Sleeping

~~Зад~~ 1 $\mathcal{L} = \langle p \rangle$, $\# p = 3$, $p \in Pred$

$\mathcal{A} = \langle \mathbb{N}, p^{\mathcal{A}} \rangle$

$p^{\mathcal{A}}(a, b, c) \leftrightarrow a.b + 1 = c^2$

Определете $\{0\}, \{1\}, \{2\}$,

$M = \leftrightharpoons \{ \langle a, b \rangle \mid a, b \in \mathbb{N} \ \& \ a = b \}$.

Докажете, че $\forall n \in \mathbb{N}$, то $\{n\}$ е определимо.

$\varphi_{\langle 0,1 \rangle}(y, z) \leftrightharpoons \forall x \, p(x, y, z)$

$x.y + 1 = z^2$

$\varphi_0(y) \leftrightharpoons \exists z \forall x \, p(x, y, z)$

$\varphi_1(z) \leftrightharpoons \exists y \forall x \, p(x, y, z)$

$\psi_0(z) \leftrightharpoons \forall x \forall y \neg p(x, y, z)$

$\chi_0(x) \leftrightharpoons \exists z \, p(x, x, z)$ // $x.x + 1 = z^2 \leftrightarrow x^2 + 1 = z^2$

$\mathcal{L} = \langle m \rangle$, $m \in \mathbb{P}red$, $\# m = 3$

$\mathcal{A} = \langle \mathbb{N}, m^{\mathcal{A}} \rangle$ и имаме ф. р $\doteq$

$\underline{a, b, c \in \mathbb{N}}$ $m^{\mathcal{A}}(a, b, c) \iff c = a \cdot b$

Определете $M_{prime} = \{ p \mid p \in \mathbb{N}$ & "p е просто число"$\}$

$M_{m|n} = \{ \langle m, n \rangle \mid m, n \in \mathbb{N}$ & $m \mid n \}$

$\varphi_{m|n}(x, y) \leftrightharpoons \exists z\, m(x, z, y)$ // $y = x \cdot z$

$\varphi_{m|n}(x, y)$ е true $\iff x \mid y$

$\varphi_{prime}(x) \leftrightharpoons \forall y \forall z (m(y, z, x) \Rightarrow y \doteq x \vee z \doteq x)$

$\underline{\& \underbrace{\neg \varphi_1(x)}_{\neg p(x,x,x)} \& \neg \varphi_0(x)}$

$\psi_{prime}(x) \leftrightharpoons \forall y (\neg \varphi_{m|n}(x, y) \vee \varphi_1(y) \vee x \doteq y)$ &

$\neg \varphi_0(x)$ & $\neg \varphi_1(y)$

$p^{+}(a,b,c) \iff \underbrace{a.b+1=c^2}$

φ_0, φ_1

$a.b = c^2 - 1$, $c = 2$

$2^2 - 1 = 3 = \underline{a.b}$

$\boxed{a.b = (c-1).(c+1)}$

$(a=3$ и $b=1)$ или

$(a=1$ и $b=3)$

$\{2\} \to \varphi_2(x) \leftrightharpoons \forall y \forall z (\underline{p(y,z,x)} \Rightarrow \varphi_1(y) \vee \varphi_1(z)) \mathbin{\&} \neg\varphi_0(x) \mathbin{\&} \neg\varphi_1(x)$

$y.z+1 = x^2 \iff$

$x^2 - 1 = y.z \iff$

$(x-1).(x+1) = y.z$

$\underline{\{3\} \to \varphi_3(x)} \leftrightharpoons \exists y \exists z (\varphi_{\underline{1}}(y) \mathbin{\&} \varphi_{\underline{2}}(z) \mathbin{\&} p(y,x,z))$

φ_1, φ_2
ги познаваме

$\boxed{x = n \iff x.(n-2)+1 = (n-1)^2}$

$\underset{\sim}{3}.1 + 1 = 2^2$

Индукция по n, ~~зада~~ да, че $\{n\}$ е определ.

