$c_1 \doteq \forall x \forall y (\neg p(x,y) \Rightarrow \forall z (p(x,z) \Rightarrow \neg p(z,y)))$

$c_2 \doteq \forall x \exists y \forall z ((p(x,z) \& p(y,z)) \Rightarrow (p(z,x) \& \neg p(z,x)))$

$c_3 \doteq \forall x \forall y (\forall x \exists y (p(x,y) \vee \neg p(x,y)) \Leftarrow \forall z \exists t (p(y,t) \& p(x,t) \& p(z,t)))$

$\{c_1, c_2\} \models c_3$? $\longleftrightarrow$ $\{c_1, c_2, \neg c_3\}$ непрот. ?

$c \Rightarrow \psi \vdash \neg(\varphi \vee \psi) \dashv \psi$

c_2'

$c_2' \doteq \forall x \exists y \forall z \neg (p(x,z) \& p(y,z))$

$c_3' \doteq \forall x \forall y \forall z \exists t (p(y,t) \& p(x,t) \& p(z,t))$

$\{c_1, c_2', c_3'\}$ непрот. ?

$\psi \doteq \neg c_3' \vdash \exists x \exists y \exists z \forall t (\neg p(y,t) \vee \neg p(x,t) \vee \neg p(z,t))$
$\quad\quad a \quad b \quad c$

$c_1' \doteq$

$c_1 \vdash (\forall x \forall y (p(x,y) \vee \forall z (\neg p(x,z) \vee \neg p(z,y))))$

$\forall x \forall y \forall z (p(x,z) \& p(z,y) \Rightarrow p(x,y))$

$c_2'' \doteq \forall x \forall y \exists z (p(x,z) \& p(y,z))$.

$c_1'' \doteq \forall x \forall y \forall z (p(x,y) \vee \neg p(x,z) \vee \neg p(z,y))$.

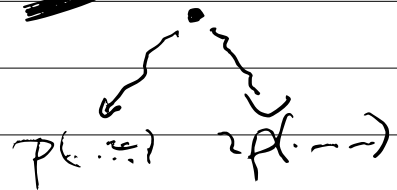$\psi^S \doteq \forall t (\neg p(a,t) \vee \neg p(b,t) \vee \neg p(c,t))$.

$c_2^S \doteq \forall x \forall y (p(x, f(x,y)) \& p(y, f(x,y)))$.

$$C_1'' \leq \forall x \forall y \forall z \, (p(x,y) \lor \lnot p(x,z) \lor \lnot p(z,y))$$

$$\psi^5 \leq \forall t \, (\lnot p(b,t) \lor \lnot p(a,t) \lor \lnot p(c,t))$$

$$C_2^3 \leq \forall x \forall y \, (p(x, f(x,y)) \,\&\, p(y, f(x,y))).$$

$D_1 = \{\, p(x_1, y_1),\ \lnot p(x_1, z_1),\ \lnot p(z_1, y_1) \,\}$

$D_2 = \{\, \lnot p(b,t),\ \lnot p(a,t),\ \lnot p(c,t) \,\}$

$D_3 = \{\, p(x_3, f(x_3, y_3)) \,\}$

$D_4 = \{\, p(y_4, f(x_4, y_4)) \,\}$

$p(\cdots)$ $\lnot p(\cdots)$

① Posn. D_1 D_3:

$$D_5 = Res\left(D_1 \left\{ x_1/x_3,\ z_1/f(x_3,y_3) \right\},\ D_3 \right) =$$
$$= \{\, p(x_3, y_1),\ \lnot p(f(x_3, y_3), y_1) \,\}$$

② Posn. D_5 D_2:

$$D_6 = Res\left(D_2,\ D_5 \left\{ x_3/b,\ y_1/t \right\} \right) = \{\, \lnot p(a,t),\ \lnot p(c,t),$$
$$\lnot p(f(b,y_3), t) \,\}$$

$\boxed{D_1} = \{ \boxed{p(x_1, \boxed{y_1})}, \neg p(x_1, \boxed{z_1}), \neg p(z_1, y_1) \}$

