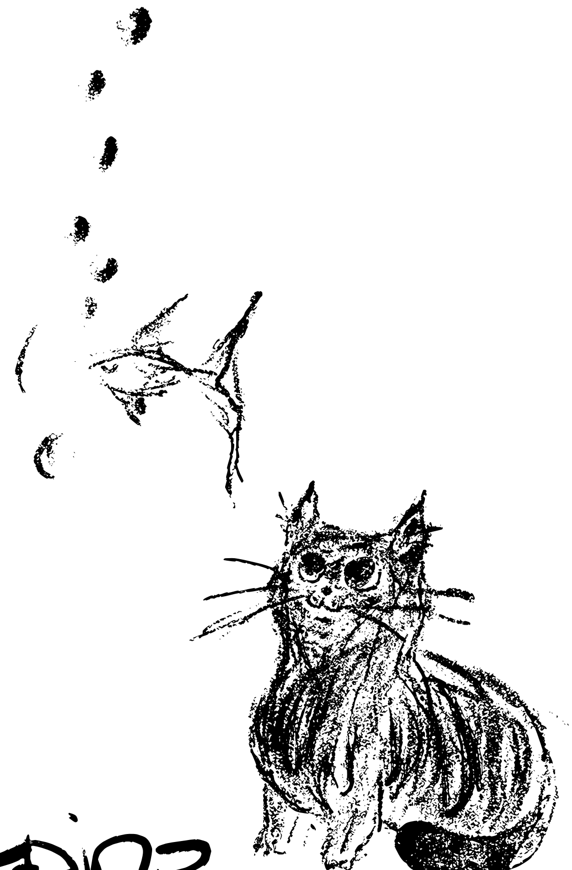


Sleeping

Зад 3

$\varphi_1 \leftrightharpoons \forall x(p(x) \Rightarrow \forall y\, p(y))$

$\varphi_2 \leftrightharpoons \forall x \forall y(q(y,y) \vee \neg q(x,y))$

$\varphi_3 \leftrightharpoons \exists x \neg q(x,x)$

$\varphi_4 \leftrightharpoons \exists x \forall y\, q(y,x)$

$\varphi_5 \leftrightharpoons \forall x \forall y \forall z((p(x) \Leftrightarrow r(x,y,z)) \Leftrightarrow q(y,z))$

Намерете модел за ф-лите горе.

$$\boxed{\varphi_1 \models\!\dashv \exists x\, p(x) \Rightarrow \forall y\, p(y)}$$

$\cancel{\forall x \neg p(x) \models\!\dashv}$

$\neg \exists x\, p(x)$

$\varphi_1 \models\!\dashv \forall x(\neg p(x) \vee \forall y\, p(y)) \models\!\dashv (\forall x \neg p(x) \vee \forall y\, p(y)) \models\!\dashv$

$x \notin Var^{free}[\forall y\, p(y)] = \emptyset$

$\models\!\dashv (\neg \exists x\, p(x) \vee \forall y\, p(y)) \models\!\dashv \exists x\, p(x) \Rightarrow \forall y\, p(y)$

Т.е. или $p^{\mathcal{A}} = A$, или $p^{\mathcal{A}} = \emptyset$

$$\boxed{\forall x(\varphi \vee \psi) \models\!\dashv (\forall x \varphi \vee \psi) \quad x \notin Var^{free}[\psi]}$$

$\varphi_1 \coloneqq \forall x\,(p(x) \Rightarrow \forall y\, p(y))$

$\dashv\vdash \forall x\,(\neg p(x) \vee \forall y\, p(y))$

$\dashv\vdash (\forall x\, \neg p(x) \vee \forall y\, p(y))$

$\dashv\vdash (\neg \exists x\, p(x) \vee \forall y\, p(y))$

$\dashv\vdash (\exists x\, p(x) \Rightarrow \forall y\, p(y))$

$\forall x\,(\varphi \vee \psi)$

$x \notin \mathrm{Var}^{\mathrm{free}}[\psi]$

$\forall x\,(\varphi \vee \psi) \dashv\vdash (\forall x\, \varphi \vee \psi)$

$\forall x\, \neg p(x) \dashv\vdash \neg \exists x\, p(x)$

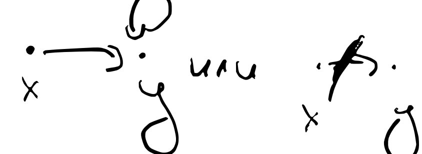
$\varphi_1 \leftrightharpoons \forall x(p(x) \Rightarrow \forall y\, p(y)) \longrightarrow p^{A} = \emptyset$ или $p^{A} = A$

