382/ $\mathcal{L} = \langle p \rangle$, $p \in Pred_{\mathcal{L}}$, $\#(p) = 3$

$\mathcal{A} = \langle \mathbb{N}, p^{\mathcal{A}} \rangle$, където

$p^{\mathcal{A}}(k, n, m) \iff k + n = m + 2$

a) Док. че всеки синглетон е определим

b) Да се определят $=$ и $<$.

??? $\varphi(x) \leftrightharpoons p(x, x, x)$. // $x + \cancel{x} = \cancel{x} + 2$, $x \stackrel{?}{=} 2$

$\mathcal{A} \models_v \varphi(x) \iff v(x) = 2$

$\varphi_=(x, y) \leftrightharpoons \exists z(\varphi_2(z) \,\&\, p(x, z, y))$ // $x + \cancel{2} = y + \cancel{2}$, $x \stackrel{?}{=} y$

$\varphi_0(x) \leftrightharpoons \neg\exists t\, p(x, x, t)$. // $0 + 0 = 0 \neq t + 2$, $t \geq 0$

$\varphi_1(x) \leftrightharpoons \exists t(p(x, x, t) \,\&\, \varphi_0(t))$

$1 + 1 = 0 + 2 = 2$

(База) $\varphi_0, \varphi_1, \varphi_2$

(И.П.) Нека за $n \geq 3$ то дъл. се опр. от ф-лата φ_n

(Стъпка) $\varphi_{n+1}(x) \leftrightharpoons \exists y \exists z \, (\varphi_1(y) \,\&\, \varphi_n(z) \,\&\, p(x, y, z))$.

$\underbrace{(n+1)}\underbrace{+1} = \underbrace{n+2}$

$\varphi_<(x, y) \leftrightharpoons \exists z \, (p(x, z, y) \,\&\, \neg\varphi_0(z) \,\&\, \neg\varphi_1(z) \,\&\, \neg\varphi_2(z))$

$x + z = y + 2 \to x + \cancel{2} + z_1 = y + \cancel{2}$

$z > 2 \to z = 2 + \underbrace{z_1}_{\geq 1}$ $\qquad x + \underbrace{z_1} = y$

$\underline{z_1 > 0}$

3ад/ Нека $\mathcal{L} = \langle \underline{II} \rangle$, $\underline{II} \in Preds$, $\#(\underline{II}) = 3$.

Нека $\mathcal{A} = \langle \mathbb{R}^2, \underline{II}^{\mathcal{A}} \rangle$, където:

$$\underline{II}^{\mathcal{A}}(A, B, C) \leftrightharpoons A \neq B \text{ и } A \neq C \text{ и } \angle BAC = 30^\circ$$

Да се докаже, че в стр. $\mathcal{A}$ са определими:

- $Eq \leftrightharpoons \{\langle A, A\rangle \mid A \in \mathbb{R}^2\}$
- $Col \leftrightharpoons \{\langle A, B, C\rangle \mid$ лежат на една права$\}$
- $Circ \leftrightharpoons \{\langle A, B, C\rangle \mid C$ лежи на окръжност с диаметър $AB\}$

Вярно ли е, че в $\mathcal{A}$ са опр. м-вата и зад.?

- $Mid \leftrightharpoons \{\langle A, B, C\rangle \mid C$ е среда на отс. $AB\}$
- $Seg \leftrightharpoons \{\langle A, B, C\rangle \mid C$ лежи на отс. $AB\}$

Зад | $\mathcal{L} = \langle p \rangle$, $\# p = 3$, $p \in \mathrm{Pred}$

$\mathcal{A} = \langle \mathbb{N}, p^{\mathcal{A}} \rangle$

$p^{\mathcal{A}}(a, b, c) \iff a \cdot b + 1 = c^2$

Докажете, че:

$M = \{\langle a, b \rangle \mid a, b \in \mathbb{N} \,\&\, a = b\}$ е

определимо м-во и

$(\forall n \in \mathbb{N})$ [$\{n\}$ е определимо м-во]

- $\varphi_0(x) \leftrightharpoons \exists y (\varphi_1(y) \,\&\, p(x, x, y))$.
- $\varphi_1(x) \leftrightharpoons \exists y (p(x, y, x))$ // $x \cdot y + 1 = x^2 \iff 1 = x^2 - x \cdot y \iff 1 = x \cdot (x - y) \iff x = 1$ и $y = 0$
- $p(x, y, x)$
- $\varphi_{\langle 0, 1 \rangle}(y, x) \leftrightharpoons p(x, y, x)$

- $p(x,x,y)$

  $x^2+1=y^2 \iff x=0,\ y=1$

- $p(y,x,x)$

  $y.x+1=x^2 \iff x=1,\ y=0$

- $\varphi_0(x) \leftrightharpoons \neg\exists y \exists z\ p(y,z,x).$
- $f_{\langle 0,1\rangle}(x,y) \leftrightharpoons \forall z\ (p(x,z,y))$ зачер. z

  $x.z+1=y^2$

  $x=0,\ y=1$

2.2.4

$p^A(a,b,c) \iff a.b+1=c^2 \iff$

$a.b=c^2-1=(c-1).(c+1) \iff$

$\varphi_2(x) \leftrightharpoons \forall y \forall z\,(p(y,z,x) \Rightarrow$

$\varphi_1(y) \vee \varphi_1(z))\ \&$

$\neg\varphi_0(x)\ \&\ \neg\varphi_1(x).$

$a.b=\underbrace{(c-1)}_{1}.\underbrace{(c+1)}_{3}$

$c=2$

3 просто число

защо $3=1.3=3.1$

$\underbrace{2}.4=8=1.8 \Rightarrow$ $c=3$

(=) 2ny 3a $n > 2$

$p^{A}(a, b, c)$

2 3y : $3 = 1 \cdot x + 1 = 2^2 \Leftrightarrow$

$x = 2^2 - 1 = 3$

$C_3(x) \leftrightharpoons \exists y \exists z \,(C_1(y) \,\&\, C_2(z) \,\&\, p(y, x, z))$.

