петък: 8:00 упражнение

$\Sigma \models \varphi$, Σ' е неизпълнимо

$\Sigma \cup \{\neg\varphi\}$ е

неизпълнимо

ПНФ, СНФ, КНФ, Дизюнкти $\xrightarrow{\text{Резолюция}}$ $\blacksquare$

$$\exists x \forall y (q(x,y) \Leftrightarrow \forall z (q(y,z) \Rightarrow (q(z,y) \Rightarrow$$

$$\forall x (q(x,y) \,\&\, \neg q(x,y)))))) \models$$

неизпълнимо?

$$\varphi \leftrightarrows \exists x \forall y (q(x,y) \Leftrightarrow \forall z (q(y,z) \overset{F}{\Rightarrow} \neg q(z,y)))$$

① Вкарваме отрицанието до атомарни ф-ли и разписваме „⇔" и „⇒".

Ползваме: $\neg\forall x\varphi \models\!\mid \exists x\neg\varphi$

$\neg\exists x\varphi \models\!\mid \forall x\neg\varphi$

$\neg(\varphi\vee\psi) \models\!\mid \neg\varphi \,\&\, \neg\psi$

$\neg(\varphi\,\&\,\psi) \models\!\mid \neg\varphi\vee\neg\psi$

$\neg\neg\varphi \models\!\mid \varphi$

$\varphi\Rightarrow\psi \models\!\mid \neg\varphi\vee\psi$

$\varphi\Leftrightarrow\psi \models\!\mid (\varphi\Rightarrow\psi)\,\&\,(\psi\Rightarrow\varphi) \models\!\mid (\neg\varphi\vee\psi)\,\&\,(\neg\psi\vee\varphi)$.

$\varphi \models\!\mid \exists x\forall y(q(x,y)\Leftrightarrow\forall z(\neg q(y,z)\vee\neg q(z,y)))$

$\models\!\mid \exists x\forall y((\neg q(x,y)\vee\forall z(\neg q(y,z)\vee\neg q(z,y)))\,\&$

$(\exists z(q(y,z)\,\&\,q(z,y))\vee q(x,y)))$

$\varphi' \leftrightarrows$

$t \qquad t \qquad t$

② ПНФ

$\varphi \equiv \overbrace{Q_1 x_1 \dots Q_n x_n}^{\text{кванторен префикс}} \theta$ → матрица, където:

- $Q_i \in \{\exists, \forall\}$, $1 \le i \le n$
- θ е безкв. ф-ла
- $i \ne j \to x_i \ne x_j$ [redacted]

$x \notin Var^{free}[\psi]$

$\underline{\exists x} \varphi \vee \psi \models\!\mid \exists x (\varphi \vee \psi)$

$\exists x \varphi \,\&\, \psi \models\!\mid \exists x (\varphi \,\&\, \psi)$

$\forall x \varphi \vee \psi \models\!\mid \forall x (\varphi \vee \psi)$

$\forall x \varphi \,\&\, \psi \models\!\mid \forall x (\varphi \,\&\, \psi)$

Тн за вариантно.

$Var^{free}[\varphi] \cap \underline{Var^{bd}[\varphi]} \ne \emptyset$

$\forall y (\underset{\exists}{\forall x} p(x,y) \underset{\&}{\vee} \exists z\, p(z,z)) \models\!\mid$ [redacted]

$\underline{\forall y \exists z \forall x} (p(x,y) \vee p(z,z))$

$\varphi'' \leftrightharpoons \exists x \forall y \exists t \forall z ((\neg q(x,y) \vee \neg q(z,y) \vee \neg q(y,z)) \&$

$(q(x,y) \vee (q(t,y) \& q(y,t))))$

$c \quad f$

③ <u>СНФ</u> $\quad \psi$ инд

едносъпкова

$\varphi_{\underbrace{s s \ldots s}}$ $\longrightarrow$ отляво надясно по

- $\psi \equiv \exists x \varphi \rightsquigarrow c_\varphi \rightsquigarrow \mathcal{L}'$ с нова ИК. c_φ

  $\psi_s \leftrightharpoons \varphi[x/c_\varphi]$

- $\psi \equiv \forall y_1 \ldots \forall y_n \exists x \varphi \rightsquigarrow f_\varphi \rightsquigarrow \mathcal{L}'$ с нова ф.с. f_φ

  $n > 0$

  $\psi_S \leftrightharpoons \forall y_1 \ldots \forall y_n \varphi[x/f_\varphi(y_1, \ldots, y_n)]$

$\varphi'' \not\models \varphi^S$

$\psi^S \equiv \psi_{\underbrace{ss\ldots s}}$

колкото кв. за $\exists$ се срещат

$\varphi^S \leftrightharpoons \forall y \forall z ((\neg q(c,y) \vee \neg q(z,y) \vee \neg q(y,z)) \&$

$(q(c,y) \vee (q(f(y),y) \& q(y,f(y)))))$

4. $\varphi \vee (\psi \& \chi) \vDash (\varphi \vee \psi) \& (\varphi \vee \chi)$

$\varphi^{fin} \leftrightharpoons \forall y \forall z ((\neg q(c, y) \vee \neg q(z, y) \vee \neg q(y, z)) \&$
$(q(c, y) \vee q(f(y), y)) \&$
$(q(c, y) \vee q(y, f(y))))$.

