38з) Нека $A = \langle$ крайни непрекъснати интервали от реални числа, $p^A \rangle$ е стр. с двуместен предикатен символ с интерпретация:

$\langle i, j \rangle \in p^A \Longleftrightarrow$ десния край на интервала i (= $R(i)$) съвпада с левия край на интервала j (= $L(j)$)

Определете:

- $\{\langle i, j \rangle \mid L(i) = L(j)\}$
- $\{\langle i, j \rangle \mid R(i) = R(j)\}$
- $\{\langle i, j \rangle \mid i = j\}$
- $\{\langle i, j \rangle \mid L(i) < L(j)\}$

- $\{\langle i, j \rangle \mid R(i) < R(j)\}$
- Два интервала да имат непразно сечение
- интервал i да е подинтервал на интервал j

39) Нека $A = \langle \mathbb{Q}, p^A \rangle$, където интерпретацията на триместния предикатен символ е:

$$\langle n, m, k \rangle \in p^A \iff n^5 \cdot m = k$$

Определете: $\{0\}$, $\{1\}$, $\{-1\}$, $\{\langle x, x\rangle \mid x \in \mathbb{Q}\}$, $\{\langle x, y\rangle \mid x \cdot y = 1\}$, $\{\langle x, y, z\rangle \mid x \cdot y = z\}$.

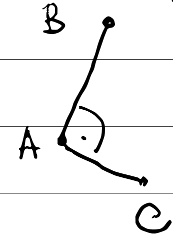
Докажете, че $\{5\}$ не е определимо.

380. Нека $\mathcal{L} = \langle p \rangle$, за $p \in Pred_{\mathcal{L}}$, $\#(p) = 3$.

Нека $\mathcal{A} = \langle \mathbb{R}^2, p^{\mathcal{A}} \rangle$, където:

$$p^{\mathcal{A}}(A, B, C) \leftrightarrows A \neq B \text{ и } A \neq C \text{ и } \angle BAC = 90^\circ$$

$A, B, C \in \mathbb{R}^2$



// имплицитно и $B \neq C$.

Да се док., че в $\mathcal{A}$ са определими:

(i) $Eq = \{\langle A, A \rangle \mid A \in \mathbb{R}^2\}$

(ii) $Col = \{\langle A, B, C \rangle \mid A, B, C \in \mathbb{R}^2$ лежат на една права$\}$

(iii) $Circ = \{\langle A, B, C \rangle \mid C$ лежи на окръж. с диаметър $AB\}$

Вярно ли е, че в $\mathcal{A}$ са определими и двата и защо?:

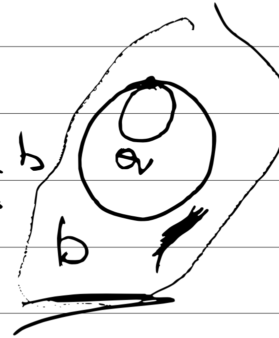
$Mid = \{\langle A, B, C \rangle \mid C$ е среда на отсечката $AB\}$

$Seg = \{\langle A, B, C \rangle \mid C$ лежи на отсечката $AB\}$

Намерете поне два различни автоморфизма в $\mathcal{A}$.

