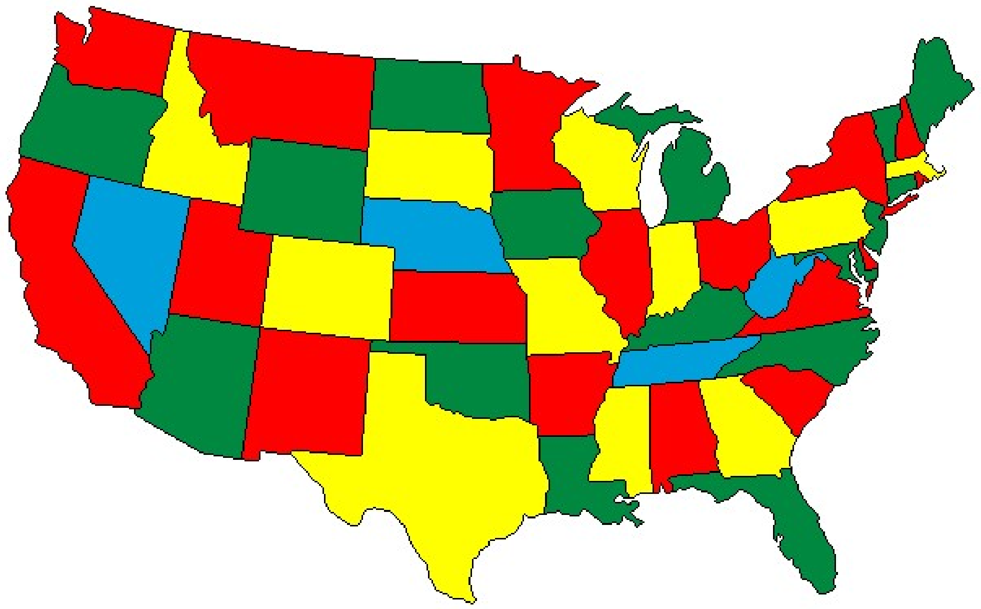
CS 4810 Artificial Intelligence
CS 6810 Topics in Artificial Intelligence

Constraint Satisfaction Problems



8			4		6			7
						4		
	1					6	5	
5		9		3		7	8	
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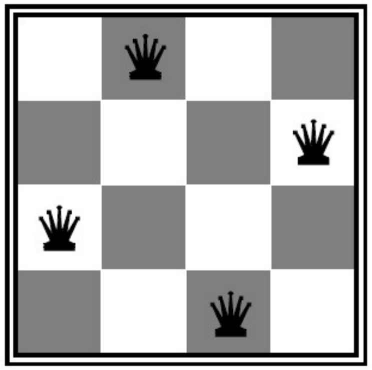


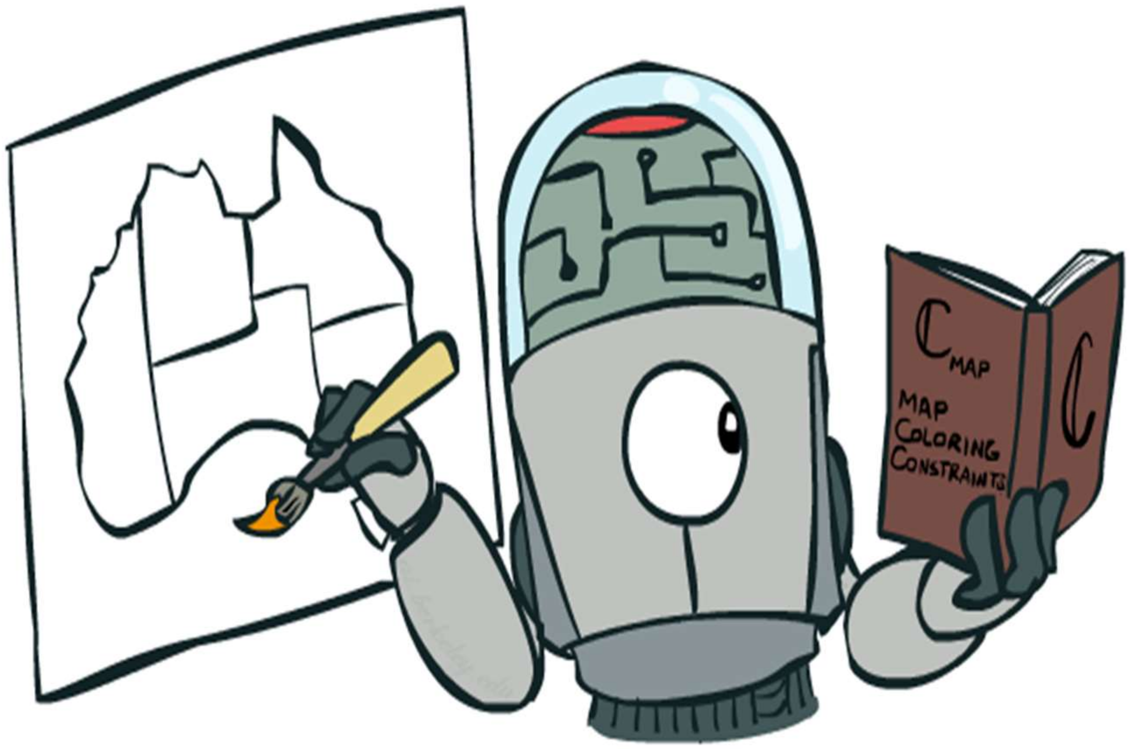
What is Search For?

