петък: 8:00 упражнение

$\Sigma \models \varphi$, Σ' е неизпълнимо

$\Sigma \cup \{\neg\varphi\}$ е неизпълнимо

ПНФ, СНФ, КНФ, Дизюнкти $\xrightarrow{\text{Резолюция}} \boxed{}$

$\exists x \forall y (q(x,y) \Leftrightarrow \forall z (q(y,z) \Rightarrow (q(z,y) \Rightarrow \forall x (q(x,y) \,\&\, \neg q(x,y)))))) \models$

неизпълнимо?

$\varphi \leq \exists x \forall y (q(x,y) \Leftrightarrow \forall z (q(y,z) \overset{F}{\Rightarrow} \neg q(z,y)))$

① Вкарваме отрицанието в атомарни ф-ли и разписваме „⇔" и „⇒".

Ползваме: $\neg\forall x\varphi \models\mid \exists x\neg\varphi$

$\neg\exists x\varphi \models\mid \underline{\forall x\neg}\varphi$

$\neg(\varphi\vee\psi) \models\mid \neg\varphi \,\&\, \neg\psi$

$\neg(\varphi\,\&\,\psi) \models\mid \neg\varphi\vee\neg\psi$

$\neg\neg\varphi \models\mid \varphi$

$\varphi\Rightarrow\psi \models\mid \neg\varphi\vee\psi$

$\varphi\Leftrightarrow\psi \models\mid (\varphi\Rightarrow\psi)\,\&\,(\psi\Rightarrow\varphi) \models\mid$
$(\neg\varphi\vee\psi)\,\&\,(\neg\psi\vee\varphi)$.

$\varphi \models\mid \exists x\forall y(q(x,y) \Leftrightarrow \forall z(\neg q(y,z)\vee\neg q(z,y)))$

$\models\mid \exists x\forall y((\neg q(x,y) \vee \forall z(\neg q(y,z)\vee\neg q(z,y)))\,\&$

$(\exists z(q(y,z)\,\&\,q(z,y)) \vee q(x,y)))$

φ'

t t t

② ПНФ кванторен префикс

$\varphi \equiv \underbrace{Q_1x_1 \dots Q_nx_n}_{} \theta$, където: ($\theta$ → матрица)

- $Q_i \in \{\exists, \forall\}$, $1 \le i \le n$
- θ е безкв. ф-ла
- $i \neq j \rightarrow x_i \neq x_j$

$x \notin Var^{free}[\psi]$

$\exists x \varphi \vee \psi \models\!\mid \exists x(\varphi \vee \psi)$

$\exists x \varphi \& \psi \models\!\mid \exists x(\varphi \& \psi)$

$\forall x \varphi \vee \psi \models\!\mid \forall x(\varphi \vee \psi)$

$\forall x \varphi \& \psi \models\!\mid \forall x(\varphi \& \psi)$

Тн за вариантна.

$Var^{free}[\varphi] \cap Var^{bd}[\varphi] \neq \emptyset$

$\forall y(\underset{\exists}{\forall} x p(x,y) \underset{\&}{\vee}$ [redacted] $\exists z p(z,z)) \models\!\mid$

$\forall y \exists z \forall x (p(x,y) \vee p(z,z))$

$\varphi'' \leftrightharpoons \exists x \forall y \exists t \forall z ((\neg q(x,y) \vee \neg q(z,y) \vee \neg q(y,z)) \,\&$
$(q(x,y) \vee (q(t,y) \,\&\, q(y,t))))$

c f

③ <u>Скол.</u> ψ инд.

едностъпкова

$\varphi^{\underbrace{S \dots S}}$ отляво надясно по

• $\psi \equiv \exists x \varphi \rightsquigarrow c_\varphi \rightsquigarrow \mathcal{L}'$ с нова ик. c_φ

$\psi_S \leftrightharpoons \varphi[x/c_\varphi]$

• $\psi \equiv \forall y_1 \dots \forall y_n \exists x \varphi \rightsquigarrow f_\varphi \rightsquigarrow \mathcal{L}'$ с нова ф.с. f_φ

$n > 0$

$\psi_S \leftrightharpoons \forall y_1 \dots \forall y_n \varphi[x/f_\varphi(y_1, \dots, y_n)]$

$\varphi'' \not\models \varphi^S$ $\quad \psi^S \equiv \psi_{\underbrace{SS \dots S}}$

колкото кв. за ∃ се срещат

$\varphi^S \leftrightharpoons \forall y \forall z ((\neg q(c,y) \vee \neg q(z,y) \vee \neg q(y,z)) \,\&$
$(q(c,y) \vee (q(f(y),y) \,\&\, q(y,f(y)))))$

④ $\varphi \vee (\psi \& \chi) \dashv\vdash (\varphi \vee \psi) \& (\varphi \vee \chi)$

$\varphi^{fin} \leftrightharpoons \forall y \forall z ((\neg q(c,y) \vee \neg q(z,y) \vee \neg q(y,z)) \,\&$
$(q(c,y) \vee q(f(y),y)) \,\&$
$(q(c,y) \vee q(y,f(y)))).$

КНФ, СНФ и ННФ.

