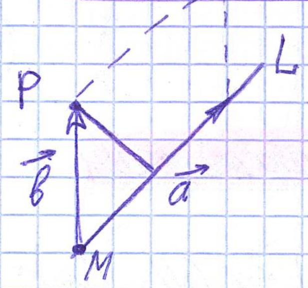
Семинар 8. 21/10/19

Тема 6. Нахождение различных расстояний.

Расстояние от точки до прямой.

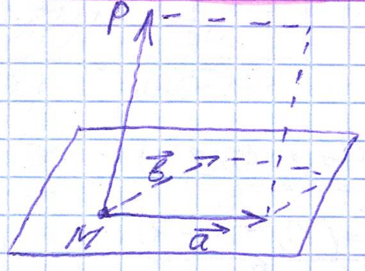


$$d(P,L) = \frac{|[\vec{a} \times \overrightarrow{MP}]|}{|\vec{a}|}$$

$$d(P,L) = \frac{S_{\square}(\vec{a}, \overrightarrow{MP})}{|\vec{a}|}$$

$$S_{\square}(\vec{a}, \overrightarrow{MP}) = |\vec{a}| \cdot h$$

Расстояние от точки до плоскости.

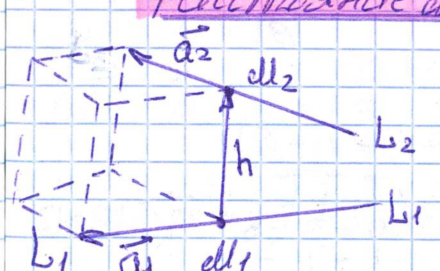


$$V(\vec{a}, \vec{b}, \overrightarrow{MP}) = |(\vec{a}, \vec{b}, \overrightarrow{MP})|$$

$$V = S_{\square}(\vec{a}, \vec{b}) \cdot h$$

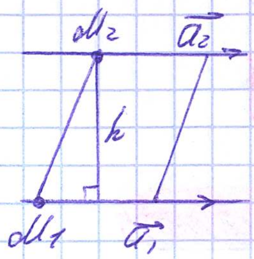
$$d(P,Q) = \frac{|(\vec{a}, \vec{b}, \overrightarrow{MP})|}{|[\vec{a} \times \vec{b}]|}$$

Расстояние между скрещ. прямыми.



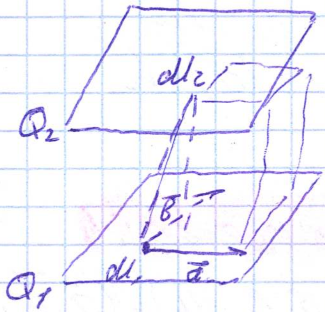
$$d(L_1, L_2) = \frac{|(\vec{a}_1, \vec{a}_2, \overrightarrow{M_1M_2})|}{|[\vec{a}_1 \times \vec{a}_2]|}$$

Расстояние между парал. прямыми.



$$d(L_2, L_1) = d(M_2, L_1) = \frac{|[\vec{a} \times \overrightarrow{M_1M_2}]|}{|\vec{a}|}$$

Расстояние между парал. плоскостями.



$$d(Q_1, Q_2) = d(M_2, Q_1) = \frac{|(\vec{a}, \vec{b}, \overrightarrow{M_1M_2})|}{|[\vec{a} \times \vec{b}]|}$$

Задача.

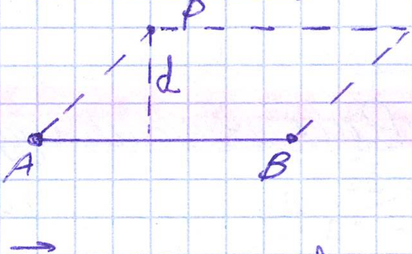
$A = (2, 5, 7)$
$B = (2, 2, 4)$
$C = (-2, -2, 2)$
$D = (1, 0, -2)$
$P = (1, 1, 1)$

Найти:

1) $d(P, AB)$
2) $d(P, ABC)$
3) $d(AB, CD)$

Решение.