- Assumptions about the world: <u>a single agent</u>, <u>deterministic actions</u>, <u>fully observed state</u>, <u>discrete state space</u>

- ***<u>Planning</u>***: sequences of actions
  - The path to the goal is the important thing
  - Paths have various costs, depths
  - Heuristics give problem-specific guidance

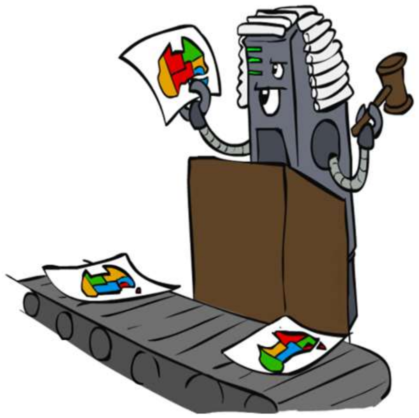
- ***<u>Identification</u>***: assignments to variables
  - The goal itself is important, not the path
  - All paths at the same depth (for some formulat
  - CSPs are specialized for identification problem

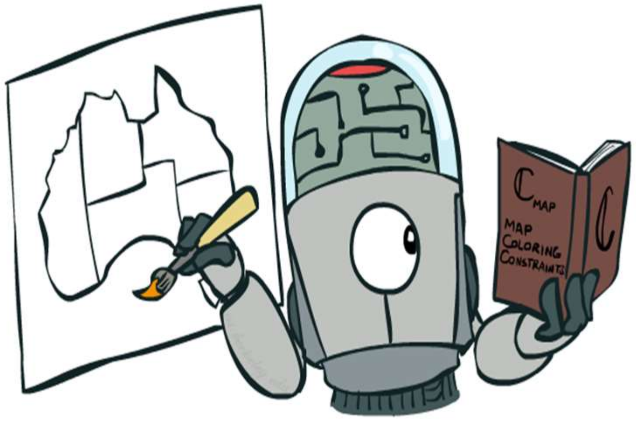
Constraint Satisfaction Problems



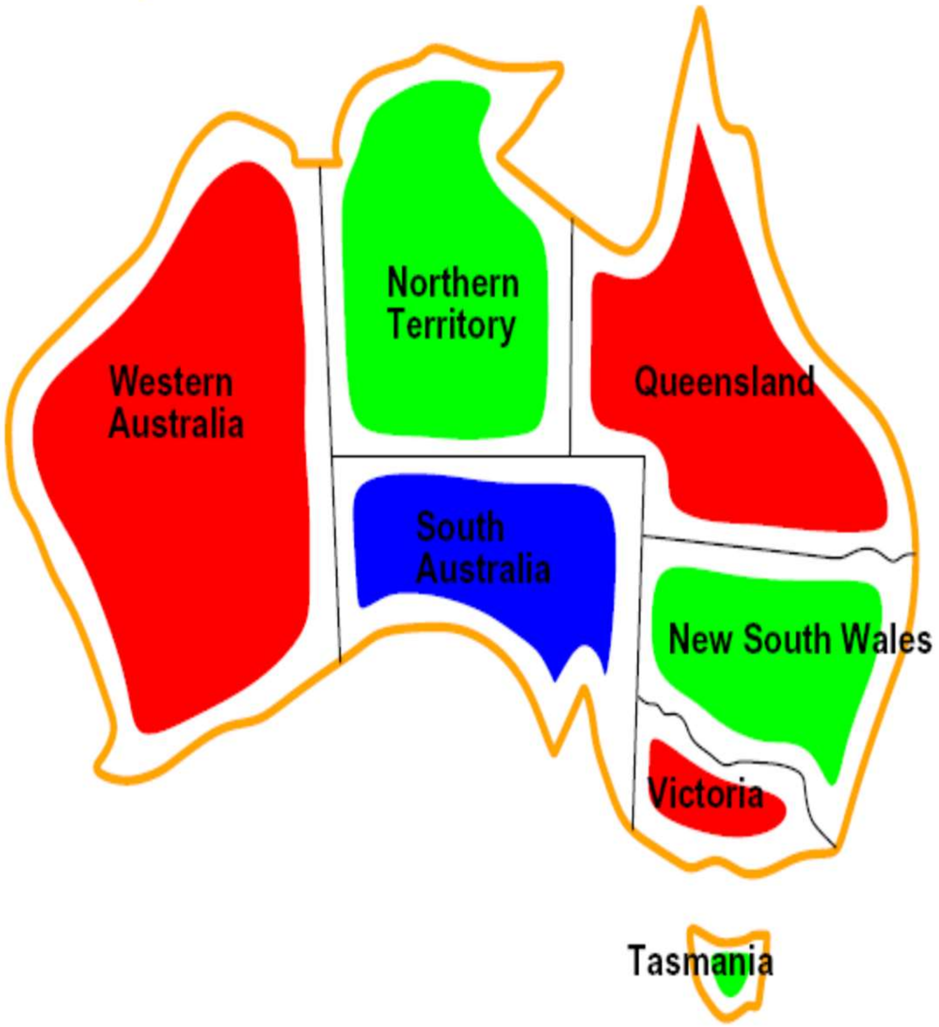
Constraint Satisfaction Problems

- Standard search problems:
  - State is a "black box": arbitrary data structure
  - Goal test can be any function over states
  - Successor function can also be anything
- Constraint satisfaction problems (CSPs):
  - A special subset of search problems
  - State is defined by variables X_i with values from a domain D (sometimes D depends on i)
  - Goal test is a set of constraints specifying allowable combinations of values for subsets of variables
- Simple example of a *formal representation language*
- Allows useful general-purpose algorithms with more power than standard search algorithms





