$\varphi_1 \leftrightharpoons \forall x \forall y (\neg p(x,y) \Rightarrow \forall z (p(x,z) \Rightarrow \neg p(z,y)))$

$\varphi_2 \leftrightharpoons \forall x \neg \exists y \forall z ((p(x,z) \,\&\, p(y,z)) \Rightarrow (p(z,x) \,\&\, \neg p(z,x)))$

$\varphi_3 \leftrightharpoons \forall x \forall y (\forall x \exists y (p(x,y) \vee \neg p(x,y)) \Rightarrow \forall z \exists t (p(y,t) \,\&\, p(x,t) \,\&\, p(z,t)))$

$\varphi_1, \varphi_2 \models \varphi_3$? $\Longleftrightarrow$ $\{\varphi_1, \varphi_2, \neg\varphi_3\}$ неизп. ?

$\varphi \Rightarrow \psi \models \neg\varphi \vee \psi \models \psi$

$\wedge$

a_2 II

$\varphi_2' \leftrightharpoons \forall x \exists y \forall z \neg(p(x,z) \,\&\, p(y,z))$

$\varphi_3' \leftrightharpoons \forall x \forall y \forall z \exists t (p(y,t) \,\&\, p(x,t) \,\&\, p(z,t))$

$\{\varphi_1, \varphi_2', \overset{\psi}{\neg\varphi_3'}\}$ неизп. ?

$\psi \leftrightharpoons \neg\varphi_3' \models \underbrace{\exists x}_{a} \underbrace{\exists y}_{b} \underbrace{\exists z}_{c} \forall t (\neg p(y,t) \vee \neg p(x,t) \vee \neg p(z,t))$

$\varphi_1' \leftrightharpoons$

$\varphi_1 \models \forall x \forall y (p(x,y) \vee \forall z (\neg p(x,z) \vee \neg p(z,y))) \models$

$\forall x \forall y \forall z (p(x,z) \,\&\, p(z,y) \Rightarrow p(x,y))$

$\varphi_2'' \leftrightharpoons \forall x \forall y \exists z (p(x,z) \,\&\, p(y,z))$.

$\varphi_1'' \leftrightharpoons \forall x \forall y \forall z (p(x,y) \vee \neg p(x,z) \vee \neg p(z,y))$.

$\psi^S \leftrightharpoons \forall t (\neg p(b,t) \vee \neg p(a,t) \vee \neg p(c,t))$.

$\varphi_2^S \leftrightharpoons \forall x \forall y (p(x, f(x,y)) \,\&\, p(y, f(x,y)))$.

$\varphi_1'' \leq \forall x \forall y \forall z \, (p(x,y) \vee \neg p(x,z) \vee \neg p(z,y))$

$\psi^S \leq \forall t \, (\neg p(b,t) \vee \neg p(a,t) \vee \neg p(c,t))$.

$\varphi_2^S \leq \forall x \forall y \, (p(x, f(x,y)) \,\&\, p(y, f(x,y)))$.

$D_1 = \{ p(x_1,y_1), \neg p(x_1,z_1), \neg p(z_1,y_1) \}$

$D_2 = \{ \neg p(b,t), \neg p(a,t), \neg p(c,t) \}$

$D_3 = \{ p(x_3, f(x_3,y_3)) \}$

$D_4 = \{ p(y_4, f(x_4,y_4)) \}$

$p(\dots)$ $\neg p(\dots)$

(1) Rozst. D_1, D_3:

$D_5 = Res(D_1 \{ x_1/x_3, z_1/f(x_3,y_3) \}, D_3) =$

$= \{ p(x_3,y_1), \neg p(f(x_3,y_3), y_1) \}$

(2) Rozst. $D_5, \cancel{D_2}$:

$D_6 = Res(D_2, D_5 \{ x_3/b, y_1/t \}) = \{ \neg p(a,t), \neg p(c,t),$
$\neg p(f(b,y_3), t) \}$