1) $\overrightarrow{AB} = (0, -3, -3)$



$\overrightarrow{AP} = (-1, -4, -6)$

$$[\overrightarrow{AB} \times \overrightarrow{AP}] = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -3 & -3 \\ -1 & -4 & -6 \end{pmatrix} = \vec{i}(18 - 12$$

$$-\vec{j}(-3)+\vec{k}\cdot(-3) = 6\vec{i}+3\vec{j}-3\vec{k}$$

$$|\vec{AB}| = \sqrt{9+9} = 3\sqrt{2}$$

$$d(P, AB) = \frac{|[\vec{AD}\times\vec{AB}]|}{|\vec{AB}|} = \frac{\sqrt{36+18}}{3\sqrt{2}} = \frac{3\sqrt{6}}{3\sqrt{2}} = \underline{\underline{\sqrt{3}}}$$

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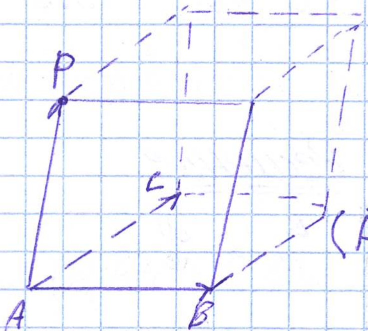
(2)



$\vec{AB} = (0, -3, -3)$

$\vec{AC} = (-4, -7, -5)$

$\vec{AP} = (-1, -4, -6)$

$$(\vec{AB}, \vec{AC}, \vec{AP}) = \begin{vmatrix} 0 & -3 & -3 \\ -4 & -7 & -5 \\ -1 & -4 & -6 \end{vmatrix} =$$

$$= +3\cdot(\overset{19}{24-5}) - 3\cdot(\overset{9}{16-7}) =$$

$$= +57 - 27 = 30$$

$$[\vec{AC}\times\vec{AB}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & -7 & -5 \\ 0 & -3 & -3 \end{vmatrix} =$$

$$= \vec{i}\cdot(\overset{6}{21-15}) - \vec{j}(12) + \vec{k}(12) =$$

$$= 6\cdot\vec{i} - 12\cdot\vec{j} + 12\vec{k}$$

$$d(P, ABC) = \frac{|(\vec{AB}, \vec{AC}, \vec{AP})|}{|[\vec{AC}\times\vec{AB}]|} = \frac{30}{\sqrt{36+288}}$$

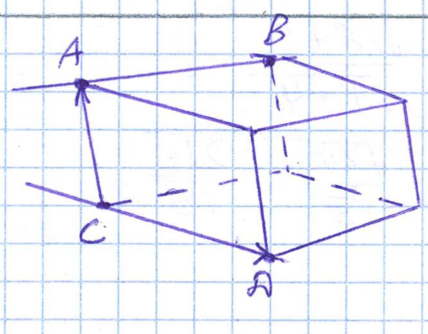
$$= \frac{30}{18} = \frac{10}{6} = \underline{\underline{\frac{5}{3}}}$$

(3.)

$\vec{AB} = (0, -3, -3)$

$\vec{CD} = (3, 2, -4)$

$\vec{CA} = (4, 7, 5)$



$$(\vec{AB}, \vec{CD}, \vec{CA}) = \begin{vmatrix} 0 & -3 & -3 \\ 3 & 2 & -4 \\ 4 & 7 & 5 \end{vmatrix} =$$

$$= 3(\overset{31}{15+16}) - 3(\overset{13}{21-8}) = 93 - 39 = 54$$

45

$$[\vec{AB}\times\vec{CD}] = \begin{vmatrix} \vec i & \vec j & \vec k \\ 0 & -3 & -3 \\ 3 & 2 & -4 \end{vmatrix} = \vec i\cdot(12+6) - \vec j(9) + \vec k(9) = 18\vec i - 9\vec j + 9\vec k$$

$$d(\vec{AB}, \vec{CD}) = \frac{|(\vec{AB}, \vec{CD}, \vec{CA})|}{|[\vec{AB}\times\vec{CD}]|} = \frac{54}{\sqrt{486}} = \frac{54}{9\sqrt6} = \frac{6}{\sqrt6} = \sqrt6$$

Задача.