CSP Examples



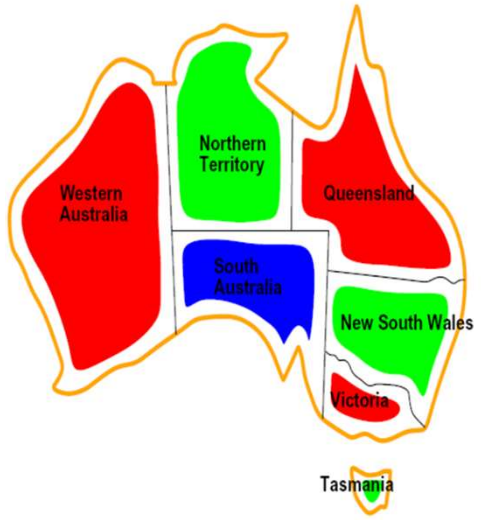
Example: Map Coloring

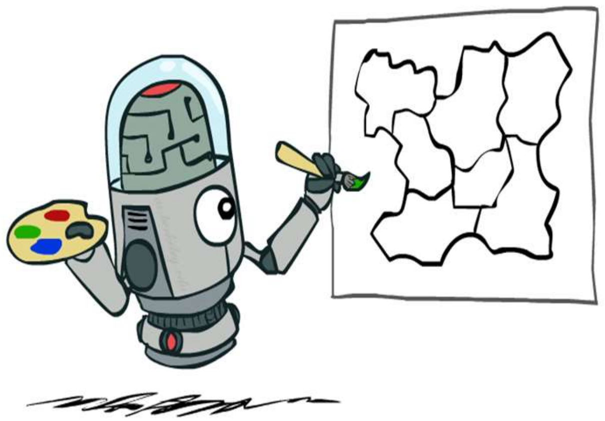
- Variables: WA, NT, Q, NSW, V, SA, T
- Domains: $D = \{\text{red}, \text{green}, \text{blue}\}$
- Constraints: adjacent regions must have different colors

  Implicit: $WA \neq NT$

  Explicit: $(WA, NT) \in \{(\text{red}, \text{green}), (\text{red}, \text{blue}), \ldots\}$
- Solutions are assignments satisfying all constraints, e.g.:

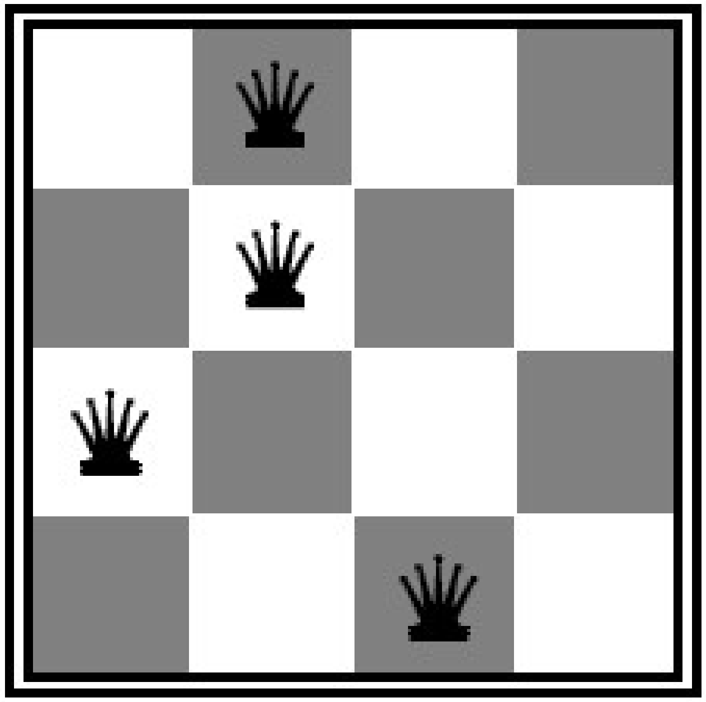
  {WA=red, NT=green, Q=red, NSW=green, V=red, SA=blue, T=green}