$D_2 = \{ \neg p(\boxed{b}, t), \neg p(a, t), \neg p(c, t) \}$

$\boxed{D_3} = \{ p(x_3, f(x_3, y_3)) \}$

$D_4 = \{ p(y_4, f(x_4, y_4)) \}$

① Разр. D_1, D_3

$D_7 = Res \left(D_1 \{ z_1/x_3, y_1/f(x_3, y_3) \}, D_3 \right) =$
$$\{ p(x_1, f(x_3, y_3)), \neg p(x_1, x_3) \}$$

② Разр. D_4, D_7

$\boxed{D_8} = Res \left(D_4, D_7 \{ x_1/y_4, x_3/f(x_4, y_4) \} \right) = $
$$= \{ p(y_4, f(f(x_4, y_4), y_3)) \}$$

③ Разр. D_4, D_2

$D_9 = Res \left(D_4 \{ y_4/b \}, D_2 \{ t/f(x_4, b) \} \right) =$
$$= \{ \neg p(a, f(x_4, b)), \neg p(c, f(x_4, b)) \}$$

④ Разр. D_1, D_9:

$D_{10} = Res \left(D_1 \{ x_1/a, y_1/f(x_4, b) \}, D_9 \right) =$

④ Резол. D_1, D_9:

$$D_{10} = Res\left(D_1\{x_1/a,\ y_1'(f(x_4, b)\},\ D_9\right) =$$
$$= \{\ \neg p(a, c_1),\quad \neg p(c, f(x_4, b)),$$
$$\neg p(c, f(x_4, b))\ \}$$

$D_3 = \{\ p(x_3,\ f(x, y_3))\ \} \quad\longrightarrow\quad p(x_3',\ f(x_3', y_3'))$

$D_4 = \{\ p(y_4,\ f(x, y_4))\ \}$

⑤ Резол: D_3 и D_{10}

$$D_{11} = Res\left(D_{10}\{z_1/x_3,\ x_4\wedge x_3\},\ D_3\{y_3/b\}\right) =$$
$$= \{\ \neg p(a, x_3),\ \neg p(c,\ f(x_3, b))\ \}$$
$$f(\cdot, c)$$

⑥ Резол: P_4 и D_{11}

$$D_{12} = Res\left(P_{11}\{x_3/f(a, y')\},\ D_3\{x_5'/a\}\right) =$$
$$= \{\ \neg p(c,\ f(f(a, y_3'),\ b))\ \}$$

$$D_8 = Res\left(D_4 \left\{ D_7 \left\{ x_1/y_4, \; x_3/f(x_4,y_4) \right\} \right)\right) =$$
$$= \left\{ p(y_4) \; f(f(x_4,y_4), y_3))) \right\},$$

(5) Рассм. P_{11} и D_{11}
$$D_{12} = Res\left(P_{11} \left\{ x_3/f(a, y_3') \right\} D_3 \left\{ x_5'/a \right\} \right) =$$
$$= \left\{ \neg p(c, \; f(f(a, y_3'), b)) \right\}$$

$$D_1 = Res\left(D_8 \left\{ y_4/c, \; x_4/a, \; y_3/b \right\}, D_{12} \left\{ y_3'/c \right\} \right)$$

$\varphi_1 \coloneqq \forall x \exists y \, (q(y,x) \,\&\, \forall z \, (q(y,z) \Rightarrow r(z,x)))$

$\varphi_2 \coloneqq \forall x \, (\exists y \, q(x,y) \Rightarrow \exists y \, (q(x,y) \,\&\, \neg \exists z \, (q(y,z) \,\&\, q(x,z))))$

$\varphi_3 \coloneqq \forall x \forall y \forall z \, (q(y,x) \,\&\, r(z,y) \Rightarrow q(z,x))$

$\varphi_4 \coloneqq \forall x \neg q(x,x)$

$$\varphi_1, \varphi_2, \varphi_3 \models \varphi_4 \, \text{?} \qquad c$$

$\psi \coloneqq \neg \varphi_4 \models \exists x \, q(x,x)$

$\varphi_1' \coloneqq \forall x \exists y \, (q(y,x) \,\&\, \forall z \, (\neg q(y,z) \vee r(z,x)))$

$\varphi_2' \coloneqq \forall x \, (\forall y \neg q(x,y) \vee \exists y \, (q(x,y) \,\&\, \forall z \, (\neg q(y,z) \vee \neg q(x,z))))$

$\varphi_3' \coloneqq \forall x \forall y \forall z \, (\neg q(y,x) \vee \neg r(z \ldots$

