$$\varphi_1^f \leftrightharpoons \forall x [(s(f(x)) \vee \neg r(x)) \& (\neg p(f(x), x) \vee \neg r(x))]$$

$$\varphi_2^f \leftrightharpoons \forall x [\neg s(x) \vee \neg q(x, a)]$$

$$\psi^f \leftrightharpoons \forall x \forall z [r(g(x)) \& (p(z, g(x)) \vee q(z, x))]$$

Да извлечем дизюнктите!!!

$$\varphi_1^{fin} \leftrightharpoons [\underbrace{(s(f(x)) \vee \neg r(x))} \& \underbrace{(\neg p(f(x), x) \vee \neg r(x))}]$$

$$\varphi_2^{fin} \leftrightharpoons [\underbrace{\neg s(x) \vee \neg q(x, a)}]$$

$$\psi^{fin} \leftrightharpoons [\underbrace{r(g(x))} \& \underbrace{(p(z, g(x)) \vee q(z, x))}]$$

$$D_1 \leftrightharpoons \{ s(f(x_1)), \neg r(x_1) \}$$

$$D_2 \leftrightharpoons \{ \neg p(f(x_2), x_2), \neg r(x_2) \}$$

$$D_3 \leftrightharpoons \{ \neg s(x_3), \neg q(x_3, a) \}$$

$$D_4 \leftrightharpoons \{ r(g(x_4)) \}$$

$$D_5 \leftrightharpoons \{ p(z_5, g(x_5)), q(z_5, x_5) \}$$

def/ ~~Субституция~~

Крайно м-во от вида $\{x_1/\tau_1, \dots, x_n/\tau_n\}$

$x_1, \dots, x_n \in Var$ различни помежду си и

$\tau_1, \dots, \tau_n \in \mathcal{T}_L$ и $x_i \neq \tau_i$ за всяко i: $1 \le i \le n$.

$x_i \to x_i$

Нека $\xi \leftrightharpoons \{x_1/\tau_1, \dots, x_n/\tau_n\}$

Инд. по постр. на терма $\tau \in \mathcal{T}_L$

base
- $x \in Var$, то $x^{\xi} = \begin{cases} x, & x \notin \{x_1, \dots, x_n\} \\ \tau_i, & x \in \{x_1, \dots, x_n\} \end{cases}$
- $c \in Const_L$, то $c^{\xi} = c$

(i.h.) вход $\mathcal{H}_i^{\xi} \dots$

step
- $f \in Func_L$, $\#f = n$, ~~$\mathcal{H}_1, \dots, \mathcal{H}_n \in$~~ $\mathcal{T}_L$, то $f(\mathcal{H}_1, \dots, \mathcal{H}_n)^{\xi} = f(\mathcal{H}_1^{\xi}, \dots, \mathcal{H}_n^{\xi})$

$\{x\} \mapsto x$

$\{x\} \mapsto y$

Променливи заменяме с термове!

$\tau\xi = \tau[x_1/\tau_1, \dots, x_n/\tau_n]$

едновременна замяна на $x_1, \dots, x_n$ с термове $\tau_1, \dots, \tau_n$.

$(\neg p(x,y))$ $\quad \xi = \{x/f(y), y/c\}$

$(\neg p(x,y))\xi = (\neg p(x\xi, y\xi)) = (\neg p(f(y), c))$

<u>def</u>/ <u>Унификатор</u>

Нека $\underline{\mathcal{E} \neq \emptyset}$ <u>м-во от термове</u> т.е. $\mathcal{E} \subseteq T_L$ и нека σ субституция. Казваме, че σ е унификатор за $\mathcal{E}$: $\underline{\mathcal{E}\sigma} = \{\tau\sigma \mid \tau \in \mathcal{E}\}$, $\underline{\overline{\overline{\mathcal{E}\sigma}} = 1}$ е синглетон.

$\underline{\mathcal{E}} = \{\underline{g(f(x), c)}, \underline{g(y, z)}, \underline{g(t, v)}\}$, $\quad \sigma = \{y/f(x), t/f(x), z/c, v/c\}$

$\underline{\mathcal{E}\sigma = \{g(f(x), c)\}}$

① Резолвента

Нека D_i и D_j са дизюнкти, които $D_i = \{L\} \cup D_i'$, а $D_j = \{\neg L'\} \cup D_j'$.

Б.О.О нека D_i и D_j нямат общи променливи помежду си (иначе с преименуваща субстит. оттипа $x_{нов} \mapsto y_{нов}$ ги правим различни).