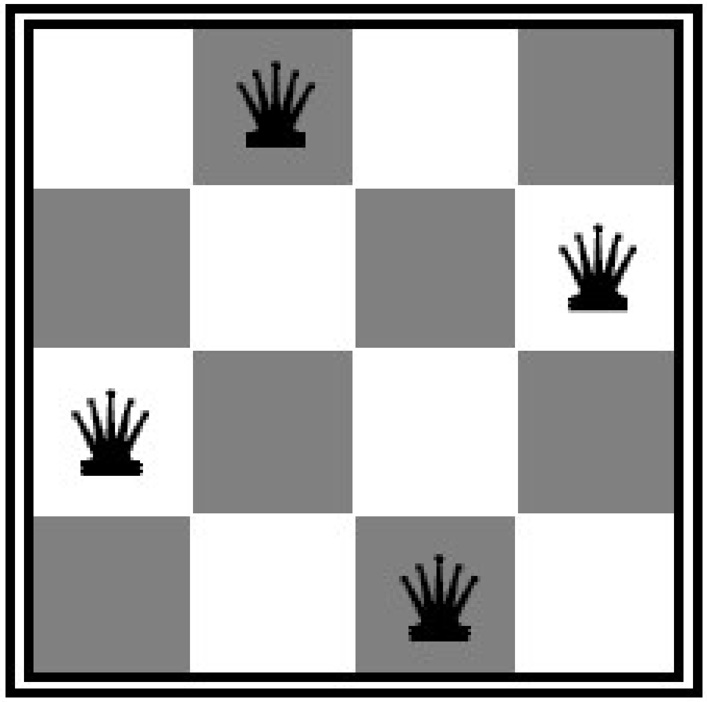




Example: *n*-queens problem

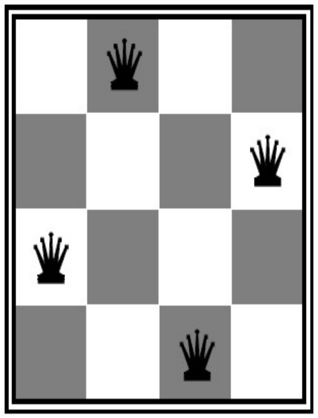
- Put *n* queens on an *n* × *n* board with no two queens on the same row, column, or diagonal

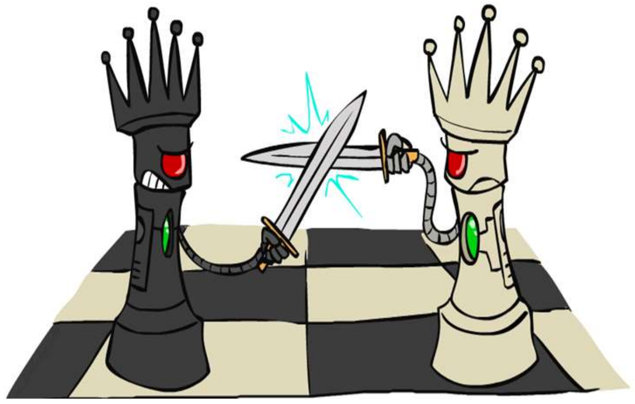




Example: N-Queens

- Formulation 1:
  - Variables: X_{ij}
  - Domains: $\{0,1\}$
  - Constraints



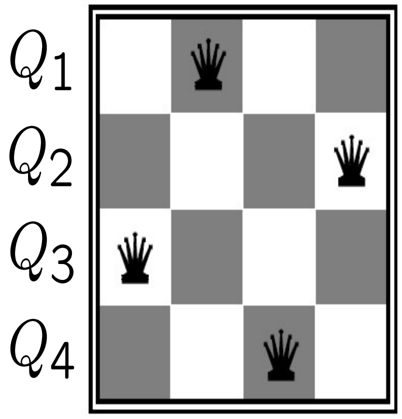


$$\forall i,j,k \;\; (X_{ij}, X_{ik}) \in \{(0,0),(0,1),(1,0)\}$$
$$\forall i,j,k \;\; (X_{ij}, X_{kj}) \in \{(0,0),(0,1),(1,0)\}$$
$$\forall i,j,k \;\; (X_{ij}, X_{i+k,j+k}) \in \{(0,0),(0,1),(1,0)\}$$
$$\forall i,j,k \;\; (X_{ij}, X_{i+k,j-k}) \in \{(0,0),(0,1),(1,0)\}$$

$$\sum_{i,j} X_{ij} = N$$

Example: N-Queens

- Formulation 2:
  - Variables: Q_k
  - Domains: $\{1, 2, 3, \dots N\}$
  - Constraints:

