ПИ ИНФ 2018 Резолюция

$c_1 \leftrightharpoons \forall x \forall y (\neg p(x,y) \Rightarrow$
$\forall z (p(x,z) \Rightarrow \neg p(z,y)))$

$c_2 \leftrightharpoons \forall x \neg \exists y \forall z ((p(x,z) \& p(y,z)) \Rightarrow$
$(p(z,x) \& \neg p(z,x)))$

$c_3 \leftrightharpoons \forall x \forall y (\forall x \exists y (p(x,y) \lor \neg p(x,y)) \Leftarrow$
$\forall z \exists t (p(y,t) \& p(x,t) \& p(z,t)))$

$\boxed{c_1, c_2 \overset{?}{\vDash} c_3}$

$c_2 \vdash \forall x \neg \exists y \forall z (\neg (p(x,z) \& p(y,z)))$
$c_3 \vdash \forall x \forall y \forall z \exists t (p(y,t) \& p(x,t) \& p(z,t))$
$c \vdash \forall x \forall y \forall z (p(x,y) \& p(y,z) \Rightarrow p(x,z))$

① $c_1 \vdash \forall x \forall y \forall z (\neg p(x,y) \lor \neg p(y,z) \lor p(x,z))$
$\quad c_1'$

$c_2 \vdash \forall x \forall y \exists z (p(x,z) \& p(y,z))$
$\quad c_2'$

$\psi \leftrightharpoons \neg c_3 \vdash \exists x \exists y \exists z \forall t (p(y,t) \lor \neg p(x,t) \lor$
$\neg p(z,t))$

② ✓ ПНФ⊄

③ СНФ⊄

$$\varphi_1^S \leq \forall x \forall y \forall z \, (\neg p(x,y) \vee \neg p(y,z) \vee p(x,z))$$

$$\varphi_2^S \leq \forall x \forall y \, (p(x, f(x,y)) \,\&\, p(y, f(x,y)))$$

$$\psi^S \leq \forall t \, (\neg p(b,t) \vee \neg p(a,t) \vee \neg p(c,t))$$

④ ∪

$D_1 = \{ \neg p(x_1, y_1), \, \neg p(y_1, z_1), \, p(x_1, z_1) \}$

$D_2 = \{ p(x_2, f(x_2, y_2)) \}$

$D_3 = \{ p(y_3, f(x_3, y_3)) \}$

$D_4 = \{ \neg p(b, t), \, \neg p(a, t), \, \neg p(c, t) \}$

① $D_5 = Res(D_1 \{y_1/x_2, z_1/f(x_2, y_2)\}, D_2) =$
$= \{ \neg p(x_1, x_2), \, p(x_1, f(x_2, y_2)) \}$

② $D_6 = Res(D_5 \{x_1/c\}, D_4 \{t/f(x_2, y_2)\}) =$
$= \{ \neg p(b, f(x_2, y_2)), \, \neg p(a, f(x_2, y_2)), \, \neg p(c, x_2) \}$

$D_1 = \{ \neg p(x_1, y_1), \neg p(y_1, z_1), p(x_1, z_1) \}$

$D_2 = \{ p(x_2, f(x_2, y_2)) \}$

$D_3 = \{ p(y_3, f(x_3, y_3)) \}$

$\underline{D_4} = \{ \neg p(b, t), \neg p(a, t), \neg p(c, t) \}$

$D_5 = \{ \neg p(x_1, x_2), p(x_1, f(x_2, y_2)) \}$

$D_6 = \{ \neg p(b, f(x_2, y_2)), \neg p(a, f(x_2, y_2)),$
$\quad \neg p(c, x_2) \}$

③ $D_7 = \mathrm{Res}(D_6 \{y_2/b\}, D_3 \{y_3/b\}) =$
$\quad = \{ \neg p(a, f(x_2, b)), \neg p(c, x_2) \}$

④ $D_8 = \mathrm{Res}(D_7 \{x_2/f(c, y_2)\}, D_2 \{x_2/c\}) =$
$\quad = \{ \neg p(a, f(f(c, y_2), b)) \}$

⑤ $D_9 = \mathrm{Res}(D_1 \{x_1/y_3, y_1/f(x_3, y_3)\}, D_3) =$
$\quad = \{ \neg p(f(x_3, y_3), z_1), p(y_3, z_1) \}$

⑥ $D_{10} = \mathrm{Res}(D_9 \{z_1/f(f(x_3, y_3), y_2)\}, D_2 \{x_2/f(x_3, y_3)\}) =$
$\quad = \{ p(y_3, f(f(x_3, y_3), y_2)) \}$

⑦ $\boxdot = \mathrm{Res}(D_8 \{y_2/a\}, D_{10} \{y_3/a, x_3/c, y_2/b\})$