$\varphi_2 \leftrightharpoons \forall x \forall y (q(y,y) \vee \neg q(x,y))$

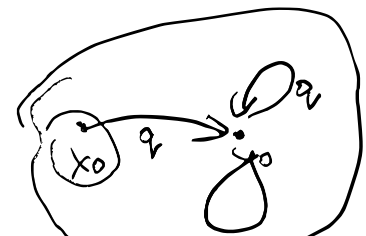
$\varphi_3 \leftrightharpoons \exists x \neg q(x,x)$

$\varphi_4 \leftrightharpoons \exists x \forall y\, q(y,x)$

$\varphi_5 \leftrightharpoons \forall x \forall y \forall z ((p(x) \Leftrightarrow r(x,y,z)) \Leftrightarrow q(y,z))$

φ_2: 

$\varphi_2 \models \forall x \forall y (q(x,y) \Rightarrow q(y,y))$



φ_3: връх без примка

φ_4:  или връх сочи от всеки друг

$\Gamma_1 \leftrightharpoons \{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$, то $A_1 \models \Gamma_1$

$A_1 = \langle \{x_0, y_0\}, p^{A}, q^{A} \rangle$

$\langle x_0, x_0, x_0 \rangle \in r^{A}$

$\langle x_0, x_0, y_0 \rangle \notin r^{A}$

$\langle x_0, y_0, x_0 \rangle \in r^{A}$

$\langle y_0, x_0, x_0 \rangle \in r^{A}$

$\langle y_0, x_0, y_0 \rangle \notin r^{A}$

$\langle y_0, y_0, x_0 \rangle \in r^{A}$

$\langle y_0, y_0, y_0 \rangle \notin r^{A}$

$\langle x_0, y_0, y_0 \rangle \notin r^{A}$

$p^{A}(x_0)$ } not

$p^{A}(y_0)$ } true

Когато имаме $p(x)$

x/y зависими —> не слагаме

$\delta(q(y,z))$ тройки в r

$\langle x, y, z \rangle$

$$q^{A}(y,z) \equiv \mathbb{T} \rightarrow r^{A}(x,y,z) \equiv \mathbb{F}$$

$$q^{A}(y,z) \equiv \mathbb{F} \Rightarrow r^{A}(x,y,z) \equiv \mathbb{T}$$

$$\varphi_5 \leftrightharpoons \forall x \forall y \forall z ((p(x) \Leftrightarrow r(x,y,z)) \Leftrightarrow q(y,z))$$

$A = \langle \{0,1\}, p^{A}, r^{A}, q^{A} \rangle$ $p^{A} = \emptyset$, $q^{A} = \{\langle 0,1 \rangle, \langle 1,1 \rangle\}$

$r^{A} = \{\langle 0,0,0 \rangle, \langle 0,1,0 \rangle, \dots\}$

Зад 4

$\varphi_1 \leftrightharpoons \forall x \forall y \forall z \,(f(x, f(y,z)) \doteq f(f(x,y),z))$

$\varphi_2 \leftrightharpoons \forall x \exists y \,(f(x,y) \doteq x \,\&\, f(y,x) \doteq x)$

$\varphi_3 \leftrightharpoons \neg \exists y \forall x \,(f(x,y) \doteq x \,\&\, f(y,x) \doteq x)$

$\varphi_4 \leftrightharpoons \forall x \exists y \forall z \,(f(y,z) \neq x)$

Намерете модел за ф-лите.

φ_1 - асоциативност

φ_2 - имаме единица за всеки елемент (не непрем. различна)

φ_3 - нямаме универсална единица

$f^{\mathcal{A}}(a,b) \leftrightharpoons a \cap b$

$A \leftrightharpoons \{\underbrace{(0;n)}_{\text{интервали на } \mathbb{R}} \subseteq \mathbb{R} \mid n \in \mathbb{N}_{>0}\} \cup \{\underline{(0;1/n)} \subseteq \mathbb{R} \mid n \in \mathbb{N}_{>0}\}$