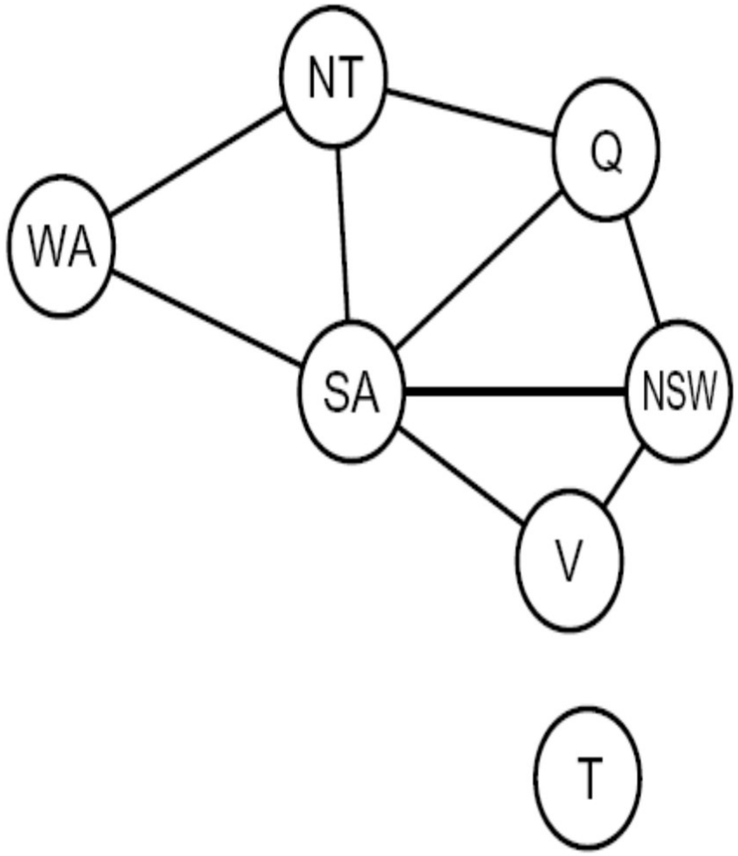


Implicit: $\forall i, j$ non-threatening(Q_i, Q_j)

Explicit: $(Q_1, Q_2) \in \{(1,3), (1,4), \dots\}$

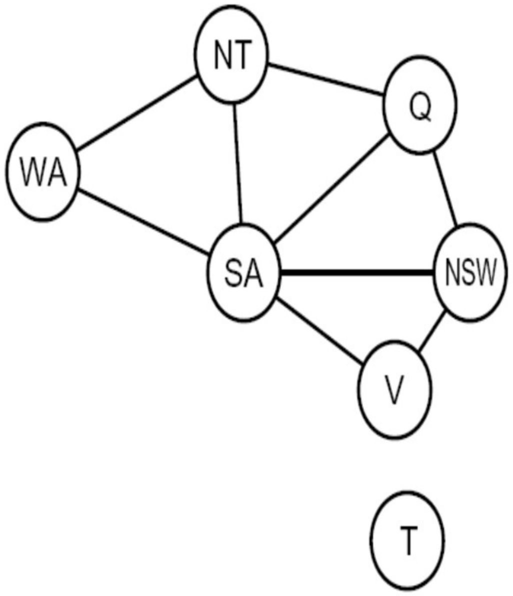
$\dots$

Constraint Graphs



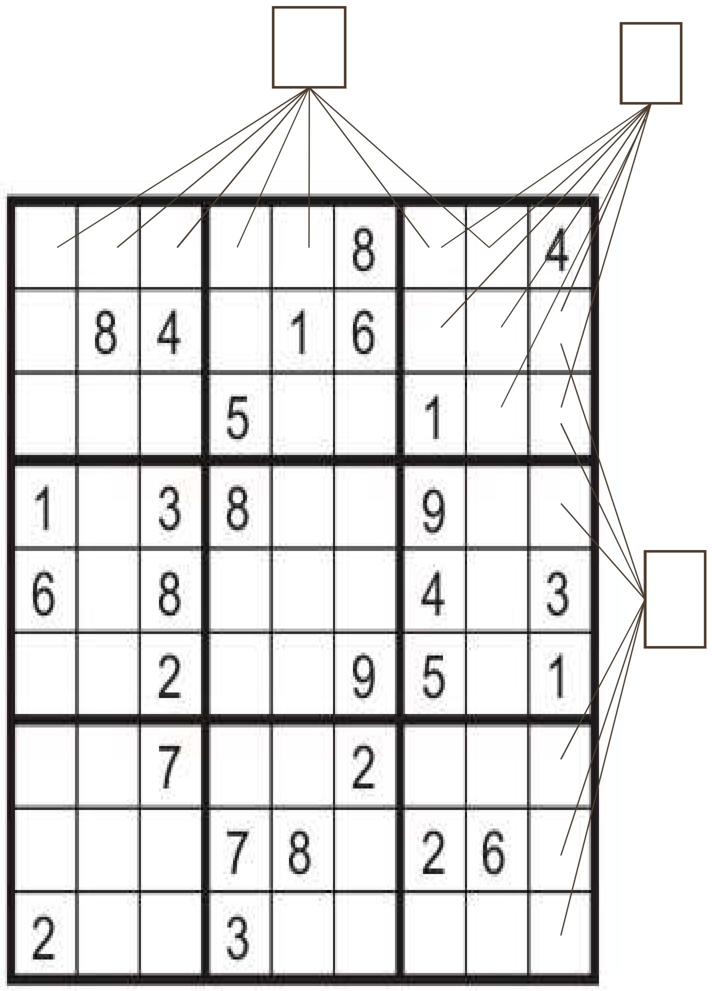
Constraint Graphs

- Binary CSP: each constraint relates (at most) two variables
- Binary constraint graph: nodes are variables, arcs show constraints
- General-purpose CSP algorithms use the graph structure to speed up search. E.g., Tasmania is an independent subproblem!



