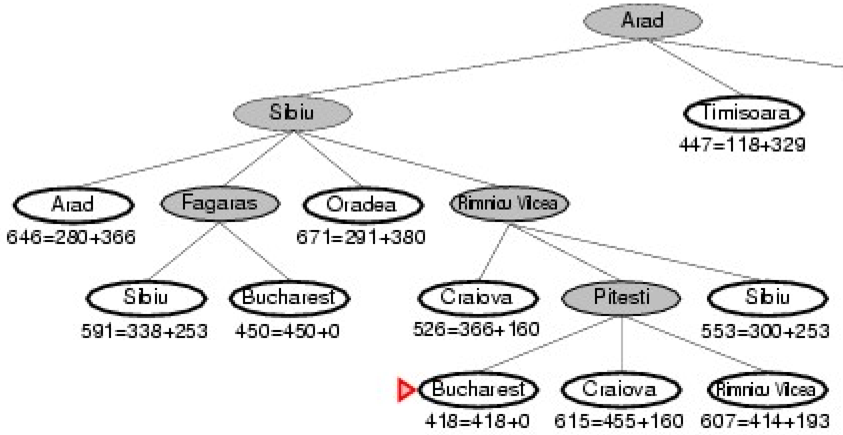

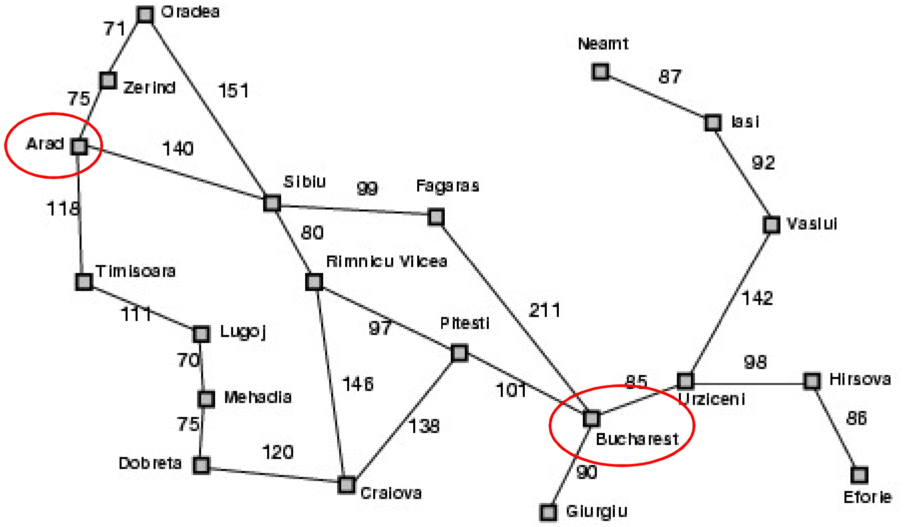

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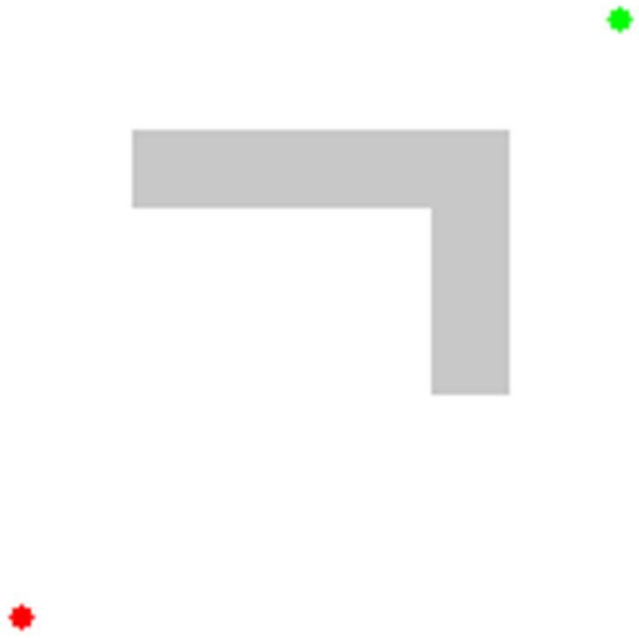
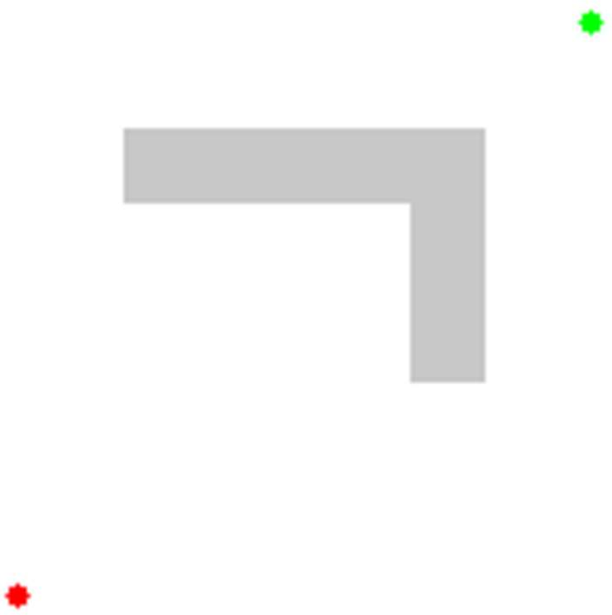
Another example