$D_1 = \{\neg q(c,y_1), \neg q(z_1,y_1), \neg q(y_1,z_1)\}$
$D_2 = \{q(c,y_2), q(f(y_2),y_2)\}$
$D_3 = \{q(c,y_3), q(y_3,f(y_3))\}$

Правила на играта

$\{x_1/x_2\}$

① Правило за резолюцията

Нека D_1, D_2 са дизюнкти без общи променливи.

Нека $D_1 = D_1' \cup \{L\}$, $D_2 = D_2' \cup \{\neg L'\}$. Търсим субституция σ т.ч. $L\sigma = L'\sigma$. Тогава

$$Res(D_1, D_2) = D_1'\sigma \cup D_2'\sigma = (D_1\sigma \setminus \{L\sigma\}) \cup (D_2\sigma \setminus \{\neg L'\sigma\}).$$

② Колапс на дизюнкт. $n>1$

Нека $D = D' \cup \{L_1, \ldots, L_n\}$. Търсим субст. σ т.ч. $L_1\sigma = L_2\sigma = \ldots = L_n\sigma$.

Тогава $Collapse(D) = D'\sigma \cup \{L_1\sigma\} = D\sigma$

$D_1 = \{\neg q(c, y_1), \neg q(z_1, y_1), \neg q(y_1, z_1)\}$

$D_2 = \{q(c, y_2), q(f(y_2), y_2)\}$

$D_3 = \{q(c, y_3), q(y_3, f(y_3))\}$

$\left| \begin{array}{l} c = z_1 \\ z_1 = y_1 \end{array} \right. \quad | y_1 = c$

① Розгл. D_1.

$\text{Collapse}(D_1 \{y_1/c, z_1/c\}) = \{\neg q(c, c)\} = D_4$

② Розгл. D_4 и D_2 $\quad \sigma = \sigma_1 \cup \sigma_2$

$\text{Res}(D_4, D_2\{y_2/c\}) = \{q(f(c), c)\} = D_5$

③ Розгл. D_3 и D_4

$\text{Res}(D_4, D_3\{y_3/c\}) = \{q(c, f(c))\} = D_6$

④ Розгл. D_6 и D_1

$\text{Res}(D_6, D_1\{y_1/f(c), z_1/c\}) = \{\neg q(f(c), c)\} = D_7$

⑤ $\text{Res}(D_7, D_5) = \blacksquare$

~~$\{p(x), \neg p(x)\}$~~ $\rightarrow$ нохоме

more $\underbrace{\mathcal{D}_1}_{\text{important!!!!!}} \subseteq \mathcal{D}_2$ $\qquad D_1 = \{p(x)\},\ D_2 = \{p(x), q(x)\}$

зад $\varphi_1 \& \varphi_2 \Rightarrow \varphi_3$? $\{\varphi_1, \varphi_2\} \models \varphi_3 \Leftrightarrow \{\varphi_1, \varphi_2, \neg\varphi_3\}$ до е неизп. $\Leftrightarrow$ $\varphi_1 \& \varphi_2 \& \neg\varphi_3$ до е неизп.

$\varphi_1 \leftrightharpoons \forall x [\forall y (S(y) \Rightarrow p(y,x)) \Rightarrow \neg r(x)]$

$\varphi_2 \leftrightharpoons \exists y \forall x [\neg s(x) \vee \neg q(x,y)]$

$\varphi_3 \leftrightharpoons \exists x \forall y [r(y) \Rightarrow \exists z (\neg p(z,y) \& \neg q(z,x))]$

$\neg\psi \leftrightharpoons \forall x \exists y [r(y) \& \forall z (p(z,y) \vee q(z,x))]$

$\varphi_1 \models\!\mid \forall x [\forall y (\neg s(y) \vee p(y,x)) \Rightarrow \neg r(x)] \models 1$

$\varphi_1' \leftrightharpoons \forall x [\exists y (S(y) \& \neg p(y,x)) \vee \neg r(x)]$

$\varphi_1'' \leftrightharpoons \forall x \exists y [(S(y) \& \neg p(y,x)) \vee \neg r(x)]$

$\psi'' \leftrightharpoons \forall x \exists y \forall z [r(y) \& (p(z,y) \vee q(z,x))]$

$\varphi_1^S \leftrightharpoons \forall x \, [(s(f(x)) \,\&\, \neg p(f(x), x)) \vee \neg r(x)]$

$\varphi_2^S \leftrightharpoons \forall x \, [\neg s(x) \vee \neg q(x, c)] \vee$

$\psi^S \leftrightharpoons \forall x \forall z \, [r(g(x)) \,\&\, (p(z, g(x)) \vee q(z, x))]$.

