Билет №8.02.

№1.

$P(A) = \frac{|A|}{|\Omega|}$

Формулы: Перестановки: $P_n = n!$; Размещение: $A_n^m = \frac{n!}{(n-m)!}$; сочетание: $C_n^m = \frac{n!}{(n-m)!m!}$

Задачи

$$P(A) = \frac{|A|}{|\Omega|} = \frac{3 \cdot C_4^2 \cdot C_4^1 C_4^1}{C_{12}^4} = \frac{3 \cdot 4 \cdot 8 \cdot 4 \cdot 4 \cdot 1 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 11 \cdot 10 \cdot 9 \cdot 5} = \frac{32}{55}$$

№2.

Равномерное распределение на отрезке

$$f_\xi = \begin{cases} \frac{1}{b-a}, & x \in [a,b] \\ 0, & \text{иначе} \end{cases} \quad M\xi = \frac{a+b}{2} \quad D\xi = \frac{(b-a)^2}{12}$$

$$F\xi = \begin{cases} \frac{x-a}{b-a}, & a \le x \le b \\ 0, & x < a \\ 1, & x > b \end{cases}$$

ξ - р.р. $[-1, 5]$ $\quad D\xi = \frac{36}{12} = 3 \quad \sigma_\xi = \sqrt{3}$

$$P(-\sigma_\xi < \xi < \sigma_\xi) = P(-\sqrt{3} < \xi < \sqrt{3}) = F(\sqrt{3}) - F(-\sqrt{3}) = \frac{\sqrt{3}+1}{6} - \frac{1-\sqrt{3}}{6} = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$$

№3.

	$P(H_i)$	$P(A\|H_i)$
H_1	$\frac{3}{18}$	$\frac{25}{30}$
H_2	$\frac{6}{18}$	$\frac{20}{30}$
H_3	$\frac{9}{18}$	$\frac{15}{30}$

$$P(A) = \sum_{i=1}^{3} P(H_i) \cdot P(A|H_i) = \frac{5}{36} + \frac{2}{9} + \frac{9}{36} = \frac{22}{36} = \frac{11}{18}$$

$$P(H_2|A) = \frac{P(H_2) \cdot P(A|H_2)}{P(A)} = \frac{2}{9} \cdot \frac{18}{11} = \frac{4}{11}$$

$$P(H_3|A) = \frac{P(H_3) P(A|H_3)}{P(A)} = \frac{9}{36} \cdot \frac{18}{11} = \frac{9}{22}$$

$$P(B) = P(H_2|A) + P(H_3|A) = \frac{4}{11} + \frac{9}{22} = \frac{17}{22}$$

№4.

$\xi_1 \backslash \xi_2$	-1	0	1
-1	$\frac{1}{9}$ (0)	$\frac{1}{9}$ (-1)	$\frac{1}{9}$ (0)
0	$\frac{1}{9}$ (1)	$\frac{1}{9}$ (0)	$\frac{1}{9}$ (1)
1	$\frac{1}{9}$ (0)	$\frac{1}{9}$ (1)	$\frac{1}{9}$ (2)

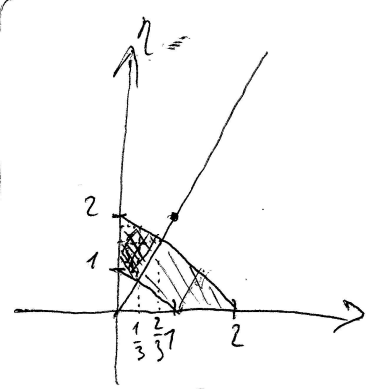
η_1	-2	-1	0	1	2
P	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{9}$

η_2	-1	0	1
P	$\frac{2}{9}$	$\frac{5}{9}$	$\frac{2}{9}$

$$cov(\eta_1, \eta_2) = M(\eta_1 \cdot \eta_2) - M(\eta_1) M(\eta_2)$$

$$= 0 - 0 \cdot 0 = 0$$

$\eta_1 \backslash \eta_2$	-2	-1	0	1	2
-1	0	0	$\frac{1}{9}$ (0)	0	0
0	0	$\frac{2}{9}$ (0)	$\frac{1}{9}$ (0)	$\frac{2}{9}$ (0)	0
1	$\frac{1}{9}$ (-1)	0	$\frac{1}{9}$ (0)	0	$\frac{1}{9}$ (1)



$S_G = \frac{3}{2}$

№5

$D_z = \{(x,y) \mid y \ge 2x + z\}$

$S_1 = \frac{1}{3} \cdot 1 = \frac{1}{3}$

$S_2 = \frac{1}{2} \cdot 1 \cdot \frac{1}{3} = \frac{1}{6}$

$S_{D_z \cap G} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

$F_\psi(z) = P(\psi \le z) = P(2\xi - \eta \le z) =$

$= \frac{S_{D_z \cap G}}{S_G} = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}.$