$D_1 = \{ p(x_1, y_1), \neg p(x_1, z_1), \neg p(z_1, y_1) \}$

$D_2 = \{ \neg p(b, t), \neg p(a, t), \neg p(c, t) \}$

$D_3 = \{ p(x_3, f(x_3, y_3)) \}$

$D_4 = \{ p(y_4, f(x_4, y_4)) \}$

① Разгл. D_1, D_3

$D_7 = Res(D_1\{z_1/x_3, y_1/f(x_3, y_3)\}, D_3) =$

$\{ p(x_1, f(x_3, y_3)), \neg p(x_1, x_3) \}$

② Разгл. D_4, D_7

$D_8 = Res(D_4, D_7\{x_1/y_4, x_3/f(x_4, y_4)\}) =$

$= \{ p(y_4, f(f(x_4, y_4), y_3))) \}$

③ Разгл. D_4, D_2

$D_9 = Res(D_4\{y_4/b\}, D_2\{t/f(x_4, b)\}) =$

$= \{ \neg p(a, f(x_4, b)), \neg p(c, f(x_4, b)) \}$

④ Разгл. D_1, D_9:

$D_{10} = Res(D_1\{x_1/a, y_1/f(x_4, b)\}, D_9) =$

④ Разгл. D_1, D_8:

$$D_{10} = Res\left(D_1\{x_1/a,\ y_1/(f(x_4,b))\},\ D_8\right) =$$

$$= \{\neg p(a, z_1),\ \neg p(z_1, f(x_4, b)),\ \neg p(c, f(x_4, b))\}$$

$$D_3 = \{p(x_3, f(x_3, y_3))\} \to p(x_3', f(x_3', y_3'))$$

$$D_4 = \{p(y_4, f(x_4, y_4))\}$$

⑤ Разгл.: D_3 и D_{10}

$$D_{11} = Res\left(D_{10}\{z_1/x_3,\ x_4/x_3\},\ D_3\{y_3/b\}\right) =$$

$$= \{\neg p(a, x_3),\ \neg p(c, f(x_3, b))\}$$

$f(\cdot, c)$

⑥ Разгл.: D_4 и D_{11}

$$D_{12} = Res\left(D_{11}\{x_3/f(a, y_3')\},\ D_3\{x_3'/a\}\right) =$$

$$= \{\neg p(c, f(f(a, y_3'), b))\}$$

$D_8 = Res(D_4\{\}, D_7\{x_1/y_4, x_3/f(x_4, y_4)\}) =$

$= \{p(y_4, f(f(x_4, y_4), y_3))\}$

(6) Резол: D_4 и D_{11}

$D_{12} = Res(D_{11}\{x_3/f(a, y_3')\}, D_3\{x_5'/a\}) =$

$= \{\neg p(c, f(f(a, y_3'), b))\}$

[illegible] $= Res(D_8\{y_4/c, x_4/a, y_3/b\}, D_{12}\{y_3'/c\})$

$\varphi_1 \leqslant \forall x \exists y \, (q(y,x) \,\&\, \forall z \, (q(y,z) \Rightarrow r(z,x)))$

$\varphi_2 \leqslant \forall x \, (\exists y \, q(x,y) \Rightarrow \exists y \, (q(x,y) \,\&\, \neg \exists z \, (q(y,z) \,\&\, q(x,z))))$

$\varphi_3 \leqslant \forall x \forall y \forall z \, (q(y,x) \,\&\, r(z,y) \Rightarrow q(z,x))$

$\varphi_4 \leqslant \forall x \, \neg q(x,x)$

$\varphi_1, \varphi_2, \varphi_3 \models \varphi_4 \, ? \quad c$

$\psi \leqslant \neg \varphi_4 \equiv \exists x \, q(x,x)$

$\varphi_1' \leqslant \forall x \exists y \, (q(y,x) \,\&\, \forall z \, (\neg q(y,z) \lor r(z,x)))$

$\varphi_2' \leqslant \forall x \, (\forall y \neg q(x,y) \lor \exists y \, (q(x,y) \,\&\, \forall z \, (\neg q(y,z) \lor \neg q(x,z))))$