КНФ, СНФ и ПНФ.

$D_1 = \{\neg q(c, y_1), \neg q(z_1, y_1), \neg q(y_1, z_1)\}$
$D_2 = \{q(c, y_2), q(f(y_2), y_2)\}$
$D_3 = \{q(c, y_3), q(y_3, f(y_3))\}$

Правила на играта

$\{x_1/x_2\}$

① Правило за резолюцията

Нека D_1, D_2 са дизюнкти без общи променливи.
Нека $D_1 = D_1' \cup \{L\}$, $D_2 = D_2' \cup \{\neg L'\}$. Търсим субституция σ т.ч. $L\sigma = L'\sigma$. Тогава
$\mathrm{Res}(D_1, D_2) = D_1'\sigma \cup D_2'\sigma = (D_1\sigma \setminus \{L\sigma\}) \cup (D_2\sigma \setminus \{\neg L'\sigma\})$.

② Колапс на дизюнкт. $n>1$

Нека $D = D' \cup \{L_1, \ldots, L_n\}$. Търсим субст. σ т.ч. $L_1\sigma = L_2\sigma = \ldots = L_n\sigma$.
Тогава $\mathrm{Collapse}(D) = D'\sigma \cup \{L_1\sigma\} = D\sigma$

$D_1 = \{\neg q(c, y_1), \neg q(z_1, y_1), \neg q(y_1, z_1)\}$

$D_2 = \{q(c, y_2), q(f(y_2), y_2)\}$

$D_3 = \{q(c, y_3), q(y_3, f(y_3))\}$

$\left| \begin{array}{l} c = z_1 \\ z_1 = y_1 \end{array} \right. \quad | \, y_1 = c$

1) Розгл. D_1.

$\text{Collapse}(D_1\{y_1/c, z_1/c\}) = \{\neg q(c, c)\} = D_4$

2) Розгл. D_4 и D_2 $\quad \sigma = \sigma_1 \cup \sigma_2$

$\text{Res}(D_4, D_2\{y_2/c\}) = \{q(f(c), c)\} = D_5$

3) Розгл. D_3 и D_4

$\text{Res}(D_4, D_3\{y_3/c\}) = \{q(c, f(c))\} = D_6$

4) Розгл. D_6 и D_1

$\text{Res}(D_6, D_1\{y_1/f(c), z_1/c\}) = \{\neg q(f(c), c)\} = D_7$

5) $\text{Res}(D_7, D_5) = \blacksquare$

~~$\{p(x), \neg p(x)\}$~~ $\Rightarrow$ не може

more $\underbrace{D_1}$ $\subseteq D_2$

important!!!!!

$D_1 = \{p(x)\}$, $D_2 = \{p(x), q(x)\}$

зад. $\varphi_1 \& \varphi_2 \Rightarrow \varphi_3$? $\{\varphi_1, \varphi_2\} \models \varphi_3 \Leftrightarrow \{\varphi_1, \varphi_2, \neg\varphi_3\}$ да е неизп.

$\varphi_1 \leftrightharpoons \forall x [\forall y (S(y) \Rightarrow p(y,x)) \Rightarrow \neg r(x)]$

$\varphi_2 \leftrightharpoons \exists y \forall x [\neg S(x) \vee \neg q(x,y)]$

$\varphi_3 \leftrightharpoons \exists x \forall y [r(y) \Rightarrow \exists z (\neg p(z,y) \& \neg q(z,x))]$

$\varphi_1 \& \varphi_2 \& \neg\varphi_3$ да е неизп.