База: $\varphi_0, \varphi_1, \varphi_2, \varphi_3$

ИП: Нека за $m < n+2$, то $\{m\}$ е определимо с формула $\varphi_m(x)$, $m > 2$.

Стъпка: $\varphi_{n+2}(x) \leftrightharpoons \exists y \exists z (\varphi_n(y) \,\&\, \varphi_{n+1}(z) \,\&$
$p(x, y, z))$

Защо не работи за 2?

Умножаваме по 0 !?.

Зад | Нека $\mathcal{L} = \langle q, r \rangle$, $\#(q)=3$, $\#(r)=2$ и $q, r \in Pred_{\mathcal{L}}$.

Нека $\mathcal{A} = \langle \mathbb{R}, q^{\mathcal{A}}, r^{\mathcal{A}} \rangle$, за

$\langle a, b, c \rangle \in q^{\mathcal{A}} \Leftrightarrow c = a.b$

$\langle a, b \rangle \in r^{\mathcal{A}} \Leftrightarrow b = a+2$

Опр: $\{2\}$, $\{1/2\}$, $\{\sqrt{2}\}$, $\{\sqrt[3]{2}\}$, $\{a \mid a > 1\}$

hint: $a>0$ и нитредвот

$\varphi_0(z) \leftrightharpoons \forall x\, q(x, z, z)$

$\varphi_2(z) \leftrightharpoons \exists x(\varphi_0(x) \,\&\, r(x, z))$

$\psi_1(x) \leftrightharpoons \forall y\, q(x, y, y)$

$\psi_0(x) \leftrightharpoons q(x, x, x) \,\&\, \neg\psi_1(x)$

$\varphi_{1/2}(x) \leftrightharpoons \exists y \exists z(\varphi_1(y) \,\&\, \varphi_2(z) \,\&\, \underline{q(x, z, y)})$

$x.z = y$

$x.2 = 1$

$x = 1/2$

$\{\sqrt{2}\} \xrightarrow{?}$

$$\varphi_{>0}(x) \leftrightharpoons \exists y\,(\neg\varphi_0(y) \;\&\; q(y,y,x))$$

$$(\exists y \in \mathbb{R})[\,y \neq 0 \;\&\; y.y = y^2 = x\,]$$

$$\varphi_{\geq 0}(x) \leftrightharpoons \exists y\,(q(y,y,x))$$

$$\varphi_{\sqrt{2}}(x) \leftrightharpoons \varphi_{\geq 0}(x) \;\&\; \exists y\,(\varphi_2(y) \;\&\; \underbrace{q(x,x,y)}_{})$$

$x.x = x^2 = y = 2$

Т.е. $x^2 = 2 \Leftrightarrow$

$$\boxed{x \geq 0 \;\text{ и }\; x = \sqrt{2}}$$

$\{\sqrt[3]{2}\}$? $\longrightarrow$

$$\varphi_{\sqrt[3]{2}}(x) \leftrightharpoons \exists y\,\exists z\,(\varphi_2(y) \;\&\; \underline{q(x,x,z)} \;\&\; \underline{q(x,z,y)})$$

$z = x^2$

$y = 2$

$x.x^2 = y \Leftrightarrow x^3 = y \Leftrightarrow x = \sqrt[3]{2}$

$$q^{A}(a,b,c) \leftrightarrow \underline{c = a \cdot b}$$

$$r^{A}(a,b) \leftrightarrow \underline{b = a + 2}$$

$$\varphi_{2},\ \varphi_{1/2},\ \varphi_{\sqrt{2}},\ \varphi_{\sqrt[3]{2}},\ \varphi_{>0},\ \varphi_{\geq 0}$$

$$U \leftrightharpoons \{ a \mid a \in \mathbb{R} \ \&\ \underline{a > 1} \}$$

$$\varphi_{>2}(x) \leftrightharpoons \exists z [\varphi_{>0}(z) \ \&\ \underline{r(z,x)}]$$

$$x = z + 2$$

$$0 < z \leftrightarrow 2 < x$$

$$\varphi_{>1}(x) \leftrightharpoons \exists z [\varphi_{>2}(z) \ \&\ \exists y [\varphi_{1/2}(y) \ \&\ \underline{q(z,y,x)}]]$$