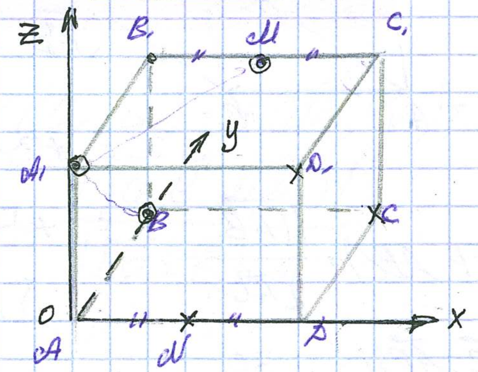
$ABCDA_1B_1C_1D_1$ – куб с ребром 2

M – сер. B_1C_1

N – сер. AD

1) $d(A_1M, CN)$
2) $d(A_1BM, CD_1N)$
3) $d(A_1M, CD_1)$

Решение.



1. $B=(0,2,0)$ $D_1=(2,0,2)$

$A_1=(0,0,2)$ $C=(2,2,0)$

$M=(1,2,2)$ $N=(1,0,0)$

$\vec{A_1M}=(1,2,0)$ $\vec{CN}=(-1,-2,0)$

$\vec{NA_1}=(-1,0,2)$ $\vec{NC}=(1,2,0)$

$$d(\vec{A_1M}, \vec{CN}) = \frac{|[\vec{NA_1}\times\vec{NC}]|}{|\vec{NC}|} = \frac{\sqrt{16+8}}{\sqrt{1+4}} = \frac{\sqrt{24}}{\sqrt5} = \frac{2\sqrt6}{\sqrt5}$$

$$[\vec{NA_1}\times\vec{NC}] = \begin{vmatrix} \vec i & \vec j & \vec k \\ -1 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix} = \vec i(-4) - \vec j(-2) + \vec k(-2) = -4\vec i + 2\vec j - 2\vec k$$

(2) $\overrightarrow{A_1M} = (1,2,0)$

$\overrightarrow{A_1B} = (0,2,-2)$

$(\overrightarrow{A_1M}, \overrightarrow{A_1B}, \overrightarrow{A_1D_1}) = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 2 & -2 \\ 2 & 2 & -2 \end{vmatrix} =$

$= 1\cdot(-4+4) - 2(\cdot 4) = -8$

$\overrightarrow{A_1C} = (2,2,-2)$

$[\overrightarrow{A_1M} \times \overrightarrow{A_1B}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 0 & 2 & -2 \end{vmatrix} = \vec{i}\cdot(-4) - \vec{j}\cdot(-2) + \vec{k}\cdot(2)$

$= -4\vec{i} + 2\cdot\vec{j} + 2\vec{k}$

$d(A_1BM, CD_1N) = \dfrac{|(\overrightarrow{A_1M}, \overrightarrow{A_1B}, \overrightarrow{AD_1})|}{[\overrightarrow{A_1M} \times \overrightarrow{A_1B}]} = \dfrac{8}{\sqrt{16+8}} = \dfrac{8}{2\sqrt{6}} =$

$= \dfrac{4}{\sqrt{6}}$

(3) $\overrightarrow{A_1M} = (1,2,0)$

$\overrightarrow{CD_1} = (0,-2,2)$

$\overrightarrow{MC} = (1,0,-2)$

$(\overrightarrow{A_1M}, \overrightarrow{CD_1}, \overrightarrow{MC}) = \begin{vmatrix} 1 & 2 & 0 \\ 0 & -2 & 2 \\ 1 & 0 & -2 \end{vmatrix} =$

$= 1\cdot(4) - 2\cdot(-2) = 4+4 = 8$

$[\overrightarrow{A_1M} \times \overrightarrow{CD_1}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 0 & -2 & 2 \end{vmatrix} =$

$= \vec{i}\cdot(4) - \vec{j}\,(2) + \vec{k}\cdot(-2) =$

$= 4\cdot\vec{i} - 2\cdot\vec{j} - 2\vec{k}$

$d(A_1M, CD_1) =$

$= \dfrac{8}{\sqrt{16+8}} = \dfrac{8}{2\sqrt{6}}$

$= \dfrac{4}{\sqrt{6}}$