$\neg\psi \leftrightharpoons \forall x \exists y [r(y) \& \forall z (p(z,y) \vee q(z,x))]$

$\varphi_1 \dashv\vdash \forall x [\forall y (\neg S(y) \vee p(y,x)) \Rightarrow \neg r(x)] \models 1$

$\varphi_1' \leftrightharpoons \forall x [\exists y (S(y) \& \neg p(y,x)) \vee \neg r(x)]$

$\varphi_1'' \leftrightharpoons \forall x \exists y [(S(y) \& \neg p(y,x)) \vee \neg r(x)]$

$\psi'' \leftrightharpoons \forall x \exists y \forall z [r(y) \& (p(z,y) \vee q(z,x))]$

$$\varphi_1^S \leftrightharpoons \forall x\, [(s(f(x)) \,\&\, \neg p(f(x), x)) \vee \neg r(x)]$$

$$\varphi_2^S \leftrightharpoons \forall x\, [\neg s(x) \vee \neg q(x, c)] \vee$$

$$\psi^S \leftrightharpoons \forall x \forall z\, [r(g(x)) \,\&\, (p(z, g(x)) \vee q(z, x))].$$

$$\varphi_1^{fin} \leftrightharpoons \forall x [(s(f(x)) \vee \neg r(x)) \,\&\, (\neg p(f(x), x) \vee \neg r(x))].$$

$D_1 = \{ s(f(x_1)), \neg r(x_1) \}$

$D_2 = \{ \neg p(f(x_2), x_2), \neg r(x_2) \}$

$D_3 = \{ \neg s(x_3), \neg q(x_3, c) \}$

$D_4 = \{ r(g(x_4)) \}$

$D_5 = \{ p(z_5, g(x_5)), q(z_5, x_5) \}$

(5) $Res(D_9\{x_5/x_4\}, D_7) =$

$= \square$

(1) $Res(D_1\{x_1/g(x_4)\}, D_4) = \{ s(f(g(x_4))) \} = D_6$

(2) $Res(D_2\{x_2/g(x_4)\}, D_4) = \{ \neg p(f(g(x_4)), g(x_4)) \} = D_7$

(3) $Res(D_3\{x_3/f(g(x_4))\}, D_6) = \{ \neg q(f(g(x_4)), c) \} = D_8$

(4) $Res(D_5\{z_5/f(g(x_4))\}, D_8) = \{ p(f(g(x_4)), g(x_5)) \} = D_9$

D_1, D_2 са дизюнкти.

$L \in D_1$, $\bar{L} \in D_2$

$I \models D_1, D_2$

$D_1' = D_1 \setminus \{L\}$, $D_2' = D_2 \setminus \{\bar{L}\}$

$I \models D_1' \cup D_2'$ резолвента на D_1 и D_2 по L ($\bar{L}$)

$\{p, q\}$

$\{\neg p, r, s\}$

$\{q, r, s\}$

$D_1\sigma_1$, $D_2\sigma_2$

резолвенти

1. Някои пациенти уважават докторите.
2. Никой пациент не уважава шарлатаните.
3. Докторите не са шарлатани.

$\text{п}(x), \text{д}(x), \text{ш}(x), \text{ув}(x,y)$

$\varphi_1 \leftrightharpoons \exists x(\text{П}(x) \wedge \forall y(\text{д}(y) \to \text{ув}(x,y)))$

$\varphi_2 \leftrightharpoons \forall x(\text{П}(x) \to \forall y(\text{ш}(y) \to \neg \text{ув}(x,y)))$

$\varphi_3 \leftrightharpoons \neg \forall y(\text{д}(y) \to \neg \text{ш}(y)) \models\mid \exists y(\text{д}(y) \wedge \text{ш}(y))$

$\{\varphi_1, \varphi_2, \varphi_3\}$ е неизпълнимо.

$\varphi_1 \models\mid \exists x \forall y(\text{П}(x) \wedge (\neg \text{д}(y) \vee \text{ув}(x,y)))$

$\varphi_2 \models\mid \forall x \forall y(\neg \text{П}(x) \vee \neg \text{ш}(y) \vee \neg \text{ув}(x,y))$

$\text{СНФ}(\varphi_1) \leftrightharpoons \forall y(\text{П}(c) \wedge (\neg \text{д}(y) \vee \text{ув}(c,y)))$

$\text{СНФ}(\varphi_3) \leftrightharpoons \text{д}(d) \wedge \text{ш}(d)$

$\{\text{д}(d)\}, \{\text{ш}(d)\}, \{\text{П}(c)\}, \{\neg \text{д}(y), \text{ув}(c,y)\}$

$\{\neg \text{П}(x), \neg \text{ш}(y), \neg \text{ув}(x,y)\}$

$\underset{D_1}{\{g(d)\}}, \underset{D_2}{\{w(d)\}}, \underset{D_3}{\{\pi(c)\}}, \underset{D_4}{\{\neg g(y), \neg b(c,y)\}}$

$D_5 = \{\neg\pi(x), \neg w(y), \neg yb(x,y)\}$

$D_6 = Res(D_1, D_4\{y/d\}) = \{yB(c,d)\}$

$D_7 = Res(D_2, D_5\{y/d\}) = \{\neg\pi(x), \neg yB(x,d)\}$

$D_8 = Res(D_7\{x/c\}, D_3) = \{\neg yB(c,d)\}$

$= Res(D_6, D_8)$