[Demo: CSP applet (made available by

Example: Sudoku



- Variables:
  - Each (open) square
- Domains:
  - {1,2,…,9}
- Constraints:

9-way alldiff for each column

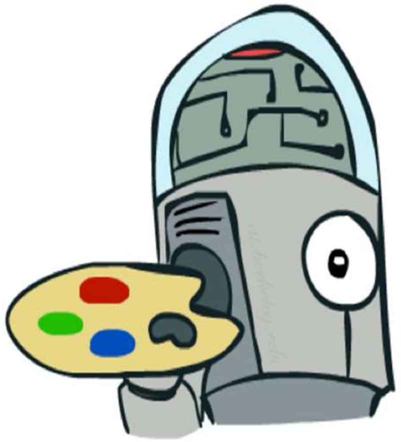
9-way alldiff for each row

9-way alldiff for each region

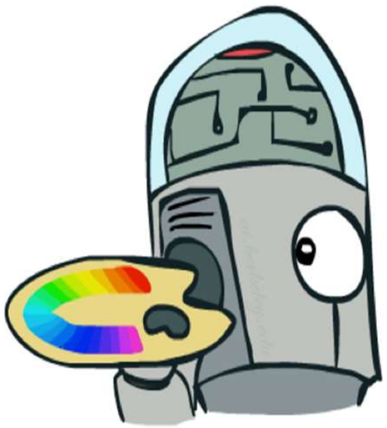
(or can have a
bunch of pairwise
inequality
constraints)

Varieties of CSPs

- Discrete Variables
  - Finite domains
    - Size d means $O(d^n)$ complete assignments
    - E.g., Boolean CSPs, including Boolean satisfiability (NP-complete)
  - Infinite domains (integers, strings, etc.)
    - E.g., job scheduling, variables are start/end times for each job
    - Linear constraints solvable, nonlinear undecidable



- Continuous variables
  - E.g., start/end times for Hubble Telescope observations
  - Linear constraints solvable in polynomial time by LP methods



Varieties of Constraints

- Varieties of Constraints
  - Unary constraints involve a single variable (equivalent to reducing domains), e.g.:

$$SA \neq green$$

  - Binary constraints involve pairs of variables, e.

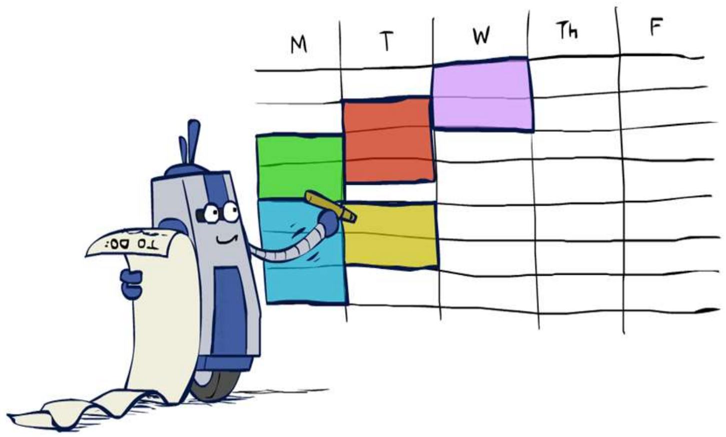
$$SA \neq WA$$

  - Higher-order constraints involve 3 or more variables:

- Preferences (soft constraints):
  - E.g., red is better than green
  - Often representable by a cost for each variable assignment
  - Gives constrained optimization problems
  - (We'll ignore these until we get to Bayes' nets)