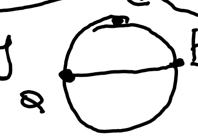
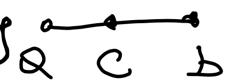
$$z > 2 \ \&\ z \cdot 1/2 = x$$

$$\leftrightarrow$$

$$x > 1$$

Зад/ $S = \langle \mathbb{R}^2, p^S \rangle$ точки в $\mathbb{R}^2$, $p \in Pred$, $\#p = 3$

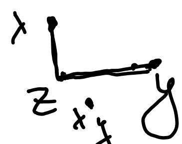
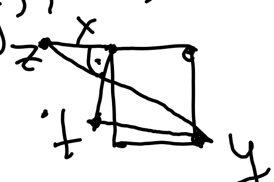
$a,b,c \in \mathbb{R}^2$ $p^S(a,b,c) \iff a \neq b$ и $b \neq c$ и $\angle bac = 90^\circ$

- $Eq = \{\langle a,a\rangle \mid a \in \mathbb{R}^2\}$
- $Col = \{\langle a,b,c\rangle \mid a,b,c \in \mathbb{R}^2$ и лежат на една и съща права$\}$
- $Circ = \{\langle a,b,c\rangle \mid c$ лежи на окр. с диам. $ab\}$ 
- Опр. ли са?:
  - $Mid\ \{\langle a,b,c\rangle \mid c$ е среда на $ab\}$ 
  - $Seg = \{\langle a,b,c\rangle \mid c$ лежи на $ab\}$ 

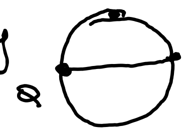
* Намерете два разл. автоморфизма в S.

$\varphi_{Eq}(x,y) \leftrightharpoons \forall z \forall t (p(x,z,t) \iff p(y,z,t))$

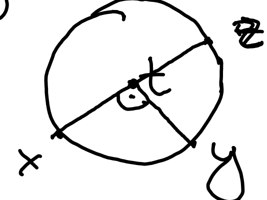
$\psi_{Eq}(x,y) \leftrightharpoons \forall z \neg p(z,x,y)$

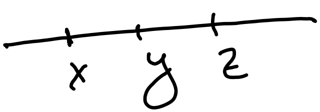
- $Col = \{\langle a, b, c\rangle \mid a, b, c \in \mathbb{R}^2$ и лежат на една и съща права$\}$
- $Circ = \{\langle a, b, c\rangle \mid c$ лежи на окр. с диам. $ab\}$

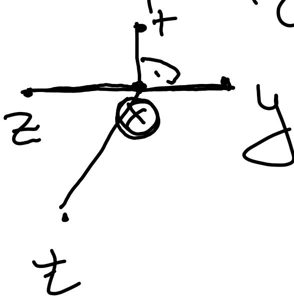


$$\varphi_{Col}(x, y, z) \leftrightharpoons \exists t\,(p(y, x, t) \,\&\, p(y, z, t))$$

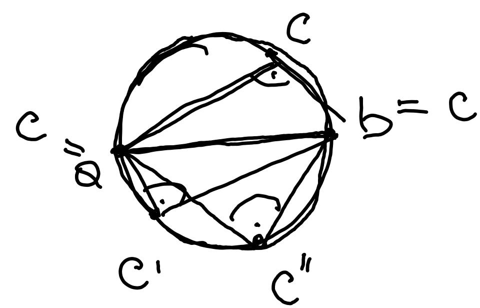


$$\varphi_{Eq}(x, y, z) \leftrightharpoons \forall t\,(p(x, y, t) \Longleftrightarrow p(x, z, t))$$





$$\varphi_{Circ}(x, y, z) \leftrightharpoons p(z, x, y) \vee \varphi_{Eq}(z, x) \vee \varphi_{Eq}(z, y).$$