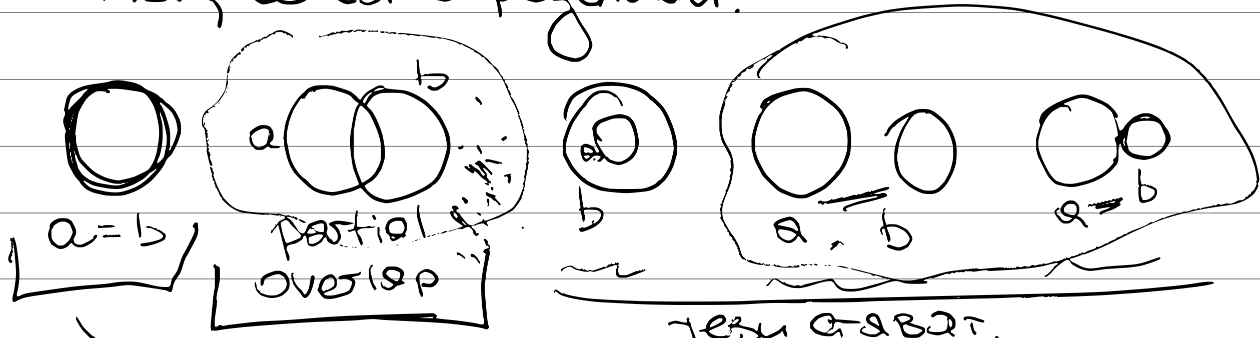
Зад. $A = \langle$ кръговете с ненулев радиус ; $p^A \rangle$, където

$\langle a,b \rangle \in p^A \Leftrightarrow$ a е вътр. доп. за b



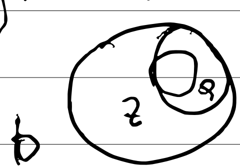
Кои са останалите отношения?

Док, че са определими.



не стават за деф. на p^A

$$\varphi_=(x,y) \leftrightharpoons \forall z(p(x,z) \Leftrightarrow p(y,z)).$$

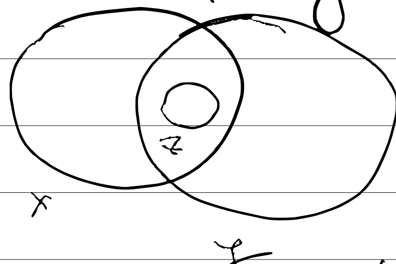


$$\varphi_C(x,y) \leftrightharpoons \exists z(p(z,y) \,\&\, p(x,z)) \,\&\, \neg p(x,y).$$

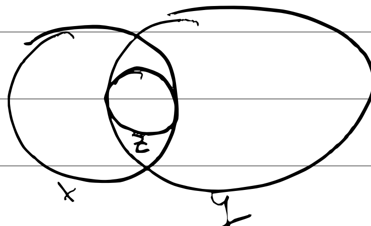
$\forall + \Rightarrow \neg BH\Lambda$

$\exists + \& = BH\Lambda$

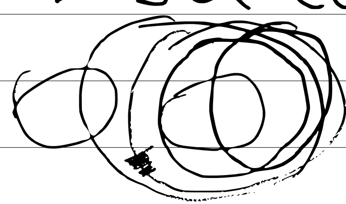
$\varphi_{PO}(x,y) \leftrightharpoons \exists z\,(\varphi_C(z,x)\ \&\ \varphi_C(z,y))\ \&$
$\neg P(x,y)\ \&\ \neg P(y,x)\ \&\ \neg \underline{\varphi_C}(x,y)\ \&\ \neg \varphi_C(y,x).$

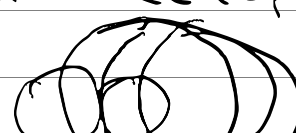


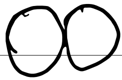
$\varphi_{PO}(x,y) \leftrightharpoons \exists z\,(P(z,x)\ \&\ P(z,y))\ \&$
$\neg P(x,y)\ \&\ \neg P(y,x)\ \&\ \neg \varphi_=(x,y).$

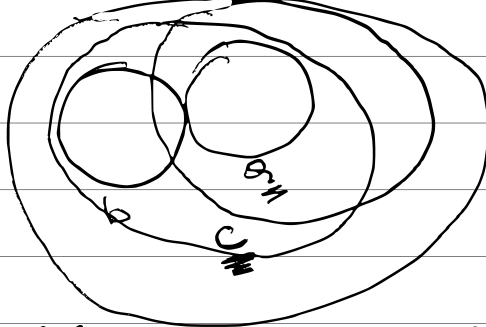


$\varphi_{OO}(x,y) \leftrightharpoons \neg \exists z\,(\varphi_C(z,x)\ \&\ \varphi_C(z,y)).$