$x = (0; n)$? $\cancel{y = (0; 1/n)}$ $\underline{n > 1}$

$x \cap y = (0; 1/n)$

$\forall ? \, z \to (0; 1/n) \subseteq z$

$x = (0; 1)$? $y = (0; 1/2)$

$x \cap y = (0; 1/2)$

$\forall ? \, z \to z \supseteq (0; 1/2) \xrightarrow{?} (0; 1/2)$

$z \subseteq (0; 1/2) \xrightarrow{?} z \neq y$

$f^{\mathcal{B}}(a,b) = a \cup b$

$\mathcal{B} = \langle \{\{0\}, \{1\}, \{0,1\}\}, f^{\mathcal{B}} \rangle$

A – безкрайно м-во

$\mathcal{C} = \langle Fin(A) \setminus \{\emptyset\}, f^{\mathcal{C}} \rangle$

$f^{\mathcal{C}}(a,b) \leq a \cup b$

$x = B \in Fin(A) \setminus \{\emptyset\}$

$\exists \quad y = B \cup \{c\} \cup z \supsetneq B$

$c \notin B$

$\mathfrak{t}' = \langle \underline{\{0,1\}}, f^{\mathfrak{t}'} \rangle$ ✓

$\underline{f^{\mathfrak{t}'}(a,b) = a}$

$$\boxed{\begin{array}{l} \mathfrak{t}'' = \langle \mathbb{Z}, f^{\mathfrak{t}''} \rangle \\ f^{\mathfrak{t}''}(a,b) = \underline{\min\{a,b\}} \end{array}}$$ ✓

$\forall y \exists x \neg (f(x,y) \doteq x \wedge f(y,x) \doteq x) \dashv\vdash$

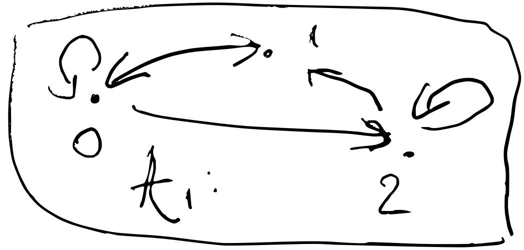
$\forall y \exists x (f(x,y) \not\doteq x \vee f(y,x) \not\doteq x)$

(4) $\varphi_1 \leftrightharpoons \forall x \exists y\, p(x,y)$

$\varphi_2 \leftrightharpoons \exists x\, p(x,x)$

$\varphi_3 \leftrightharpoons \exists x \neg p(x,x)$

$\varphi_4 \leftrightharpoons \forall x \forall y (p(x,y) \Rightarrow \exists z (p(x,z) \,\&\, p(z,y)))$



$\Gamma_1 \leftrightharpoons \{\varphi_1, \varphi_4\}$

$\mathcal{A}_2 = \langle \mathbb{R}, \underbrace{p^{\mathcal{A}_2}}_{<_{\mathbb{R}}} \rangle \models \Gamma_1$

Как да направим $\mathcal{A}_2$ модел за всички ф-ли?

$\mathcal{A}_3 = \langle \mathbb{R}, p^{\mathcal{A}_3} \rangle$, $\langle a, b \rangle \in p^{\mathcal{A}_3} \Leftrightarrow (a <_{\mathbb{R}} b) \vee (a = b = 0)$

Зад 5

$\varphi_1 \leftrightharpoons \forall x \exists y \exists z (q(x,y) \,\&\, \neg q(x,z))$

$\varphi_2 \leftrightharpoons \forall x \forall y (q(x,y) \Rightarrow r(x,y))$

$\varphi_3 \leftrightharpoons \forall x \forall y \forall z (r(x,y) \,\&\, r(y,z) \Rightarrow r(x,z))$

Докажете, че ф-лите имат модел.

$\boxed{\text{кш}}$

Зад ⑥ $\varphi_1 \rightleftharpoons \exists x \forall y (x \neq y \Rightarrow \exists z \neg p(y, z))$

$\varphi_2 \rightleftharpoons \exists x \forall y \, p(x, y)$

$\varphi_3 \rightleftharpoons \forall x \forall y (p(x, y) \Rightarrow \exists z (p(x, z) \& p(z, y)))$

$\varphi_4 \rightleftharpoons \forall x \exists y \, p(x, y)$

НШ → краен ?

→ безкраен ?

зад 7

$\varphi_1 \leftrightharpoons \neg \exists x\, p(x,x)$

$\varphi_2 \leftrightharpoons \forall x \exists y (p(x,y) \,\&\, \neg \exists z (p(x,z) \,\&\, p(z,y)))$

$\varphi_3 \leftrightharpoons \exists x \neg \exists y\, p(y,x) \dashv\vdash \exists x \forall y \neg p(y,x)$

$\varphi_4 \leftrightharpoons \exists x (\exists y\, p(y,x) \,\&\, \neg \exists y (p(y,x) \,\&\, \exists z (p(x,z) \,\&\, p(z,y))))$

φ_1 – ирефл.

