392/ $\mathcal{L} = \langle p \rangle$, $p \in Pred_{\mathcal{L}}$, $\#(p) = 3$

$\mathcal{A} = \langle \mathbb{N}, p^{\mathcal{A}} \rangle$, където

$p^{\mathcal{A}}(k, n, m) \iff k + n = m + 2$

a) Док. че всеки синглетон е определим

b) Да се определят $=$ и $<$.

$p^{\mathcal{A}}(k, n, m) \iff k + n = m + 2$

$\varphi_2(x) \leftrightharpoons p(x, x, x)$. // $x + x = \cancel{2x} \overset{?}{=} \cancel{x} + 2$

$x \overset{?}{=} 2$ $\{2\}$

$\varphi_=(x, y) \leftrightharpoons \exists z (\varphi_2(z) \,\&\, p(x, z, y))$. // $x + \cancel{2} = y + \cancel{2}$

$x \overset{?}{=} y$

$\{0\}, \{1\}$?

$2x = 2 \iff x = 1$

$\varphi_1(x) \leftrightharpoons \exists y (\varphi_0(y) \,\&\, \overline{p(x, x, y)})$

$\underline{k + n} = 0 + 2 = 2$

$k + n = 2$

$k = 0, n = 2$; $k = 2, n = 0$. $k = n$

$\{0\}$? $\quad p(k,n,m) \leftrightharpoons \underbrace{k+n}_{} = m+2$

$\varphi_0(x) \leftrightharpoons \forall y(\neg p(x,x,y))$.

$0 = 2x \neq \cancel{y}+2$

$y \geq 0$

$\{0\}, \{1\}, \{2\}, \ldots$

$\varphi_{+1}(x,y) \leftrightharpoons \exists z(\varphi_1(z) \ \& \ \underbrace{p(y,z,x)}_{})$

$y = x+1$

$y + \cancel{1} = x + \cancel{2}_1$

$y = x+1$

(base): $\varphi_0, \varphi_1, \varphi_2$

(i.h): За $n \geq 2$, нека $\{n\}$ е опр. от φ_n.

(step): $\varphi_{n+1}(x) \leftrightharpoons \exists y(\varphi_n(y) \ \& \ \varphi_{+1}(y,x))$.

$<$

$$\varphi_<(x,y) \leftrightharpoons \exists z\,(\underbrace{p(x,z,y)}_{x+z=y+2}\ \&\ \neg\varphi_0(z)\ \&\ \neg\varphi_1(z)\ \&\ \neg\varphi_2(z)).$$

$$x+\underbrace{z}_{\geq 2}=y+2 \to z=\underbrace{z_1}_{\geq 0}+2$$

$$\frac{x+\underbrace{z_1}_{\geq 0}=y}{x<y}$$

$$\varphi_\leq(x,y) \leftrightharpoons \varphi_=(x,y) \vee \varphi_<(x,y).$$

Зад./ Нека $\mathcal{L} = \langle \underline{\text{ll}} \rangle$, $\underline{\text{ll}} \in Preds$, $\#(\underline{\text{ll}}) = 3$.

Нека $\mathcal{A} = \langle \mathbb{R}^2, \underline{\text{ll}}^{\mathcal{A}} \rangle$, където:

$$\underline{\text{ll}}^{\mathcal{A}}(A, B, C) \leftrightharpoons A \neq B \text{ и } A \neq C \text{ и } \angle BAC = 30^\circ$$

Да се докаже, че в стр. $\mathcal{A}$ са определими:

- $Eq \leftrightharpoons \{\langle A, A \rangle \mid A \in \mathbb{R}^2\}$
- $Col \leftrightharpoons \{\langle A, B, C \rangle \mid$ лежат на една права$\}$
- $Circ \leftrightharpoons \{\langle A, B, C \rangle \mid C$ лежи на окръжност с диаметър $AB\}$

Вярно ли е, че в $\mathcal{A}$ са опр. м-вата и защо?

- $Mid \leftrightharpoons \{\langle A, B, C \rangle \mid C$ е среда на отс. $AB\}$
- $Seg \leftrightharpoons \{\langle A, B, C \rangle \mid C$ лежи на отс. $AB\}$

зад | $L = \langle p \rangle$, $\# p = 3$, $p \in \mathrm{Pred}$

$\mathcal{A} = \langle \mathbb{N}, p^{\mathcal{A}} \rangle$

$p^{\mathcal{A}}(a, b, c) \Leftrightarrow a.b + 1 = c^2$

Докажете, че:

$M_= \leftrightharpoons \{\langle a, b\rangle \mid a, b \in \mathbb{N} \;\&\; a = b\}$ е

определимо м-во и

$(\forall n \in \mathbb{N})$ [$\{n\}$ е определимо м-во]

