Числа на Фибоначи

Редицата $\{F_n\}_{n=0}^{\infty}$ се дефинира така:

$F_0 = 0$

$F_1 = 1$

$F_{n+2} = F_n + F_{n+1}$ за $n \geq 0$

Разгл. език $\mathcal{L}$ без ф.р. и единст. нелог. символи: $Func_{\mathcal{L}} = \{f\}$, $Pred_{\mathcal{L}} = \{p\}$, $Const_{\mathcal{L}} = \emptyset$, като $\#(f) = 2$, $\#(p) = 1$.

Нека $S = \langle \mathbb{N}; f^S, p^S \rangle$ е стр. за $\mathcal{L}$

като: $f^S(x, y) = z \iff x + F_{y+1} = z$

$p^S(x) \iff x \in \{F_n\}_{n=0}$

Да се док. че в S са определими:

a) $\{0\}$

b) $\{1\}$

c) $Eq = \{\langle a, a \rangle \mid a \in \mathbb{N}\}$.

d) Вярно ли е, че в S е опр.-во $Fib = \{\langle F_n, F_{n+1} \rangle \mid n \in \mathbb{N}\}$?

e) Да се намерят всички автоморф. на S с д-во.

$$\underline{e_0(x)} \leq \forall y\,(p(f(x,y)))$$

$$\| \quad x + F_{y+1} = z \in \{F_n\}_{n=0}^{\infty}$$

$$\underline{x = 0} \qquad z = F_{y+1} \in$$

$$\varphi_0(x) \leq \exists y\, \exists z\,(\underline{e_{eq}}(f(y,z),x))$$

$$\| \quad y + \underline{F_{z+1}} \neq 0 \quad \text{not possible}$$

$$z \geq 0$$

$$\frac{z+1 \geq 1}{F_{z+1} \geq 1}$$

$$\underline{e_1(x)} \leq \exists y\,(\varphi_0(y)\ \&\ e_{eq}(f(y,y),x))$$

$$\| \quad 0 + \underline{F_{0+1}} = x$$

$$F_1$$

$$=$$

$$1$$

$\varphi_{eq}(x,y) \leftrightharpoons \forall z\,(p(f(x,z)) \Leftrightarrow p(f(y,z)))$

$\forall z \in \mathbb{N}$: $\quad x + F_{\boxed{z}+1} \in \{F_n\}_{n=0}^{\infty} \Longleftrightarrow$

$y + F_{\boxed{z}+1} \in \{F_n\}_{n=0}^{\infty}$

$\Rightarrow x = y$

$\varphi_{+1}(x,y) \leftrightharpoons \exists z\,(\varphi_0(z) \,\&\, \varphi_{eq}(f(x,z), y))$

$y = x+1$

$x + F_{0+1} = y$, $\quad F_1 = 1$

База: φ_0

И.Х.: Нека φ_n опр. $\{n\}$

Стъпка: $\varphi_{n+1}(x) \leftrightharpoons \exists y\,(\varphi_n(y) \,\&\, \varphi_{+1}(y,x))$

$Prev = \{ \langle F_n, F_{n+1} \rangle \mid n \in \mathbb{N} \}$

$$\underline{\varphi_{prev}}(x, y) \leftrightharpoons \underline{p(x) \,\&\, p(y)} \,\&$$

$$\left(\begin{array}{l} (\underline{\varphi_0(x) \,\&\, \varphi_1(y)}) \vee \quad \} \; n = 0 \\ \exists n_x \exists n_y (\underline{\varphi_{fib}}(x, n_x) \,\& \\ \quad \varphi_{+1}(n_x, n_y) \,\&\, \varphi_{fib}(y, n_y)) \end{array} \right) \; n > 0$$

$F_0 = 0$

$F_1 = 1$

$F_{n+2} = F_{n+1} + F_n$

$f^S(x, y) = z \iff x + F_{y+1} = z$

$p^S(x) \iff x \in \{F_n\}_{n=0}^{\infty}$

$\varphi_0, \varphi_1, \varphi_{eq}, \varphi_{+1} \quad \vee$

$$\overbrace{\qquad\qquad}^{0 + F_{m+1} = x}$$

$$\underline{\varphi_{fib}(x, n)} \leftrightharpoons \exists y \exists m \, (\varphi_0(y) \,\&\, \varphi_{eq}(f(y, m), x) \,\&$$

$$\underbrace{\varphi_{+1}(m, n)}_{m+1=n}) \,\&\, \underline{p(x)}$$

$\underline{x = F_n}$

$\underline{n > 0}$

02 юли 2017 Изпълнимост

зад (3)

$\varphi_1 \leftrightharpoons \forall x\, p(x,x)$

$\varphi_2 \leftrightharpoons \forall x \exists y\, r(x,y)$

$\varphi_3 \leftrightharpoons \exists x \forall y\, \neg r(y,x)$ "няма никой под това x"