φ_2 – всяка точка има непосредствен наследник.

φ_3 – има елемент без входящи върхове

$\Gamma_1 \leftrightharpoons \{\varphi_1, \varphi_2, \varphi_3\}$

$\langle \mathbb{N}, <_{\mathbb{N}} \rangle \models \Gamma_1$

$\varphi_4 \dashv\vdash \exists x (\exists y\, p(y,x) \,\&\, \forall t (p(t,x) \Rightarrow \exists z (p(x,z) \,\&\, p(z,t))))$

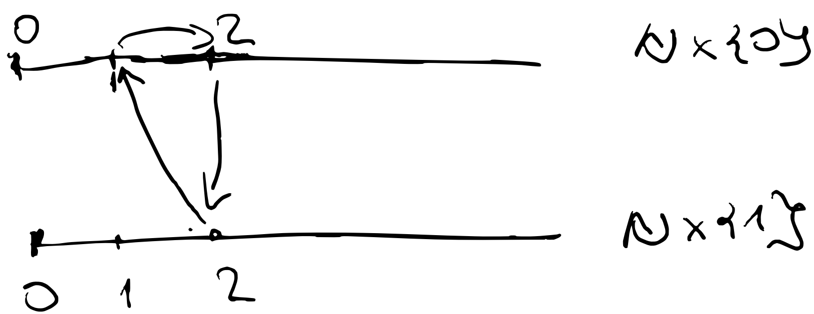
$p(1,2)$

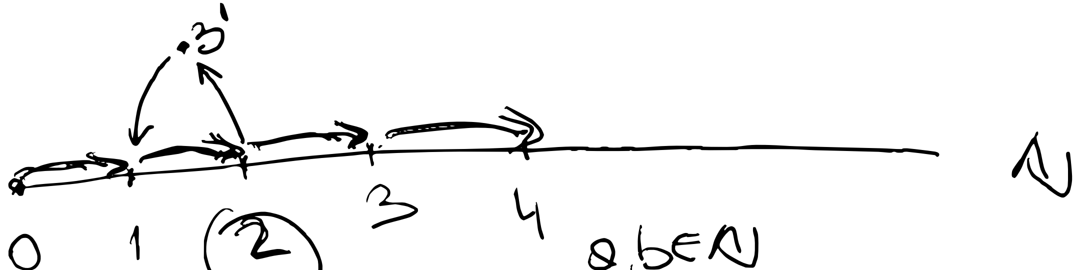
$p(0,2)$

$\mathcal{A} = \langle \mathbb{N}\times\{0\} \cup \mathbb{N}\times\{1\}, p^{\mathcal{A}} \rangle$

$p^{\mathcal{A}}(\langle a,b\rangle, \langle c,d\rangle) \Leftrightarrow (b=d \;\&\; a<c) \lor$

$(b<d \;\&\; \underline{c \leq a} \;\&\; c \neq 0 \;\&\; a \neq 0)$





$a,b \in \mathbb{N}$

$A = \mathbb{N} \cup \{3'\}$, $p^{\mathcal{A}}(a,b) \Leftrightarrow b = a+1$

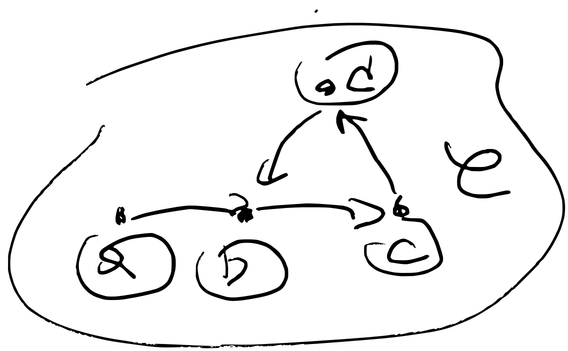
$\lor \{\langle 2,3'\rangle, \langle 3',1\rangle\}$

$\mathcal{B} = \langle \mathbb{N} \cup \{3'\}, P^{\mathcal{B}} \rangle$

$P^{\mathcal{B}}(a,b) \Leftrightarrow (a,b \in \mathbb{N} \;\&\; b = a+1) \vee$

$a,b \in \mathbb{N} \cup \{3'\} \quad (a = 2 \;\&\; b = 3') \vee$

$(a = 3' \;\&\; b = 1)$



320(8)