Source: Wikipedia

Uniform cost search vs. A* search



Source: Wikipedia

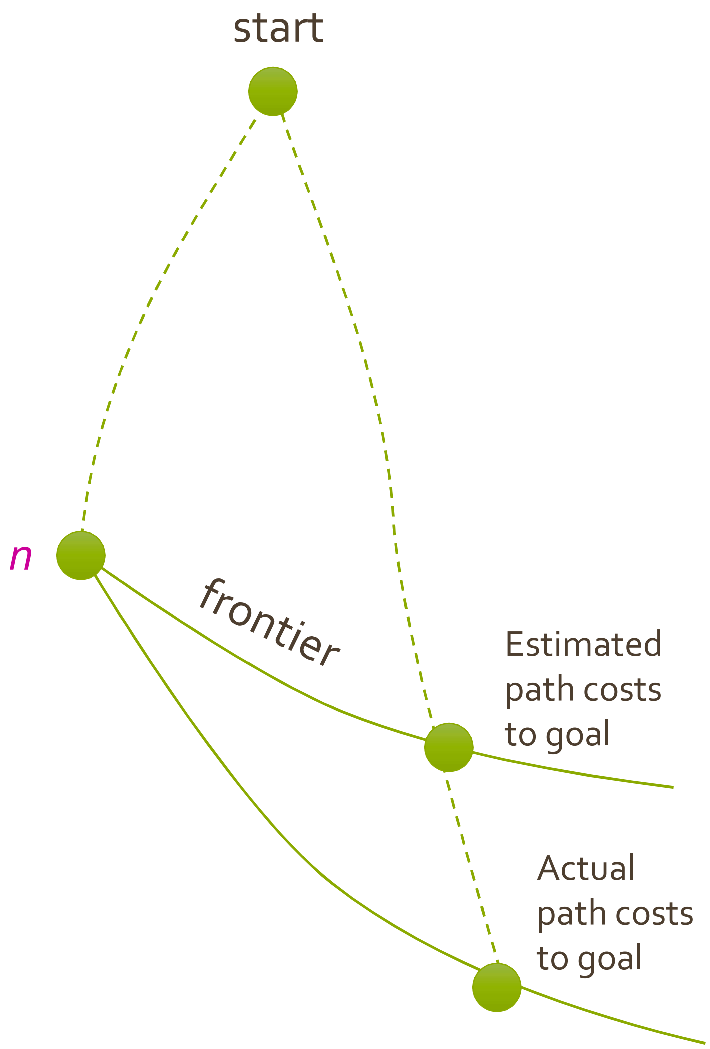
Admissible heuristics

- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^{*}(n)$, where $h^{*}(n)$ is the true cost to reach the goal state from n
- Example: straight line distance never overestimates the actual road distance
- **Theorem:** If $h(n)$ is admissible, A^{*} is optimal

Optimality of A*

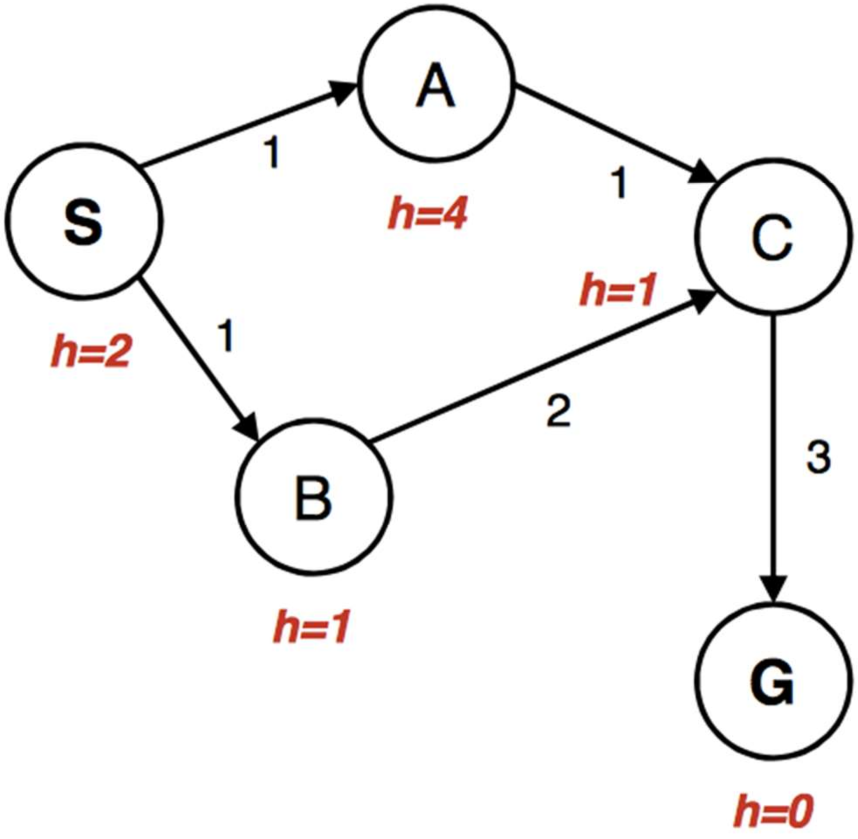
- **Theorem:** If the heuristic is admissible, A* *without repeated state detection* is optimal

