$\forall, \exists, \in$

$f(x) < y$

$\int$

$\forall \ \exists$

$\forall x \ p(x)$

$(\forall x \in A) \ p(x)$

$(\exists x \in A) \ p(x)$

$\exists x \ p(x)$ формула

$\mathfrak{A} = \langle \mathbb{N}, \underline{s}^{\mathfrak{A}}, \underline{m}^{\mathfrak{A}} \rangle$

$\langle n, m, k \rangle \in s^{\mathfrak{A}} \iff n + m = k$; $\langle n, m, k \rangle \in m^{\mathfrak{A}} \iff n \cdot m = k$

$\cancel{s(x, a, a)}$

$\cancel{\varphi_0(x) \leftrightharpoons \forall a\, s(x, a, a)}$

$\varphi_0 \ \underline{\{0\}}$

$A \leftrightharpoons \{2, 5, 98\}$

$\varphi_A(x) \iff x \in A$

$\varphi_0(x) \leftrightharpoons \forall a\, s(x, a, a).$

$\cancel{\psi_0(x) \leftrightharpoons s(x, x, x).}$

$\varphi_1(x) \leftrightharpoons \forall a\, \underline{m}(x, a, a)$. // $x \cdot a = a \iff$

$x = 1$

$\psi_1(x) \leftrightharpoons \underline{m(x, x, x)} \,\&\, \neg \psi_0(x)$

$\underline{// x \cdot x = x}$

$B\ \mathbb{N} = \{0, 1, 2, \ldots\}$ $\{n\}$

$\varphi_{\{0,1\}}(x) \leftrightharpoons m(x, x, x)$. $\&, \vee, \neg, \Rightarrow, \Leftrightarrow$

Искаме $\varphi_n(x)$

$\varphi_n(x)$ е вярна $\iff$ $\underline{x = n}$ ✓

$\cancel{m(x, \varphi_1(y), z)}$

$\underline{X = 7}$

С индукция по n.

База: $\lambda 0 y \to \varphi_0$, $\lambda 1 y \to \varphi_1$

ИП: Допускаме, че за някое $n \in \mathbb{N}_{\geq 0}$, $\underline{\varphi_n(x)}$ опр. $\lambda n y$

Стъпка: $\varphi_{n+1}(x) \leftrightharpoons \exists y(\varphi_n(y) \& \varphi_{+1}(y,x))$.

$\varphi_2(x) \leftrightharpoons \exists y(\varphi_1(y) \& S(y,y,x))$.

$\varphi_{+1}(x,y) \leftrightharpoons \exists z(\varphi_1(z) \& S(x,z,y))$.

$\varphi_{+1}(x,y) \Leftrightarrow y = x+1$

$A \Rightarrow B$

$\neg A \vee B$

$\varphi_{+1}(x,y) \leftrightharpoons \forall z(\varphi_1(z) \to S(x,z,y))$

$\exists \quad \wedge$
$\forall \quad \rightarrow$

$\mathbb{N}$ $\quad \varphi(x,y)$ $\quad \{\langle x,y\rangle \mid x,y\in A \ \ldots\}$ $\quad \overset{def}{\Longleftrightarrow}$ $\quad :=$

$\mathfrak{B} = \langle \mathbb{N}; S^{\mathfrak{B}}\rangle$

$n,m,k \in \mathbb{N} : \langle n,m,k\rangle \in S^{\mathfrak{B}} \rightleftharpoons n+m=k$

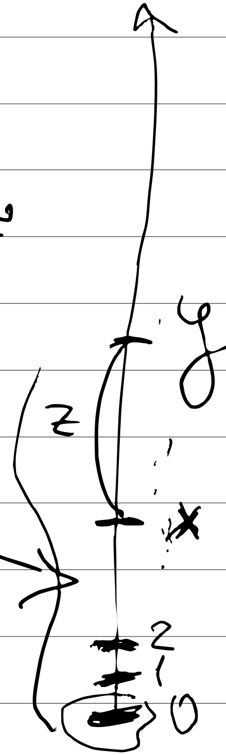
$\varphi_0(x) \rightleftharpoons S(x,x,x).$

$\varphi_1(x) \rightleftharpoons ?$

$\varphi_1(x) \rightleftharpoons \neg\varphi_0(x) \,\&\, \ldots$

$\varphi_{\le}(x,y) \rightleftharpoons \exists z\, S(x,z,y).$

$x \le y \Longleftrightarrow x+z=y$ и $z\in\mathbb{N}$



$\psi_0(x) \rightleftharpoons \forall z\, \varphi_{\le}(x,z).$

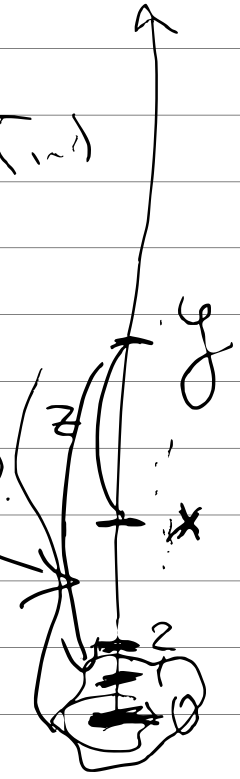
$\varphi_1(x) \rightleftharpoons \neg\varphi_0(x) \,\&\, \forall z(\varphi_<(z,x) \Rightarrow \varphi_0(z)).$