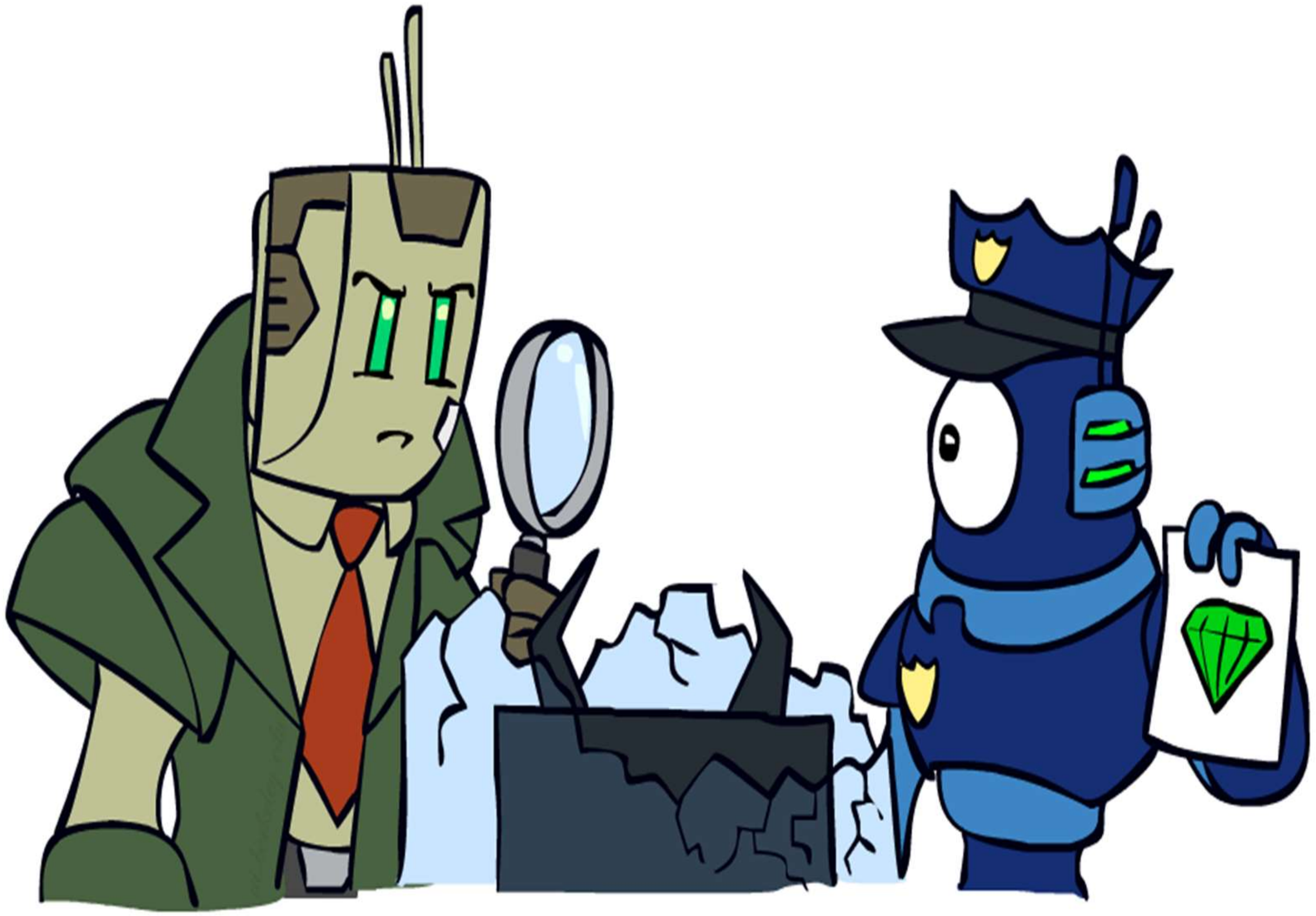
Real-World CSPs

- Assignment problems: e.g., who teaches what class
- Timetabling problems: e.g., which class is offered when and where?
- Hardware configuration
- Transportation scheduling
- Factory scheduling
- Circuit layout
- … lots more!



- Many real-world problems involve real-valued variables…

Solving CSPs

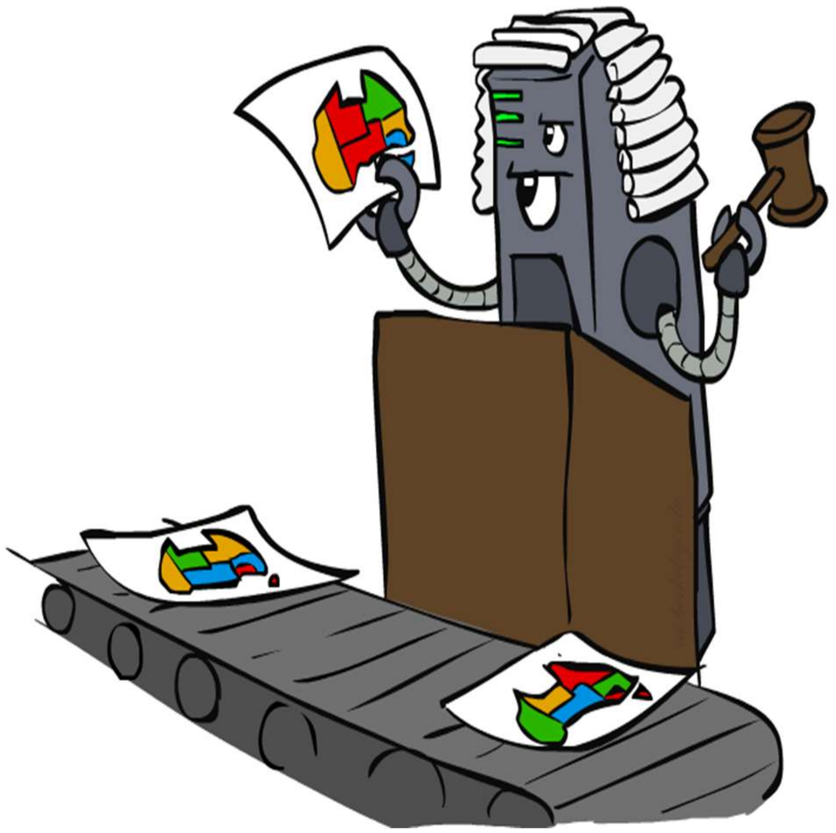


Standard Search Formulation

- Standard search formulation of CSPs

- States defined by the values assigned so far (partial assignments)
  - Initial state: the empty assignment, {}
  - Successor function: assign a value to an unassigned variable
  - Goal test: the current assignment is complete and satisfies all constraints