$\varphi_1'' \coloneqq \forall x \exists y \, (q(y,x) \,\&\, (\neg q(y,z) \vee r(z,x)))$

$\varphi_2^4 \coloneqq \forall x \exists y \forall z \, (\neg q(x,t) \vee (q(x,y) \,\&\, (\neg q(y,z) \vee \neg q(x,z))))$

$\varphi_1^5 \coloneqq \forall x \forall z \, (q(f(x),x) \,\&\, (\neg q(f(x),z) \vee r(z,x)))$

$\varphi_2^5 \coloneqq \forall x \forall z \, (\neg q(x,t) \vee (q(x,g(x)) \,\&\, (\neg q(g(x),z) \vee \neg q(x,z))))$

$\psi^5 \coloneqq q(c,c).$

$$\varphi_3' \equiv \forall x \forall y \forall z \, (\overline{\neg q(y,x) \vee \neg r(z,y) \vee q(z,x)})$$

$$\varphi_1^S \equiv \forall x \exists z \, (\overline{q(f(x),x)} \,\&\, (\neg q(f(x),z) \vee r(z,x)))$$

$$\varphi_2^S \equiv \forall x \forall t \forall z \, ((\neg q(x,t) \vee (q(x,g(x)) \,\&\, (\neg q(g(x),z) \vee \neg q(x,z)))$$

$$\psi^S \equiv q(c,c).$$

$$\varphi_2^{fun} \equiv \forall x \forall t \forall z \, ((\overline{\neg q(x,t) \vee q(x,g(x))}) \,\&\, (\neg q(x,t) \vee \neg q(g(x),z) \vee \neg q(x,z))).$$

$$D_1 = \{ q(f(x_1),x_1) \}$$

$$D_2 = \{ \neg q(f(x_2),z_2), \, r(z_2,x_2) \}$$

$$D_3 = \{ \neg q(x_3,t_3), \, q(x_3,g(x_3)) \}$$

$$D_4 = \{ \neg q(x_4,t_4), \, \neg q(g(x_4),z_4), \, \neg q(x_4,z_4) \}$$

$$D_5 = \{ \neg q(y_5,x_5), \, \neg r(z_5,y_5), \, q(z_5,x_5) \}$$

$$D_6 = \{ q(c,c) \}$$

$$D_1 = \{ \, q(f(x_1), x_1) \, \}$$

$$D_2 = \{ \, \neg q(f(x_2), z_2), \, r(z_2, x_2) \, \}$$

$$D_3 = \{ \, \neg q(x_3, t_3), \, r(x_3), \, r(x_3) \, \}$$

$$D_4 = \{ \, \neg q(x_4, t_4), \, q(x_4, z_4), \, \neg q(x_4, z_4) \, \}$$

$$D_5 = \{ \, \neg q(y_5, x_5), \, \neg r(t_5, y_5), \, \neg q(z_5, x_5) \, \}$$

$$D_6 = \{ \, q(c, c) \, \}$$

$$D_7 = Res\left(D_2 \{ z_2/z_5, \, x_2/y_5 \}, \, D_5 \right) = \{ \, \neg q(f(y_5), z_5), \, \neg q(y_5, x_5), \, q(f(y_5), x_5) \, \}$$

$$D_8 = Res\left(D_3 \{ x_3/f(y_5) \}, \, D_7 \{ z_5/q(f(y_5)) \} \right)$$
$$= \{ \, \neg q(y_5, x_5), \, q(q(f(y_5)), x_5), \, \neg q(f(y_5), t_3) \, \}$$

$$D_9 = Res\left(D_4 \{ x_4/f(y_5), \, z_4/x_5 \}, \, D_8 \right) =$$
$$= \{ \, \neg q(f(y_5), t_4), \, \neg q(x_4, x_5), \, \neg q(y_5, x_5), \, \neg q(f(y_5), t_5) \, \}$$

$$D_{10} = Collapse\left(D_9 \{ \cdot/c \} \right) = \{ \, \neg q(f(c), c), \, \neg q(c, c) \, \}$$

$$D_{11} = Res\left(D_1 \{ x_1/c \}, \, D_{10} \right) = \{ \, \neg q(c, c) \, \}$$

$$\blacksquare = Res\left(D_{11}, \, D_6 \right)$$