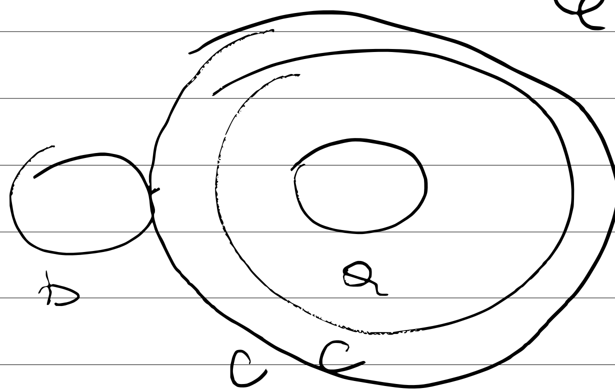








$\forall c\,(ec(a,c) \Rightarrow e_{po}(c,b) \vee p(b,c) \vee ec(b,c))\ \&\ e_{oo}(a,b).$



OO $\exists c\,(ec(a,c)\ \&\ e_{oo}(c,b)).$

(300) $\mathcal{A} = \langle \mathbb{N}; p^{\mathcal{A}} \rangle$ за $\mathcal{L} = \langle p \rangle$, $\#(p) = 2$, $p \in Pred_{\mathcal{L}}$

където $\langle n, m \rangle \in p^{\mathcal{A}} \Leftrightarrow m - n > 2$. $\to$ $m - n > 3$

Док., че:

(i) $\{\langle n, m \rangle \mid n = m\}$

(ii) $\{0\}$ и $\{1\}$

са определими.

Док., че всяко множество от вида $\{n\}$ е опр.

$\varphi_=(x, y) \leftrightharpoons \forall z (p(x, z) \Leftrightarrow p(y, z))$.

$\forall z (\underbrace{x}_{0} - z > 2 \Leftrightarrow \underbrace{y}_{1} - z > 2)$

защо $\nearrow$

не стига да кажем $\forall z (p(z, x) \Leftrightarrow p(z, y))$.

$\{0, 1, 2, 3\}$ $\quad \neg p(x, x)$

$\underbrace{x - x}_{0} \leq 2$

$$\varphi_{\{0,1,2\}}(x) \leftrightharpoons \underbrace{\forall y \neg p(y,x)}_{\forall y \; x - y \le 2}.$$

$$\varphi_{<0,3>}(y,x) \leftrightharpoons \underbrace{\wedge\, p(y,x)}_{\varphi_{\{0,1,2\}}(y)\,\&} \,\&\, \forall y' \left(p(y',x) \Rightarrow \varphi_{=}(y,y') \right).$$

$$\varphi_0(x) \leftrightharpoons \exists y\, \varphi_{<0,3>}(x,y)$$

$$\varphi_3(x) \leftrightharpoons \exists y\, \varphi_{<0,3>}(y,x).$$

$$\varphi_{\le}(x,y) \leftrightharpoons \forall z \left(p(y,z) \Rightarrow p(x,z) \right). \quad // \; \underline{x \le y}$$

$\forall x : x - y > 2 \Rightarrow x - y' > 2$ значи $y' \le y$

$$\varphi_{+1}(x,y) \leftrightharpoons \underbrace{\varphi_<(x,y)} \,\&\, \underbrace{\forall z(\varphi_<(x,z) \Rightarrow \varphi_\leq(y,z))}.$$