2 4y : $\overset{y}{2} \cdot \underset{x}{4} + 1 = \overset{z}{3}^2$

$C_4(x) \leftrightharpoons \exists y \exists z \,(C_2(y) \,\&\, C_3(z) \,\&\, p(y, x, z))$.

(~~base~~): C_0, C_1, C_2

(i.h) 3a BC. $2 < k \leq n+1$, TO C_k onp. 2ky

(step): $C_{n+2}(x) \leftrightharpoons \exists y \exists z \,(C_n(y) \,\&\, C_{n+1}(z) \,\&\, p(y, x, z))$.

$(n+1)^2 = n^2 + 2n + 1 = n \cdot (n+2) + 1$

$\varphi_=(x,y) \leftrightharpoons \forall z \forall t\,(p(z,t,x) \Leftrightarrow p(z,t,y))$

$p^A(a,b,c) \leftrightarrow a.b+1=c^2$ и за все. н. ч. сме опр.

φ

$\varphi_=(x,y) \leftrightharpoons \forall z \forall t\,(p(x,z,t) \Leftrightarrow p(y,z,t))$

2. Извод: ф-лата си е окей!
Няма контрапример, с който да я счупим!

Зад. **Числа на Фибоначи**

Редицата $\{F_n\}_{n=0}^{\infty}$ се дефинира така:

$$\begin{cases} F_0 = 0 \\ F_1 = 1 \\ F_{n+2} = F_n + F_{n+1} \quad \text{за } n \geq 0. \end{cases}$$

Разгл. език $\mathcal{L}$ без ф.р. и единст. нелог. символи: $Func_{\mathcal{L}} = \{f\}$, $Pred_{\mathcal{L}} = \{p\}$, $Const_{\mathcal{L}} = \emptyset$, като $\#(f) = 2$, $\#(p) = 1$.

Нека $S = \langle \mathbb{N}; f^S, p^S \rangle$ е стр. за $\mathcal{L}$

като:
$$f^S(x, y) = z \iff x + F_{y+1} = z$$
$$p^S(x) \iff x \in \{F_n\}_{n=0}^{\infty}$$

Да се док., че ⊃ в S са определими:

a) $\{\supset\}$

b) $\{1\}$

c) $eq = \{\langle a, a\rangle \mid a \in \mathbb{N}\}$ равенство

d) Вярно ли е, че в S е опр. м-вото

$Pred = \{\langle F_n, F_{n+1}\rangle \mid n \in \mathbb{N}\}$

e) Да се намерят всички автоморфизми на S с д-во.

$$f^S(x,y) = z \leftrightharpoons x + F_{y+1} = z$$

$$p^S(x) \leftrightharpoons x \in \{F_n\}_{n=0}$$

$\{0\}, \{1\}, =$ ~~[illegible]~~

$F_0 = 0$

$F_1 = 1$

$$\varphi_=(x,y) \leftrightharpoons \forall z \left(p(f(z,x)) \Longleftrightarrow p(f(z,y)) \right).$$

$$\varphi_0(x) \leftrightharpoons \forall y\, p(f(x,y)) \qquad // \quad \overset{0}{x} + F_{y+1} \in \{F_n\}_{n=0}^{\infty}$$

$$\varphi_1(x) \leftrightharpoons \exists y \left(\varphi_0(y) \;\&\; \varphi_=(f(y,y), x) \right).$$

$$\underbrace{0 + F_1}_{1} = x \quad x = 1$$

$$z + F_{y+1} \in \{F_n\}_{n=0}^{\infty} \Longleftrightarrow z + F_{x+1} \in \{F_n\}_{n=0}^{\infty}$$

$$z: \; 14 + F_{z+1} \in \{F_n\}_{n=0}^{\infty} \Longleftrightarrow 15 + F_{z+1} \in \{F_n\}_{n=0}^{\infty}$$

$$\mathrm{Prev} = \{ \langle F_n, F_{n+1} \rangle \mid n \in \mathbb{N} \}$$

$\langle F_0, F_1 \rangle, \langle F_1, F_2 \rangle, \ldots$

$$f^S(x, y) = z \Leftrightarrow x + F_{y+1} = z$$

$$\{0\}, \{1\} = P^S$$

$$P^S(0) \Leftrightarrow \underbrace{\exists z \in \mathbb{N} : 0 = F_z}$$

$x + F_1 = y$

$$\varphi_{+1}(x, y) \leftrightharpoons \exists z (\varphi_0(z) \,\&\, \varphi_=(f(x, z), y)).$$

$$\varphi_{getIndex}(x, ind) \leftrightharpoons P(x) \,\&\, \exists z (\varphi_0(z) \,\&$$

$x = F_z$

$$\exists t \; \varphi(f(z, t), x) \,\&\, \varphi_{+1}(t, ind))$$

$$\vee \; ((\varphi_0(ind) \,\&\, \varphi_0(x)).$$

$$\exists t \exists z (\varphi_0(z) \,\&\, 0 + F_{t+1} = \underset{F_z}{x} \;\&\, \varphi_{+1}(t, ind)).$$

$t+1 = z$

$$\mathrm{Prev}(x, y) \Leftrightarrow \varphi_{getIndex}(x, ind) \; \& \; \exists z \left(\varphi_{+1}(ind, \underbrace{z}_{ind+1}) \; \& \; \varphi_{getIndex}(y, z) \right)$$