$\varphi_1 \rightleftharpoons \forall x\,(f(x) \neq x)$

$\varphi_2 \rightleftharpoons \forall x\,(g(f(x)) = x)$

$\varphi_3 \rightleftharpoons \exists x\,(g(x) = x)$

$\varphi_4 \rightleftharpoons \forall x \forall y\,(p(x,y) \Rightarrow \neg p(y,x))$

$\varphi_5 \rightleftharpoons \forall x \forall y \forall z\,(p(x,y) \,\&\, p(y,z) \Rightarrow p(x,z))$

$\varphi_6 \rightleftharpoons \forall x \exists y\, p(x,y)$

HW

Зад 3 $\varphi_1 \leftrightharpoons \forall x \forall y \forall z (q(x,y) \,\&\, q(y,z) \Rightarrow q(x,z))$

$\varphi_2 \leftrightharpoons \exists x \exists y\, q(x,y)$

$\varphi_3 \leftrightharpoons \forall x \neg q(x,x)$

$\varphi_4 \leftrightharpoons \forall x \forall y (q(x,y) \Rightarrow \exists z (q(x,z) \,\&\, q(z,y)))$

$\varphi_5 \leftrightharpoons \forall x \forall y (q(x,y) \Rightarrow \exists z (q(z,y) \,\&\, z \neq x \,\&$
$\neg q(x,z) \,\&\, \neg q(z,x)))$

φ_1 - транзитивност

φ_2 - $q^{A} \neq \emptyset$

φ_3 - иррефлексивна

φ_4 - гъстота

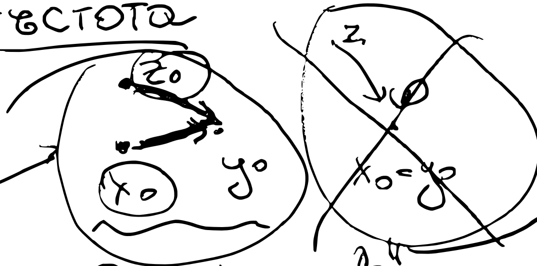
φ_5 -

поне две точки

$\Gamma_1 \leftrightharpoons \{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$

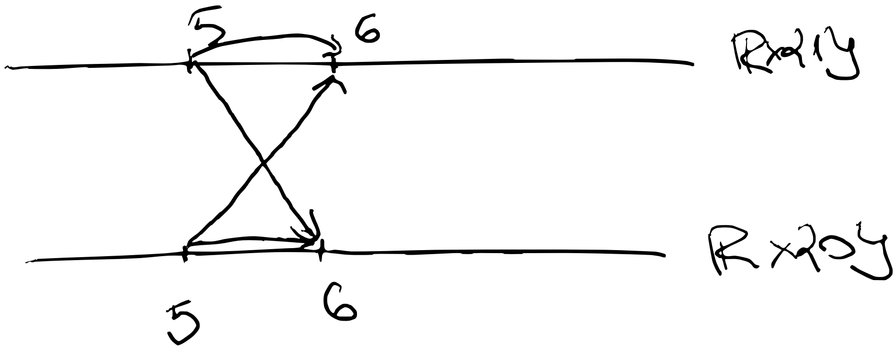
$A' = \langle \mathbb{R}, q^{A'} \rangle \models \Gamma_1$

$q^{A'} = <_{\mathbb{R}}$



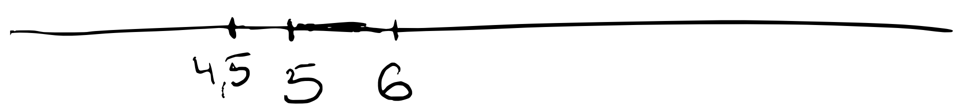
$A'' = \langle \mathbb{R} \times \{0\} \cup \mathbb{R} \times \{1\}, q^{A''} \rangle$

$q^{A''}(\langle a, b \rangle, \langle c, d \rangle) \Leftrightarrow a < c$



$\mathfrak{B} = \langle \mathbb{R}, q^{\mathfrak{B}} \rangle$

$q^{\mathfrak{B}}(a,b) \leftrightarrow a+1 \leq b$



$q^{\mathfrak{B}}(5,6) \to z = 4{,}5 \quad q^{\mathfrak{B}}(4{,}5, 6)\ \&$

$\neg q^{\mathfrak{B}}(4{,}5, 5)\ \&$

$\neg q^{\mathfrak{B}}(5, 4{,}5)$