- We'll start with the straightforward, naïve approach, then improve it

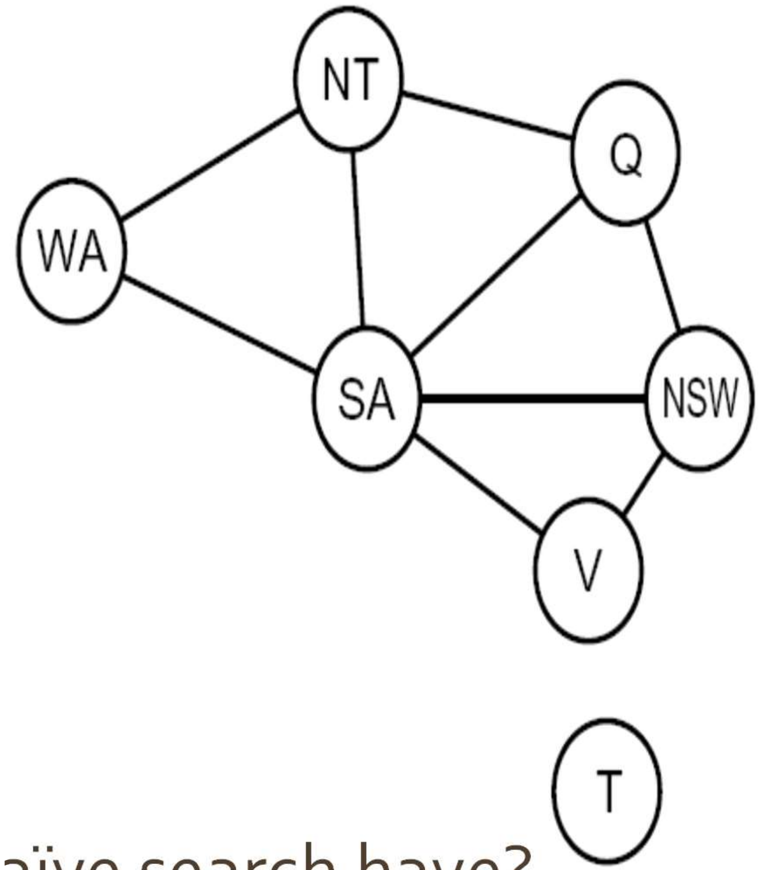


Search Methods

- What would BFS do?

- What would DFS do?

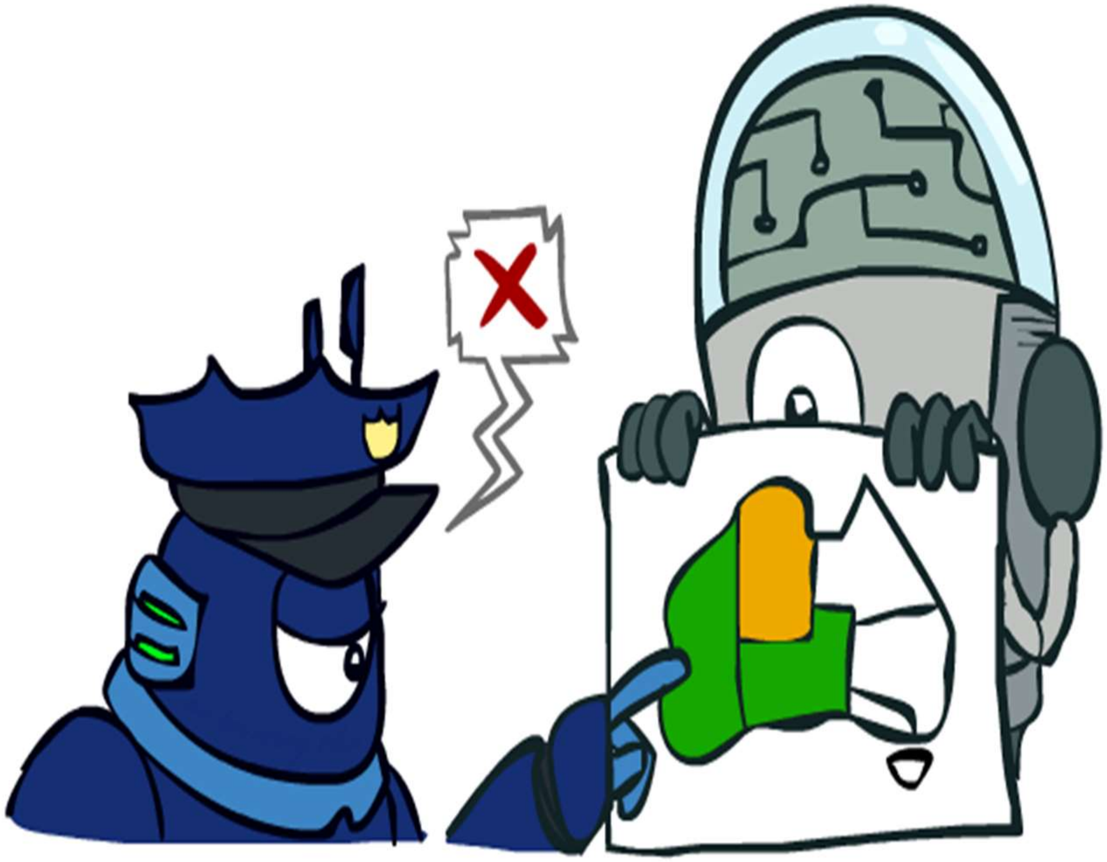
- What problems does naïve search have?



[Demo: coloring --

Video of Demo Coloring -- DFS

Backtracking Search



Backtracking Search

- Backtracking search is the basic uninformed algorithm for solving CSPs

- Idea 1: One variable at a time
  - Variable assignments are commutative, so fix ordering
  - I.e., [WA = red then NT = green] same as [NT = green then WA = red]
  - Only need to consider assignments to a single variable at each step

- Idea 2: Check constraints as you go
  - I.e. consider only values which do not conflict previous assignments
  - Might have to do some computation to check the constraints
  - "Incremental goal test"

- Depth-first search with these two improvements is called *backtracking search* (not the best name)

- Can solve n-queens for n ≈ 25