$\varphi_=(x, y) \leftrightharpoons \forall z \forall t\, (p(x, z, t) \Leftrightarrow p(y, z, t))$

$x.z + 1 = t^2 \Leftrightarrow y.z + 1 = t^2$

$x.z + \cancel{1} = y.z + \cancel{1}$

$x.\cancel{z} = y.\cancel{z}$

$z = 0$ $\quad$ $0 = 0$

$z > 0$ $\quad$ $x = y$

$p^A(a,b,c) \iff a \cdot b + 1 = c^2$ $\mathbb{N}$

$\varphi_0(x) \leftrightharpoons \forall y \forall z \neg p(y,z,x)$ $\quad y \cdot z + 1 \neq 0$

$c^2 = n^2 > 0 \to y = (n^2 - 1)$

$z = 1$

$\varphi_1(x) \leftrightharpoons \exists y (\varphi_0(y) \,\&\, p(y,y,x))$

$\psi_1(x) \leftrightharpoons \exists y \cancel{\forall z} (\varphi_0(y) \,\&\, p(y,z,x))$

$\psi_0(x) \leftrightharpoons \exists y \forall z (p(x,z,y))$

$\gamma_{\langle 0,1 \rangle}(x,y) \leftrightharpoons \forall z\, p(x,z,y).$

$\varphi_0'(x) \leftrightharpoons \exists y\, \gamma_{\langle 0,1 \rangle}(x,y).$

$\varphi_1'(x) \leftrightharpoons \exists y\, \gamma_{\langle 0,1 \rangle}(y,x).$

★ $p(a,a,c)$

$a^2+1=c^2 \Leftrightarrow a=0, c=1$

★ $p(a,b,a)$

$a.b+1=a^2 \Leftrightarrow a^2-a.b=1$

$\Rightarrow a.(a-b)=1$

$\Rightarrow a=1, b=0$

★ $p(a,b,b)$

$a.b+1=b^2 \Leftrightarrow b^2-a.b=1$

$\Rightarrow b(b-a)=1$

$\Rightarrow b=1, a=0.$

$\{2\}$?

$p(a,b,c) \Leftrightarrow a.b+1=c^2 \Leftrightarrow$

$c^2-1=a.b$

$(c-1).(c+1)=a.b$

$\underbrace{(c-1)\cdot(c+1)}_{\text{четно}} = 2\cdot \underbrace{b}_{\text{четно}}$

c - нечетно, b - четно

твърде голяма свобода

$\underbrace{\underbrace{(2-1)}_{1}\cdot\underbrace{(2+1)}_{3}} = a\cdot b$

3 е просто число

$3 = 3.1 = 1.3 \Rightarrow$ или $a = 1$,

или $b = 1$

$e_2(x) \leftrightharpoons \forall y \forall z\, (p(y,z,x) \Rightarrow e_1(y) \vee e_1(z))$

$\& \neg \underbrace{e_0(x)} \,\&\, \neg \underbrace{e_1(x)}$

$(c-1)\cdot(c+1) =$ просто число $\neq 3 = a\cdot b = \begin{cases} 1.b \\ 1.a \end{cases}$

т.е. $\underbrace{c-1 = 1}$ или $\underbrace{c+1 = 1}$

$c = 2 \vee$ $c = 0$ ↯ $\rightarrow c-1 = 0-1 = 1$

$\{3\}, \{4\}, \{5\}, \ldots$

$\varphi_3(x) \leftrightharpoons \exists y \exists z (\varphi_1(y) \,\&\, \varphi_2(z) \,\&\, p(x,y,z))$

$p^{\mathcal{A}}(a,b,c) \iff a \cdot b + 1 = c^2$ $\quad x \cdot 1 + 1 = 2^2 = 4$

$\varphi_0, \varphi_1, \varphi_2, \varphi_=$ $\quad x \cdot 1 = 3$

$x = 3$

$\varphi_4(x) \leftrightharpoons \exists y \exists z (\varphi_2(y) \,\&\, \varphi_3(z) \,\&\, p(x,y,z))$

$x \cdot 2 + 1 = 3^2 = 9$

$x \cdot 2 = 8$

$x = 4$

(base) $0, 1, 2$

(i.h.) Here ~~so~~ $2 \le k \le n+1$, φ_k опр. $\{k\}$.

(step): $\varphi_{n+2}(x) \leftrightharpoons \exists y \exists z (\varphi_n(y) \,\&\, \varphi_{n+1}(z) \,\&\, p(x,y,z))$

$x \cdot n + \cancel{1} = (n+1)^2 = n^2 + 2n + \cancel{1}$

$x = \frac{n^2 + 2n}{n}, \quad n > 0$

$x = n + 2$

Зад. Числа на Фибоначи

Редицата $\{F_n\}_{n=0}^{\infty}$ се дефинира така:

$$\left\{\begin{array}{l} F_0 = 0 \\ F_1 = 1 \\ F_{n+2} = F_n + F_{n+1} \quad \text{за } n \geq 0. \end{array}\right.$$

Разгл. език $\mathcal{L}$ без ф.р. и единст. нелог. символи: $Func_{\mathcal{L}} = \{f\}$, $Pred_{\mathcal{L}} = \{p\}$, $Const_{\mathcal{L}} = \emptyset$, като $\#(f) = 2$, $\#(p) = 1$.