- Proof sketch:
  - Let *C** be the evaluation function value (*actual* path cost) of the first goal node we select for expansion
  - Then all the other nodes on the frontier have *estimated* path costs to the goal that are at least as big as *C**
  - Because we are using an admissible heuristic, the *true* path costs to the goal for those nodes cannot be less than *C**

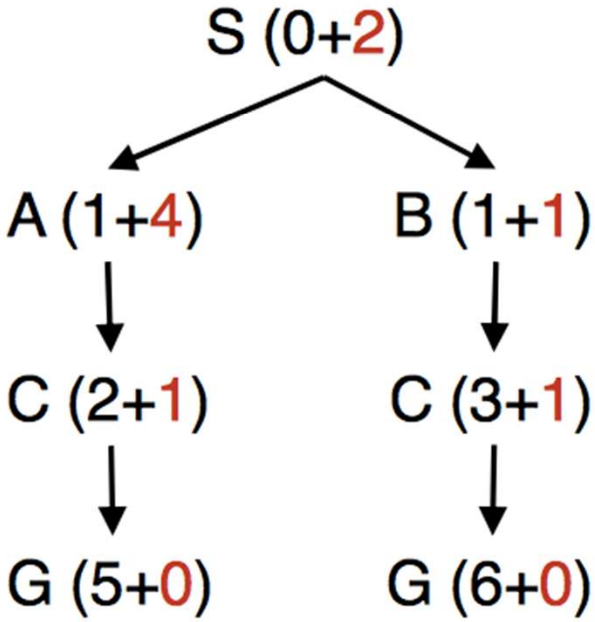


A* gone wrong?

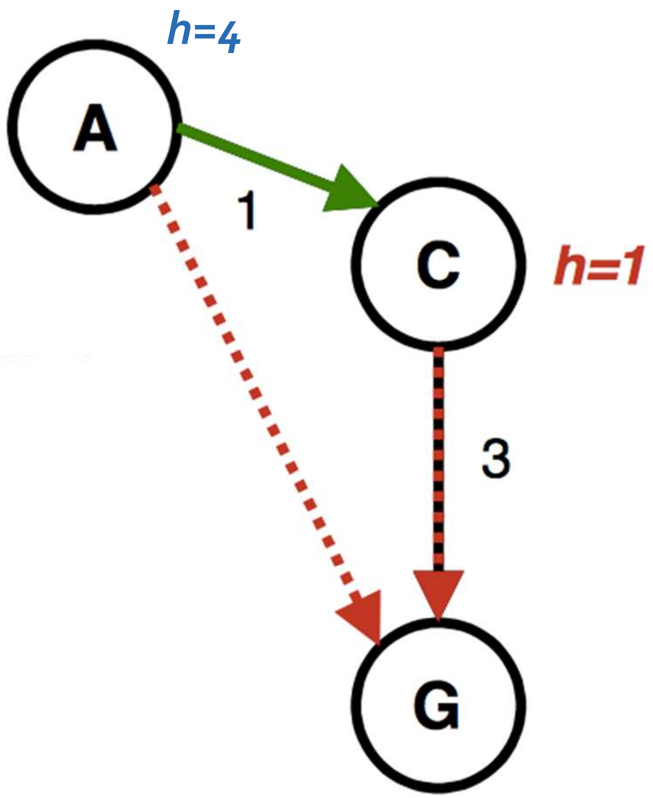
State space graph



Search tree



Consistency of heuristics



- Consistency: Stronger than admissibility
- Definition:

  cost(A to C) + h(C) ≥ h(A)

  cost(A to C) ≥ h(A) - h(C)

  real cost ≥ cost implied by heuristic

- Consequences:
  - The f value along a path never decreases
  - A* graph search is optimal

Source: Berkeley CS188x

Optimality of A*

- **Tree search** (i.e., search without repeated state detection):
  - A* is optimal if heuristic is ***admissible*** (and non-negative)
- **Graph search** (i.e., search with repeated state detection)
  - A* optimal if heuristic is ***consistent***
- Consistency implies admissibility
  - In general, most natural admissible heuristics tend to be consistent, especially if they come from relaxed problems

Source: Berkeley CS188x

Optimality of A*

- A* is *optimally efficient* – no other tree-based algorithm that uses the same heuristic can expand fewer nodes and still be guaranteed to find the optimal solution
    - A* expands all nodes for which $f(n) \leq C^*$. Any algorithm that does not risks missing the optimal solution

Properties of A*

- **Complete?**

  Yes – unless there are infinitely many nodes with *f*(*n*) ≤ *C**

- **Optimal?**

  Yes

- **Time?**

  Number of nodes for which *f*(*n*) ≤ *C** (exponential)

- **Space?**

  Exponential