Търсим унификатор σ: $\boxed{L\sigma = L'\sigma}$

$$Res_L(D_i, D_j) = D_i'\sigma \cup D_j'\sigma$$

$$D_i'\sigma = D_i\sigma \setminus \{L\sigma\}, \quad D_j'\sigma = D_j\sigma \setminus \{\neg L'\sigma\}$$

② Колапс

Нека D_i е дизюнкт, т.е. $D_i = \boxed{\{h_1, \dots, h_k\}} \cup D_i'$.

Търсим унификатор σ, т.е. $h_1\sigma = h_2\sigma = \dots = h_k\sigma$

$$Collapse(D_i) = \{h_1\sigma\} \cup D_i'\sigma.$$

$D_1 \leftrightharpoons \{ s(f(x_1)), \neg r(x_1) \}$

$D_2 \leftrightharpoons \{ \neg p(f(x_2), x_2), \neg r(x_2) \}$

$D_3 \leftrightharpoons \{ \neg s(x_3), \neg q(x_3, a) \}$

$D_4 \leftrightharpoons \{ f(g(x_4)) \}$

$D_5 \leftrightharpoons \{ p(z_5, g(x_5)), q(z_5, x_5) \}$

① Разгл. D_1 и D_4

$\| x_1 = g(x_4)$

$\sigma = \sigma_1 \cup \sigma_4$

$\sigma_1 = \{ x_1 / g(x_4) \}$

$\sigma_4 = \emptyset$

$D_6 = Res(D_1 \sigma_1, D_4) = \{ s(f(g(x_4))) \}$

② Разгл. D_3 и D_6

$\| x_3 = f(g(x_4)) \rightarrow \sigma = \sigma_3 \cup \sigma_6, \ \sigma_6 = \emptyset$

$\sigma_3 = \{ x_3 / f(g(x_4)) \}$

$D_7 = Res(D_3 \sigma_3, D_6) = \{ \neg q(f(g(x_4)), a) \}$

③ Разгл. D_5 и D_7

$\| z_5 = f(g(x_4))$
$\| x_5 = a$

$\| z_5 = f(g(x_5))$
$\| x_5 = a$
$\longleftrightarrow$
$\| z_5 = f(g(a))$
$\| x_5 = a$

пример

$D_8 = Res(D_5 \{ z_5 / f(g(x_4)), x_5 / a \}, D_7) = \longrightarrow$

$= \{ p(f(g(x_4)), g(a)) \}$

④ Разгл. D_2 и D_4

$\| \; x_2 = g(x_4)$

$D_3 = Res(D_2\{x_2 / g(x_4)\}, D_4) =$

$= \{ \neg p(f(g(x_4)), g(x_4)) \}$

⑤ Разгл. D_8 и D_3

$D_3' = D_3\{x_4/x_3\} = \{ \neg p(f(g(x_3)), g(x_3)) \}$.

$\left\| \begin{array}{l} \cancel{f}(\cancel{g}(x_4)) = \cancel{f}(\cancel{g}(x_3)) \\ \cancel{g}(a) = \cancel{g}(x_3) \end{array} \right. \Leftrightarrow \left\| \begin{array}{l} x_4 = x_3 \\ a = x_3 \end{array} \right. \Leftrightarrow \left\| \begin{array}{l} x_4 = x_3 \\ x_3 = a \end{array} \right. \Leftrightarrow \left\| \begin{array}{l} x_4 = a \\ x_3 = a \end{array} \right.$

$= Res(D_8\{x_4/a\}, D_3'\{x_3/a\})$

$\neg p(\ldots, \ldots, \ldots)$

$p(\ldots, b, \ldots)$

$\varphi_1 \leftrightharpoons \forall x \forall y (\neg p(x,y) \Rightarrow \forall z (p(x,z) \Rightarrow p(z,y)))$

$\varphi_2 \leftrightharpoons \forall x \neg \exists y \forall z ((p(x,z) \,\&\, p(y,z)) \Rightarrow (p(z,x) \,\&\, \neg p(z,x)))$

$\varphi_3 \leftrightharpoons \forall x \forall y [\forall x \exists y (p(x,y) \vee \neg p(x,y)) \Rightarrow \forall z \exists t (p(y,t) \,\&\, p(x,t) \,\&\, p(z,t))]$

Док-те $\varphi_1, \varphi_2 \models \varphi_3$

$\varphi_1 \leftrightharpoons \forall x \exists y (q(y,x) \,\&\, \forall z (q(y,z) \Rightarrow r(z,x)))$

$\varphi_2 \leftrightharpoons \forall x (\exists y\, q(x,y) \Rightarrow \exists y (q(x,y) \,\&\, \neg \exists z (q(y,z) \,\&\, q(x,z))))$

$\varphi_3 \leftrightharpoons \forall x \forall y \forall z ((q(y,x) \,\&\, r(z,y)) \Rightarrow q(z,x))$

$\varphi_4 \leftrightharpoons \neg \forall x \forall z (q(x,x) \Rightarrow (r(x,z) \,\&\, \neg r(x,z)))$

С мнор Р да се докаже, че множеството от следните четири формули е неизпълнимо.