Нека $S = \langle \mathbb{N}; f^S, p^S \rangle$ е стр. за $\mathcal{L}$ като:

$$f^S(x, y) = z \leftrightarrows x + F_{y+1} = z$$

$$p^S(x) \leftrightarrows x \in \{F_n\}_{n=0}^{\infty}$$

Да се док., че в S са определими:

a) $\{0\}$

b) $\{1\}$

c) $eq = \{\langle a, a\rangle \mid a \in \mathbb{N}\}$ равенство

d) Вярно ли е, че в S е опр. м-во $P_{fib} = \{\langle F_n, F_{n+1}\rangle \mid n \in \mathbb{N}\}$

e) Да се намерят всички автоморф. на S с д-во.

$f^S(x,y) = z \leftrightharpoons x + F_{y+1} = z$

$p^S(x) \leftrightharpoons x \in \{F_n\}_{n=0}^{\infty}$

$\varphi_0(x) \leftrightharpoons \forall y\, p(f(x,y))$

Нека $\nu(x) \in \mathbb{N}\setminus\{0\}$. 0, 1, 1, 2, 3, 5, 8, 13, 21, ...

$S \not\models_{\nu} \varphi_0(x)$?

Ако, ще $\underbrace{S \models_{\nu_n} \varphi_0(x)}$

$S \models_{\nu} \forall y\, p(f(x,y))$

За вс. $n \in \mathbb{N}$: $S \models_{\nu^y_n} p(f(x,y))$

$\underbrace{f^S(\nu^y_n(x), \nu^y_n(y))} \in p^S$

$m > 0$, $n \geq 0$: $m + F_{n+1} \in \{F_k\}_{k=0}^{\infty}$

Нека $n = 10.m > 0$: $m + F_{10.m+1} \notin \{F_k\}_{k=0}^{\infty}$

$\boxed{HW}$ За $m \in \mathbb{N}_{\geq 1}$ същ. $n \in \mathbb{N}$, т.е.

$$F_{n+1} - F_n > m$$

219: $\varphi_1(x) \leftrightharpoons \exists y(\varphi_0(y) \,\&\, \varphi_=(\underbrace{f(y,y)}_{0+F_{0+1}=1=x}, x))$

$$F_{0+1} = F_1$$

$=: \varphi_=(x,y) \leftrightharpoons \forall z(\underbrace{p(f(x,z))}_{x+F_{z+1}\in\{F_k\}_{k=0}^{\infty}} \Leftrightarrow p(f(y,z)))$

$$x+F_{z+1}\in\{F_k\}_{k=0}^{\infty} \Leftrightarrow y+F_{z+1}\in\{F_k\}_{k=0}^{\infty}$$

$Prev = \{\langle F_n, F_{n+1}\rangle \mid n \in \mathbb{N}\}$

$$f^S(x,y) = z \Leftrightarrow x + F_{y+1} = z$$

$$p^S(x) \Leftrightarrow x \in \{F_k\}_{k=0}^{\infty}$$

$\varphi_{getIndex}(x, ind_x) \leftrightharpoons p(x) \,\&\, ((\varphi_0(x) \,\&\, \varphi_0(ind_x)) \vee$

$(\neg\varphi_0(x) \,\&\, \exists z \exists t (\varphi_0(z) \,\&$

$\varphi_=(f(z, t), x) \,\&$

$\varphi_{+1}(t, ind_x)))).$

$f(x, y) = x + F_{y+1}$

$ind_x > 0$

$t = ind_x - 1 \geq 0$

$\varphi_{+1}(x, y) \leftrightharpoons \exists z(\varphi_0(z) \,\&\, \varphi_=(y, \underbrace{f(x, z)}_{x + F_{1 = 0+1}}))$

$\mathfrak{U} \leftrightharpoons \{\langle F_n, n \rangle \mid n \in \mathbb{N}\}$

$y = x + 1$

$Prev(x, y) \leftrightharpoons \exists z \exists t (\varphi_{getIndex}(x, z) \,\&\, \varphi_{+1}(z, t) \,\&$

$\varphi_{getIndex}(y, t)).$

$u \simeq \langle ? \rangle$

$\forall n \in \mathbb{N}$ no ?ny eonj. $\rightarrow$ $Aut(S) = \{Id_S\}$