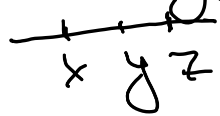


- Опр. ли са?:
  - $Mid = \{\langle a,b,c\rangle \mid c$ е средата на $ab\}$

  - $Seg = \{\langle a,b,c\rangle \mid c$ лежи на $ab\}$

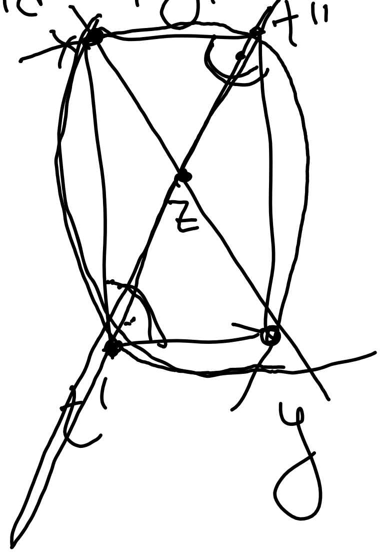
* Намерете ~~две~~ разл. автоморфизма в S.

$\varphi_{eq}(x,y)$, $\varphi_{col}(x,y,z)$, $\varphi_{circ}(x,y,z)$



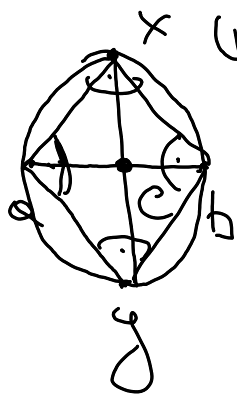
$$\varphi_{seg}(x,y,z) \leftrightharpoons \exists t\,(p(z,x,t) \,\&\, p(z,y,t) \,\&\, \varphi_{circ}(x,y,t)) \vee \varphi_{eq}(x,y) \vee \varphi_{eq}(x,z)$$

$$\varphi_{mid}(x,y,z) \leftrightharpoons Col(x,z,y) \,\&\, \exists t' \exists t'' \big(\big(Col(t',z,t'') \,\&\, \varphi_{circ}(x,y,t') \,\&\, \varphi_{circ}(x,y,t'') \,\&\, \neg\varphi_{eq}(t',t'')\big)\big)$$



1) Id

$$Aut(A) \neq \emptyset \quad Id_A \in Aut(A)$$



$$\varphi_{mid}(a,b,c) \leftrightharpoons \exists x \exists y \big(\varphi_{circ}(a,b,x) \,\&\, \varphi_{circ}(x,y,b) \,\&\, \varphi_{circ}(x,y,a) \,\&\, \varphi_{seg}(a,b,c) \,\&\, \varphi_{seg}(x,y,c)\big)$$

Зад | Нека $\Sigma = \{0, 1\}$ и $\mathcal{P}(\Sigma^*)$
НW

$\mathcal{A} = \langle \mathcal{P}(\Sigma^*), cat^{\mathcal{A}}, sub^{\mathcal{A}} \rangle$ е.ф.р ≐

$cat^{\mathcal{A}}(L_1, L_2) \leftrightharpoons L_1 \circ L_2$

$sub^{\mathcal{A}}(L_1, L_2) \Longleftrightarrow L_1 \subseteq L_2$

Опр: $\{\emptyset\}$, $\{\Sigma^*\}$, $\{\{\varepsilon\}\}$

$single = \{\{w\} \mid w \in \Sigma^*\}$ всички едноелем. езици

$union = \{\langle L_1, L_2, L_3 \rangle \mid L_3 = L_1 \cup L_2\}$

$star = \{\langle L, L^* \rangle \mid L \subseteq \Sigma^*\}$

Никой непразен език от Σ^* не е определим т.е. $w \in \Sigma^*$, то $\{\{w\}\}$ е неопр.

Док, че $h \in Aut(\mathcal{A})$, то h запазва регулярните езици.

All is well when it ends

well

and now it has.