

# Основные неопределённые интегралы:

1.  $\int u^n du = \frac{u^{n+1}}{n+1} + C, (n \neq -1)$
2.  $\int \frac{du}{u} = \ln |u| + C$
3.  $\int a^u du = \frac{a^u}{\ln a} + C, (a > 0, a \neq 1);$   
 $\int e^u du = e^u + C$
4.  $\int \sin u du = -\cos u + C$
5.  $\int \cos u du = \sin u + C$
6.  $\int \frac{du}{\cos^2 u} = \operatorname{tg} u + C$
7.  $\int \frac{du}{\sin^2 u} = -\operatorname{ctg} u + C$
8.  $\int \frac{du}{\sin u} = \ln \left| \operatorname{tg} \frac{u}{2} \right| + C =$   
 $= \ln |\operatorname{cosec} u - \operatorname{ctg} u| + C$
9.  $\int \frac{du}{\cos u} = \ln \left| \operatorname{tg} \left( \frac{u}{2} + \frac{\pi}{4} \right) \right| + C =$   
 $= \ln |\operatorname{tg} u + \sec u| + C$
10.  $\int \frac{du}{u^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{u}{a} + C, (a \neq 0)$
11.  $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C, (a \neq 0)$
12.  $\int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{arcsin} \frac{u}{a} + C \quad |u| < |a|$
13.  $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln \left| u + \sqrt{u^2 - a^2} \right| + C,$   
 $|u| > |a| > 0$
14.  $\int \frac{du}{\sqrt{u^2 + a^2}} = \ln \left( u + \sqrt{u^2 + a^2} \right) + C,$   
 $(a \neq 0)$
15.  $\int \operatorname{sh} u du = \operatorname{ch} u + C$
16.  $\int \operatorname{ch} u du = \operatorname{sh} u + C$
17.  $\int \frac{du}{\operatorname{ch}^2 u} = \operatorname{th} u + C$

18.  $\int \frac{du}{\operatorname{sh}^2 u} = -\operatorname{cth} u + C$
19.  $\int \frac{u du}{u^2 + a^2} = \frac{1}{2} \ln |u^2 + a^2| + C$
20.  $\int \frac{u du}{a^2 - u^2} = -\frac{1}{2} \ln |a^2 - u^2| + C$
21.  $\int \frac{u du}{\sqrt{a^2 + u^2}} = \sqrt{a^2 + u^2} + C$
22.  $\int \frac{u du}{\sqrt{a^2 - u^2}} = -\sqrt{a^2 - u^2} + C$
23.  $\int \sin (au) du = -\frac{1}{a} \cos (au) + C$
24.  $\int \cos (au) du = \frac{1}{a} \sin (au) + C$
25.  $\int e^{au} du = \frac{1}{a} e^{au} + C$

Свойства неопределённых интегралов:

- $\int f(x) dx = F(x) + C$   
 $F'(x) = f(x)$
- $\left( \int f(x) dx \right)' = f(x)$
- $d \int f(x) dx = f(x) dx$
- $\int f'(x) dx = f(x) + C$
- $\int c \cdot f(x) dx = c \cdot \int f(x) dx$
- $\int (f(x) \pm g(x)) dx =$   
 $= \int f(x) dx \pm \int g(x) dx$
- $\int f(u(t)) u'(t) dt = F(u(t)) + C$

Основные производные:

- |  |   |  |
|--|---|--|
| 1. $C' = 0$                            | 9. $(\sin x)' = \cos x$                             | 17. $(\operatorname{sh} x)' = \operatorname{ch} x$               |
| 2. $x' = 1$                            | 10. $(\cos x)' = -\sin x$                           | 18. $(\operatorname{ch} x)' = \operatorname{sh} x$               |
| 3. $(x^n)' = n \cdot x^{n-1}$          | 11. $(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$   | 19. $(\operatorname{th} x)' = \frac{1}{\operatorname{ch}^2 x}$   |
| 4. $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$ | 12. $(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$ | 20. $(\operatorname{cth} x)' = -\frac{1}{\operatorname{sh}^2 x}$ |
| 5. $(e^x)' = e^x$                      | 13. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$         | 21. $(\operatorname{arcsch} x)' = \frac{1}{\sqrt{x^2+1}}$        |
| 6. $(a^x)' = a^x \ln a$                | 14. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$        | 22. $(\operatorname{arcch} x)' = \frac{1}{\sqrt{x^2-1}}$         |
| 7. $(\ln x)' = \frac{1}{x}$            | 15. $(\operatorname{arctg} x)' = \frac{1}{1+x^2}$   | 23. $(\operatorname{arcth} x)' = \frac{1}{1-x^2}$                |
| 8. $(\log_a x)' = \frac{1}{x \ln a}$   | 16. $(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$ | 24. $(\operatorname{arccth} x)' = \frac{1}{1-x^2}$               |

Правила дифференцирования.

Если  $u$  и  $v$  - некие функции, зависящие от  $x$ , то

1.  $(c \cdot u)' = c \cdot u'$
2.  $(u + v)' = u' + v'$
3.  $(u \cdot v)' = u' \cdot v + u \cdot v'$
4.  $\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$
5.  $(u(v))' = u'(v) \cdot v'$
6. Производная параметрически заданной функции:  $y'_x = \frac{y'_t}{x'_t}$
7. Производная обратной функции:  $g'_y(y) = \frac{1}{f'_x(x)}$