$y = x+1$ $\qquad\qquad$ $x < y \leq z \rightsquigarrow \underline{y = x+1}$

$$\varphi_1(x) \leftrightharpoons \varphi_{\{0,1,2\}}(x) \,\&\, \neg\varphi_0(x) \,\&\, \exists y(\varphi_{\{0,1,2\}}(y) \,\&$$
$$\neg\varphi_0(y) \,\&\, \neg\varphi_=(x,y) \,\&\, \varphi_\leq(x,y)).$$

$$\varphi_2(x) \leftrightharpoons \varphi_{\{0,1,2\}}(x) \,\&\, \neg\varphi_0(x) \,\&\, \neg\varphi_1(x).$$

$\varphi_<(x,y) \leftrightharpoons \neg\varphi_\leq(y,x).$ $\qquad$ $\underline{\neg(y \leq x)}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $y > x$

База: φ_0

ИП: φ_n

Стъпка: $\varphi_{n+1}(x) \leftrightharpoons \exists y(\varphi_n(y) \,\&\, \varphi_{+1}(y,x)).$

(300) $\mathcal{A} = \langle \mathbb{N}, p^{\mathcal{A}} \rangle$ за $\mathcal{L} = \langle p \rangle$, $\#(p) = 2$, $p \in Pred_{\mathcal{L}}$,

където $\langle n, m \rangle \in p^{\mathcal{A}} \Longleftrightarrow m + n > 2$.

Кои синглетони са определими?

$\{\langle n, m \rangle \mid n = m\}$ опр. ли е?

$$\forall z (\neg p(x, z) \Leftrightarrow \neg p(y, z))$$

$$z + x \leq 2 \qquad z + y \leq 2$$

$\varphi_{\{0,1\}}(x) \leftrightharpoons \underbrace{\neg p(x, x)}_{x + x \leq 2}.$

$\varphi_{2}(x) \leftrightharpoons \exists y \exists z (\varphi_{\{0,1\}}(y) \& \varphi_{\{0,1\}}(z) \& p(x, \underset{1}{y}) \& \neg p(x, \underset{0}{z})).$

$\varphi_{\langle 2,1,0 \rangle}(x, y, z) \leftrightharpoons \varphi_{\{0,1\}}(y) \& \varphi_{\{0,1\}}(z) \& p(x, y) \& \neg p(x, z).$

$\varphi_{1}(y) \leftrightharpoons \exists x \exists z \varphi_{\langle 2,1,0 \rangle}(x, y, z)$

$\varphi_{0}(z) \leftrightharpoons \exists x \exists y \varphi_{\langle 2,1,0 \rangle}(x, y, z).$

$\varphi_{\{0,1,2\}}(x) \leftrightharpoons \exists y\, \neg p(x,y).$

$x+y \leq 2$

$\psi_2(x) \leftrightharpoons \varphi_{\{0,1,2\}} p(x) \;\&\; \neg \varphi_{\{0,1\}}$

$\{3\}$ $\quad h_{3,4}(x) \leftrightharpoons \begin{cases} 3, & x=4 \\ 4, & x=3 \\ x, & \text{else.} \end{cases}$

Тогава $h_{3,4}(x) = h^{-1}_{3,4}(x)$ и е хим отн. p^{A}.

$\{5\}$ $\quad h_{5,6}$ $\qquad \{0\} \quad \{1\} \qquad 2+0 \not> 2 \quad 2+\overset{h(0)}{1} > 2$

$h(x) \leftrightharpoons \begin{cases} x, & x=0,1,2 \\ x+1, & x \neq 0,1,2 \;\&\; x\%2=1 \\ x-1, & x \neq 0,1,2 \;\&\; x\%2=0. \end{cases}$

$\langle a,b\rangle \in p^{A} \longleftrightarrow \langle h(a), h(b)\rangle \in p^{A}$

$a+b > 2 \longleftrightarrow h(a)+h(b) > 2$

• $h(a)=a, h(b)=b$; • БОО $h(a)\neq a$ и $h(b)=b$

Зад. Нека $\mathcal{L} = \langle p \rangle$ е ФОРЛ с "$\doteq$", където $p \in Pred_{\mathcal{L}}$, $\#(p) = 2$ шахматна дъска