$\{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$ е неизп. $\Leftrightarrow$

$\varphi_1 \,\&\, \varphi_2 \,\&\, \varphi_3 \,\&\, \varphi_4$ е неизп.

(1) $\varphi_1 \models \forall x \exists y (q(y,x) \,\&\, \forall z (\neg q(y,z) \vee r(z,x)))$

$\varphi_1' \leftrightharpoons$

$\varphi_2 \models \forall x (\forall y \neg q(x,y) \vee \exists y (q(x,y) \,\&\, \forall z (\neg q(y,z) \vee \neg q(x,z))))$

$\varphi_2' \leftrightharpoons$

$\varphi_3 \models \forall x \forall y \forall z (\neg q(y,x) \vee \neg r(z,y) \vee q(z,x))$

$\varphi_3' \leftrightharpoons$

$\varphi_4 \models \neg \forall x \forall z (\neg q(x,x) \vee \overbrace{(r(x,z) \,\&\, \neg r(x,z))}^{\text{лъжа}}) \models$

$\models \neg \forall x \forall z\, (\neg q(x,x)) \models \neg \forall x (\neg q(x,x)) \models$

$\models \exists x\, q(x,x)$

$\varphi_4' \leftrightharpoons$

(2) $\forall x \exists y (q(y,x) \,\&\, \forall z (\neg q(y,z) \vee r(z,x)))$

$\varphi_1' \leftrightharpoons$

$\varphi_1'' \leftrightharpoons \forall x \exists y \forall z\, (q(y,x) \,\&\, (\neg q(y,z) \vee r(z,x)))$

$\forall x (\forall y \neg q(x,y) \vee \exists y (q(x,y) \,\&\, \forall z (\neg q(y,z) \vee \neg q(x,z))))$

$\varphi_2' \leftrightharpoons$

$\models \forall x \exists y (\forall t \neg q(x,t) \vee (q(x,y) \,\&\, \forall z (\neg q(y,z) \vee \neg q(x,z))))$

$\models \forall x \exists y \forall t \forall z (\neg q(x,t) \vee (q(x,y) \,\&\, (\neg q(y,z) \vee \neg q(x,z))))$

$\varphi_2'' \leftrightharpoons$

$\varphi_3'' \leftrightharpoons \varphi_3' = \forall x \forall y \forall z\, (\neg q(y,x) \vee \neg r(z,y) \vee q(z,x))$

$\varphi_4'' \leftrightharpoons \varphi_4' = \exists x\, q(x,x)$

$C^A \leftrightharpoons a$, $Q_\alpha = \{a \mid a \in A,\ A \models q(a,a)\}$

(3) $\varphi_1'' \lessgtr \forall x \exists y \forall z \, (q(y,x) \,\&\, (\neg q(y,z) \vee r(z,x)))$

$\varphi_1^s \lessgtr \forall x \forall z \, (q(f(x),x) \,\&\, (\neg q(f(x),z) \vee r(z,x)))$

$\varphi_2'' \lessgtr \forall x \exists y \forall t \forall z \, (\neg q(x,t) \vee (q(x,y) \,\&\, (\neg q(y,z) \vee \neg q(x,z))))$

$\varphi_2^s \lessgtr \forall x \forall t \forall z \, (\underbrace{\neg q(x,t)}_{\chi} \vee (\underbrace{q(x,g(x))}_{\psi} \,\&\, \overbrace{(\neg q(g(x),z) \vee \neg q(x,z))}^{\varphi}))$

$\varphi_3^s \lessgtr \varphi_3'' = \forall x \forall y \forall z \, (\neg q(y,x) \vee \neg r(z,y) \vee q(z,x))$

$\varphi_4^s \lessgtr q(c,c)$

(4) $\varphi_1^f \lessgtr \varphi_1^s$

$\varphi_2^f \lessgtr \forall x \forall t \forall z \, ((\neg q(x,t) \vee q(x,g(x))) \,\&\, (\neg q(x,t) \vee \neg q(g(x),z) \vee \neg q(x,z)))$