$\varphi_1^{fin} \leftrightharpoons \forall x [(s(f(x)) \vee \neg r(x)) \,\&\, (\neg p(f(x), x) \vee \neg r(x))]$.

$D_1 = \{ s(f(x_1)), \neg r(x_1) \}$

$D_2 = \{ \neg p(f(x_2), x_2), \neg r(x_2) \}$

$D_3 = \{ \neg s(x_3), \neg q(x_3, c) \}$

$D_4 = \{ r(g(x_4)) \}$

$D_5 = \{ p(z_5, g(x_5)), q(z_5, x_5) \}$

1. $Res(D_1\{x_1/g(x_4)\}, D_4) = \{ s(f(g(x_4))) \} = D_6$
2. $Res(D_2\{x_2/g(x_4)\}, D_4) = \{ \neg p(f(g(x_4)), g(x_4)) \} = D_7$
3. $Res(D_3\{x_3/f(g(x_4))\}, D_6) = \{ \neg q(f(g(x_4)), c) \} = D_8$
4. $Res(D_5\{z_5/f(g(x_4))\}, D_8) = \{ p(f(g(x_4)), g(x_5)) \} = D_9$
5. $Res(D_9\{x_5/x_4\}, D_7) =$

$=$

D_1, D_2 са дизюнкти.

$L \in D_1, \bar{L} \in D_2$

$I \models D_1, D_2$

$D_1' = D_1 \setminus \{L\}, D_2' = D_2 \setminus \{\bar{L}\}$

$I \models D_1' \cup D_2'$ резолвента на D_1 и D_2 по L $(\bar{L})$

$\{q, r, s\}$

$D_1 \sigma_1 \quad D_2 \sigma_2$

резолвента

$\{\underline{p}, q\}$

$\{\neg\underline{p}, r, s\}$

1. Някои пациенти харесват докторите.
2. Никой пациент не харесва шарлатаните.
3. Докторите не са шарлатани.

$п(x)$, $д(x)$, $ш(x)$, $хв(x,y)$

$\varphi_1 \leftrightharpoons \exists x\,(\underline{П(x)} \wedge \forall y\,(д(y) \longrightarrow хв(x,y)))$

$\varphi_2 \leftrightharpoons \forall x\,(П(x) \longrightarrow \forall y\,(ш(y) \to \neg хв(x,y)))$

$\varphi_3 \leftrightharpoons \neg \forall y\,(д(y) \to \neg ш(y)) \models\!\mid \exists y\,(д(\cancel{y}) \wedge ш(y))$

$\{\varphi_1, \varphi_2, \varphi_3\}$ е неизпълнимо.

$\varphi_1 \models\!\mid \exists x\,\forall y\,(\underline{П(x)} \wedge (\neg д(y) \vee хв(x,y)))$

$\varphi_2 \models\!\mid \forall x \forall y\,(\neg П(x) \vee \neg ш(y) \vee \neg хв(x,y))$

$СкФ(\varphi_1) \leftrightharpoons \forall y\,(П(c) \wedge (\neg д(y) \vee хв(c,y)))$

$СкФ(\varphi_3) \leftrightharpoons д(d) \wedge ш(d)$

$\{д(d)\}$, $\{ш(d)\}$, $\{П(c)\}$, $\{\neg д(y), хв(c,y)\}$

$\{\neg П(x), \neg ш(y), \neg хв(x,y)\}$

$\overset{D_1}{\{\underline{g(d)}\}}, \overset{D_2}{\{\underline{w(d)}\}}, \overset{D_3}{\{\underline{\pi(c)}\}}, \overset{D_4}{\{\neg \underline{g}(y), \neg b(c,y)\}}$

$D_5 = \{\neg \underline{\pi(x)}, \neg \underline{w(y)}\ \neg yb(x,y)\}$

$D_6 = Res(D_1, D_4\{y/d\}) = \{yB(c,d)\}$

$D_7 = Res(D_2, D_5\{y/d\}) = \{\neg \pi(x), \neg yB(x,d)\}$

$D_8 = Res(D_7\{x/c\}, D_3) = \{\neg yB(c,d)\}$

$= Res(D_6, D_8)$