$\varphi_3' \leqslant \forall x \forall y \forall z \, (\neg q(y,x) \lor \neg r(z$

$\varphi_1'' \leqslant \forall x \exists y \forall z \, (q(y,x) \,\&\, (\neg q(y,z) \lor r(z,x)))$

$\varphi_2'' \leqslant \forall x \exists y \forall t \forall z \, (\neg q(x,t) \lor (q(x,y) \,\&\, (\neg q(y,z) \lor \neg q(x,z))))$

$\varphi_1^S \leqslant \forall x \forall z \, (q(f(x),x) \,\&\, (\neg q(f(x),z) \lor r(z,x)))$

$\varphi_2^S \leqslant \forall x \forall t \forall z \, (\neg q(x,t) \lor (q(x,g(x)) \,\&\, (\neg q(g(x),z) \lor \neg q(x,z))))$

$\psi^S \leqslant q(c,c).$

$$\varphi_3' \rightleftharpoons \forall x \forall y \forall z\, (\neg q(y,x) \vee \neg r(z,y) \vee q(z,x))$$

$$\varphi_1^S \rightleftharpoons \forall x \forall z\, (q(f(x),x) \,\&\, (\neg q(f(x),z) \vee r(z,x)))$$

$$\varphi_2^S \rightleftharpoons \forall x \forall t \forall z\, (\underbrace{\neg q(x,t)}_{\psi} \vee (\underbrace{q(x,g(x))}_{\varphi} \,\&\, \underbrace{(\neg q(g(x),z) \vee \neg q(x,z))}_{\chi}))$$

$$\psi^S \rightleftharpoons q(c,c).$$

$$\varphi_2^{fin} \rightleftharpoons \forall x \forall t \forall z\, ((\neg q(x,t) \vee q(x,g(x))) \,\&$$
$$(\neg q(x,t) \vee \neg q(g(x),z) \vee \neg q(x,z))).$$

$D_1 = \{q(f(x_1),x_1)\}$

$D_2 = \{\neg q(f(x_2),z_2),\ r(z_2,x_2)\}$

$D_3 = \{\neg q(x_3,t_3),\ q(x_3,g(x_3))\}$

$D_4 = \{\neg q(x_4,t_4),\ \neg q(g(x_4),z_4),\ \neg q(x_4,z_4)\}$

$D_5 = \{\neg q(y_5,x_5),\ \neg r(z_5,y_5),\ q(z_5,x_5)\}$

$D_6 = \{q(c,c)\}$

$D_1 = \{ q(f(x_1), x_1) \}$

$D_2 = \{ \neg q(f(x_2), z_2), r(z_2, x_2) \}$

$D_3 = \{ \neg q(x_3, t_3), q(g(x_3), x_3) \}$

$D_4 = \{ \neg q(x_4, t_4), \neg q(g(x_4), z_4), \neg q(x_4, z_4) \}$

$D_5 = \{ \neg q(y_5, x_5), \neg r(z_5, y_5), \neg q(z_5, x_5) \}$

$D_6 = \{ q(c, c) \}$

$D_7 = Res(D_2\{z_2/z_5, x_2/y_5\}, D_5) = \{ \neg q(f(y_5), z_5), \neg q(y_5, x_5), q(z_5, x_5) \}$

$D_8 = Res(D_3\{x_3/f(y_5)\}, D_7\{z_5/g(f(y_5))\}) =$

$\{ \neg q(y_5, x_5), q(g(f(y_5)), x_5), \neg q(f(y_5), t_3) \}$

$D_9 = Res(D_4\{x_4/f(y_5), z_4/x_5\}, D_8) =$

$= \{ \neg q(f(y_5), t_4), \neg q(x_4, x_5), \neg q(y_5, x_5),$

$\neg q(f(y_5), t_3) \}$

$D_{10} = Collapse(D_9\{\bullet/c\}) = \{ \neg q(f(c), c), \neg q(c, c) \}$

$D_{11} = Res(D_1\{x_1/c\}, D_{10}) = \{ \neg q(c, c) \}$

$= Res(D_{11}, D_6)^*$