$\varphi_3^f \lessgtr \varphi_3^s$

$\varphi_4^f \lessgtr \varphi_4^s$

(5) $\varphi_1^{fin} \lessgtr \left(q(f(x), x) \;\&\; (\neg q(f(x), z) \vee r(z, x)) \right)$

$\varphi_2^{fin} \lessgtr \left((\neg q(x, t) \vee q(x, g(x))) \;\&\; (\neg q(x, t) \vee \neg q(g(x), z) \vee \neg q(x, z)) \right)$

$\varphi_3^{fin} \lessgtr (\neg q(y, x) \vee \neg r(z, y) \vee q(z, x))$

$\varphi_4^{fin} \lessgtr q(c, c)$

$D_1 \lessgtr \{ q(f(x_1), x_1) \}$

$D_2 \lessgtr \{ \neg q(f(x_2), z_2), r(z_2, x_2) \}$

$D_3 \lessgtr \{ \neg q(x_3, t_3), q(x_3, g(x_3)) \}$

$D_4 \lessgtr \{ \neg q(x_4, t_4), \neg q(g(x_4), z_4), \neg q(x_4, z_4) \}$

$D_5 \lessgtr \{ \neg q(y_5, x_5), \neg r(z_5, y_5), q(z_5, x_5) \}$

$D_6 \lessgtr \{ q(c, c) \}$

$D_1 \leftrightharpoons \{q(f(x_1), x_1)\}$

$D_2 \leftrightharpoons \{\neg q(f(x_2), z_2), \cancel{r(z_2, x_2)}\}$ X

$D_3 \leftrightharpoons \{\neg q(x_3, t_3), \cancel{q(x_4, g(x_3))}\}$ X

$D_4 \leftrightharpoons \{\neg q(x_4, t_4), \neg q(g(x_4), z_4), \neg q(x_4, z_4)\}$ X

$D_5 \leftrightharpoons \{\neg q(y_5, x_5), \cancel{\neg r(z_5, y_5)}, q(z_5, x_5)\}$ X

$D_6 \leftrightharpoons \{q(c, c)\}$

① Разгл. D_2 и D_5

$\left\| \begin{array}{l} z_2 = z_5 \\ x_2 = y_5 \end{array} \right.$

$D_7 = Res(D_2\{z_2/z_5, x_2/y_5\}, D_5) = \{\cancel{\neg q(f(y_5), z_5)}, \neg q(y_5, x_5), q(z_5, x_5)\}$

② Разгл. D_7 и D_3

$\left\| \begin{array}{l} x_3 = f(y_5) \\ g(x_3) = z_5 \end{array} \right. \longleftrightarrow \left\| \begin{array}{l} x_3 = f(y_5) \\ z_5 = g(x_3) \end{array} \right. \longleftrightarrow \left\| \begin{array}{l} x_3 = f(y_5) \\ z_5 = g(f(y_5)) \end{array} \right.$

$D_8 = Res(D_3\{x_3/f(y_5)\}, D_7\{z_5/g(f(y_5))\}) =$

$= \{\neg q(f(y_5), t_3), \neg q(y_5, x_5), q(g(f(y_5)), x_5)\}$

$D_4 = \{\neg q(x_4, t_4), \neg q(g(x_4), z_4), \neg q(x_4, z_4)\}$

$D_8 = \{\neg q(f(y_5), t_3), \neg q(y_5, x_5), q(g(f(y_5)), x_5)\}$

(3) Разр. D_4 и D_8

$$\left\| \begin{array}{l} \cancel{g}(x_4) = \cancel{g}(f(y_5)) \\ z_4 = x_5 \end{array} \right. \Longleftrightarrow \left\| \begin{array}{l} x_4 = f(y_5) \\ z_4 = x_5 \end{array} \right.$$

$D_9 = Res(D_4\{x_4/f(y_5), z_4/x_5\}, D_8) =$

$= \{\neg q(f(y_5), t_4), \neg q(f(y_5), x_5), \neg q(f(y_5), t_3),$
$\neg q(y_5, x_5)\}$

(4) Разр. D_9

$D_{10} = Collapse(D_9\{t_4/x_5, t_3/x_5\}) = \{\neg q(f(y_5), x_5),$
$\neg q(y_5, x_5)\}$

(5) Разр. D_{10} и D_1

$D_{11} = Res(D_1, D_{10}\{y_5/x_1, x_5/x_1\}) = \{\neg q(x_1, x_1)\}$

(6) Разр. D_6 и D_{11}

$= Res(D_6, D_{11}\{x_1/c\})$