Нека $\mathcal{A} = \langle \{ \langle i, j \rangle \mid i, j \in \{a1, \ldots, a8, b1, \ldots, h1, \ldots, h8\} \}, p^{\mathcal{A}} \rangle$,

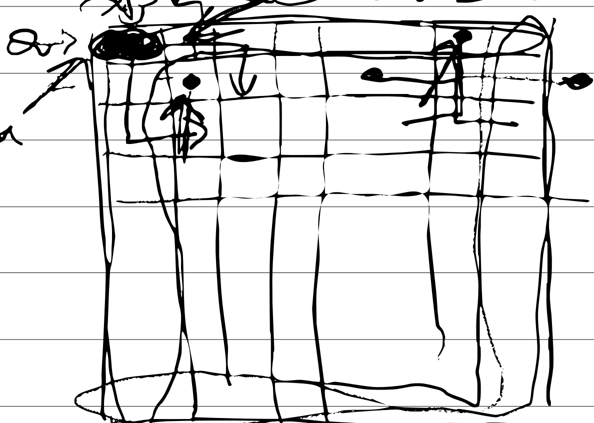
за $\langle a, b \rangle \in p^{\mathcal{A}} \iff$ от полето a с <u>кон</u> може за един ход да се отиде в полето b.

Определете:

(i) м-вото от всички ъглови полета

(ii) м-вото от всички <u>периферни</u> полета

Док. че $\{a2\}$ е неопр. в $\mathcal{A}$.



$$\varphi_{corner}(x) \leftrightharpoons \exists y \exists z (\neg(y \doteq z) \,\&\, p(x, y) \,\&\, p(x, z) \,\&\, \forall t (p(x, t) \Rightarrow t \doteq y \vee t \doteq z)).$$

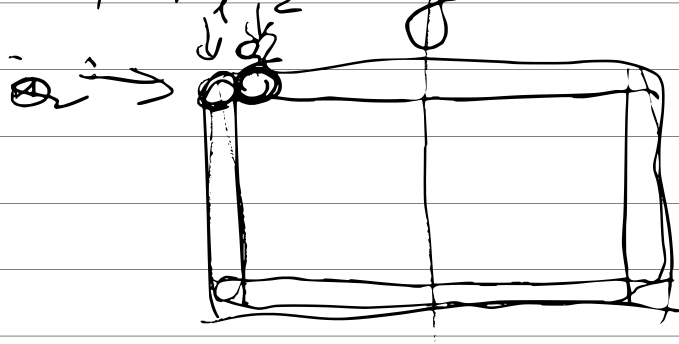
$\mathcal{C}_{3or4N}(x)$

$$\mathcal{C}_{periph.}(x) \leftrightharpoons \big(\mathcal{C}_{3or4N}(x) \,\&\, \neg \exists y \exists z (\neg(y \doteq z) \,\&\, p(x,y) \,\&\, p(x,z) \,\&\, \mathcal{C}_{3or4N}(y) \,\&\, \mathcal{C}_{3or4N}(z))\big) \vee \mathcal{C}_{corner}(x).$$

$$\mathcal{C}_{3or4N}(x) \leftrightharpoons \mathcal{C}_{3N}(x) \vee \mathcal{C}_{4N}(x)$$

$$\mathcal{C}_{3N}(x) \leftrightharpoons \exists y \exists z \exists t (\neg(y \doteq z) \,\&\, \neg(y \doteq t) \,\&\, \neg(z \doteq t) \,\&\, p(x,y) \,\&\, p(x,z) \,\&\, p(x,t) \,\&\, \forall u (p(x,u) \Rightarrow u \doteq y \vee u \doteq z \vee u \doteq t)).$$

$\mathcal{C}_{4N}(x)$ . . . .



$h(xy) = x(8-y)$

$x \in \{a, \ldots, h\}$

$y \in \{1, \ldots, 8\}$