$$\varphi_<(x,y) \leftrightharpoons \exists z\,(\neg\varphi_0(z)\ \&\ S(x,z,y)).$$

$$\cancel{\varphi(x,y) \leftrightharpoons \ldots\ \varphi_y(\ldots)\ \ \varphi_x(\ldots)}$$

$$f_<(x,y) \leftrightharpoons \neg\varphi_\le(y,x).$$

$$\varphi_2(x) \leftrightharpoons \neg\varphi_0(x)\ \&\ \neg\varphi_1(x)\ \&$$
$$\forall y\,(\varphi_<(y,x) \Rightarrow \varphi_0(y) \vee \varphi_1(y)).$$



$$\varphi_2(y) \leftrightharpoons \exists x\,(S(x,x,y)\ \&\ \varphi_1(x)).$$

$$0, 1, \ldots, n \ \mapsto\ n+1$$

$$\varphi_{n+1}(x) \leftrightharpoons \neg\varphi_0(x)\ \&\ \ldots\ \&\ \neg\varphi_n(x)\ \&$$
$$\forall y\,(\varphi_<(y,x) \Rightarrow \varphi_0(y) \vee \ldots \vee \varphi_n(y)).$$

$\varphi_1(x) \leftrightharpoons \forall y \forall z (S(y, z, x) \Rightarrow \varphi_0(y) \vee \varphi_0(z)) \,\&\, \neg \varphi_0(x).$

$B = \{2, 3, 7\}$.

$\boxed{\varphi_B(x) \text{ е вярна } \Leftrightarrow x \in B}$?

$\varphi_B(x) \leftrightharpoons \varphi_2(x) \vee \varphi_3(x) \vee \varphi_7(x).$

$\underline{\mathbb{N} \setminus B}$

$\varphi_{\mathbb{N}\setminus B}(x) \leftrightharpoons \neg \varphi_B(x).$

мн-вото на реалните числа

$|\mathbb{N}| \underset{\neq}{\leq} |\mathcal{P}(\mathbb{N})| = 2^{|\mathbb{N}|} = |\mathbb{R}|$

Има неопр. подмн-ва на $\mathbb{N}$

формулите са изброимо много

$Prime \leftrightharpoons \{ n \mid n \in \mathbb{N} \ \& \ \underbrace{\text{"n e проста"}} \}$

$$\varphi_{Prime}(x) \leftrightharpoons \forall y \forall z \, (\underbrace{m(y, z, x)}_{x = y \cdot z} \Rightarrow \varphi_1(y) \vee \varphi_1(z)) \ \& \ \neg \varphi_1(x).$$

$$\varphi_{n|m}(x, y) \leftrightharpoons \exists z \, m(x, z, y)$$

$x \mid y$

$$\varphi_{Prime}(x) \leftrightharpoons \forall y \forall z \, (m(y, z, x) \Rightarrow \varphi_{=}(y, x) \vee \varphi_{=}(z, x)) \wedge \underbrace{\neg m(x, x, x)}_{\neg \varphi_{0,1}(x)}.$$

$\&$, $\wedge$

$\varphi_{0,1}(x)$

$\neg \varphi_{0,1}(x)$

$\varphi_=(x,y) \leftrightharpoons \forall z \forall t\,(s(x,z,t) \Leftrightarrow s(y,z,t)).$

$x+z=t$

$y+z=t$

A, B

$\forall x\,(x \in A \Leftrightarrow x \in B) \Rightarrow A = B$

$\psi_=(x,y) \leftrightharpoons \forall z\,(\varphi_\leq(z,x) \Leftrightarrow \varphi_\leq(z,y)).$

$\chi_=(x,y) \leftrightharpoons \forall z \forall t\,(m(z,t,x) \Leftrightarrow m(z,t,y)).$

$\varphi'_=(x,y) \leftrightharpoons \exists z\,(\varphi_0(z)\ \&\ s(x,z,y)).$

$\varphi''_=(x,y) \leftrightharpoons \exists z\,(\varphi_1(z)\ \&\ m(x,z,y)).$

$\mathrm{Even} \leftrightharpoons \{2n \mid n \in \mathbb{N}\}$

$\varphi_{\mathrm{Even}}(x) \leftrightharpoons \exists y \exists z\,(\varphi_2(y)\ \&\ m(y,z,x)).$

$\psi_{\mathrm{Even}}(x) \leftrightharpoons \exists y\, s(y,y,x).$

$\varphi_{odd}(x) \leftrightharpoons \neg \varphi_{even}(x)$.

$t = 2.y$

$\varphi_{odd}(x) \leftrightharpoons \exists y \exists z \exists t \, (\varphi_1(z) \,\&\, \overbrace{S(y, y, t)}^{t=2.y} \,\&\, \underbrace{S(t, z, x)}_{x = t+1 = 2.y+1})$.

$\underline{2n+1}$

$x = t+1 = 2.y+1$.