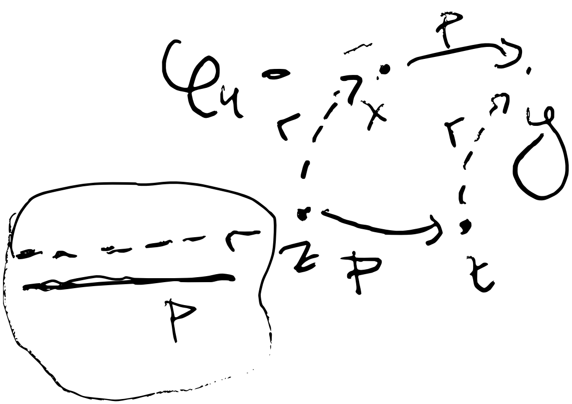
$\varphi_4 \leftrightharpoons \forall x \forall y \forall z \forall t\, (p(x,y) \,\&\, r(z,x) \,\&\, r(t,y) \Rightarrow p(z,t))$

φ_1 - рефлексивна по p

φ_2 - $outdegree(x) \geq 1$ по r сериалност неясно

φ_3 - $indegree_r(x) = 0$ по r минимален спрямо рел. r

φ_4 -



eus $\forall x \forall y \forall z \forall t\,(p(x,y) \,\&\, r(z,x) \,\&\, r(t,y) \Rightarrow p(z,t))$

$p(a,a) \,\&\, r(\ ,a) \,\&\, r(\ ,a) \Rightarrow p(\ ,\ )$

$\cancel{p(b,b) \,\&\, r(\ ,b) \,\&\, r(\ ,b) \Rightarrow p(\ ,\ )}$

$z=a,\ t=a \longrightarrow p(a,a)$ V

$z=b,\ t=a \longrightarrow p(b,a)$?

$z=a,\ t=b \longrightarrow p(a,b)$?

$z=b,\ t=b \longrightarrow p(b,b)$ V

$A = \langle \{a,b\};\ p^A, r^A \rangle$ $\quad \cancel{p(a,b) \,\&\, r(\ ,a) \,\&\, r(\ ,b)}$

$p^A = \{\langle a,a\rangle, \langle a,b\rangle, \langle b,a\rangle, \langle b,b\rangle\}$ $\quad \cancel{p(b,a) \,\&\, r(\ ,b) \,\&\, r(\ ,a)}$

$r^A = \{\langle a,a\rangle, \langle b,a\rangle\}$

27 май 2017 Изпълнимост

Зад. 1

$\varphi_1 \leftrightharpoons \forall x \forall y (q(x,y) \& q(y,x) \Rightarrow x=y)$

$\varphi_2 \leftrightharpoons \exists x \neg q(x,x)$

$\varphi_3 \leftrightharpoons \neg[\exists x \forall y \neg q(x,y)] \& \forall x \exists z\, q(z,x)$

$\varphi_4 \leftrightharpoons \exists x \exists y (q(x,y) \& q(y,x))$

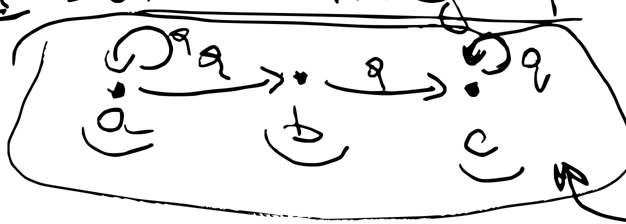
φ_1 - антисиметричност

φ_2 - имаме поне една нерефл. точка

$\varphi_3 \models \forall x \exists y\, q(x,y) \;\&\; \forall x \exists z\, q(z,x)$

outdegree$(x) \geq 1$ indegree$(x) \geq 1$

$\varphi_4 + \varphi_1 \to$ имаме поне една рефл. точка.



$\mathcal{A} = \langle \{a,b,c\}, q^{\mathcal{A}} \rangle$

$q^{\mathcal{A}}$

$\mathcal{B} = \langle \mathbb{N}, q^{\mathcal{B}} \rangle$

$q^{\mathcal{B}}(x, y) \longleftrightarrow x \leq y \ \& \ x \neq 12 \ \& \ y \neq 12$

$x, y \in \mathbb{N}$

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$$\varphi_1 \leftrightharpoons \forall x \forall y (\neg p(x,y) \Rightarrow \forall z (p(x,z) \Rightarrow \neg p(z,y)))$$

$$\varphi_2 \leftrightharpoons \forall x \neg \exists y \forall z ((p(x,z) \,\&\, p(y,z)) \Rightarrow (p(z,x) \,\&\, \neg p(z,x)))$$

$$\varphi_3 \leftrightharpoons \forall x \forall y (\forall x \exists y (p(x,y) \vee \neg p(x,y)) \Rightarrow \forall z \exists t (p(y,t) \,\&\, p(x,t) \,\&\, p(z,t)))$$

$$\boxed{\varphi_1, \varphi_2 \overset{?}{\models} \varphi_3}$$