Зад/ $\mathcal{L} = \langle p \rangle$, $p \in Pred_{\mathcal{L}}$, $\#(p) = 3$

$\mathcal{A} = \langle \mathbb{N}, p^{\mathcal{A}} \rangle$, където

$p^{\mathcal{A}}(k, n, m) \Leftrightarrow k + n = m + 2$

a) Док. че всеки синглетон е определим

b) Да се определят = и <.

$p^{\mathcal{A}}(k, n, m) \Leftrightarrow k + n = m + 2$

$\varphi_2(x) \leftrightharpoons p(x, x, x)$. // $x + x = 2x = \cancel{x} + 2$ $\quad x = 2 \quad \{2\}$

$\varphi_=(x, y) \leftrightharpoons \exists z(\varphi_2(z) \& p(x, z, y))$. // $x + \cancel{2} = y + \cancel{2}$ $\quad x = y$

$\{0\}, \{1\}$?

$2x = 2 \Leftrightarrow x = 1$

$\varphi_1(x) \leftrightharpoons \exists y(\varphi_0(y) \& \overline{p(x, x, y)})$

$\underline{k+n} = 0 + 2 = 2$

$k + n = 2$

$k = 0, n = 2$; $k = 2, n = 0$. $k = n$

$\{0\}$? $\quad p(k,n,m) \leftrightharpoons k+n = m+2$

$\varphi_0(x) \leftrightharpoons \forall y(\neg p(x,x,y))$.

$0 = 2x \neq y+2$, $\quad y \geq 0$

$\{0\}, \{1\}, \{2\}, =$

$\varphi_{+1}(x,y) \leftrightharpoons \exists z(\varphi_1(z) \;\&\; p(y,z,x))$

$y = x+1$

$y + \cancel{1} = x + \cancel{z}$, $\quad z = 1$

$y = x+1$

(base): $\varphi_0, \varphi_1, \varphi_2$

(i.h): За $n > 2$, нека $\{n\}$ се опр. от φ_n.

(step): $\varphi_{n+1}(x) \leftrightharpoons \exists y(\varphi_n(y) \;\&\; \varphi_{+1}(y,x))$.

$<$

$$\varphi_<(x,y) \leftrightharpoons \exists z\,(\underbrace{p(x,z,y)}\ \&\ \neg\varphi_0(z)\ \&\ \neg\varphi_1(z)\ \&\ \neg\varphi_2(z)).$$

$$x+\underbrace{z}_{>2}=y+2$$

$$\to z=\underbrace{z_1}_{\geq 0}+2$$

$$\frac{x+\underbrace{z_1}_{\geq 0}=y}{x<y}$$

$$\varphi_\leq(x,y) \leftrightharpoons \varphi_=(x,y) \vee \varphi_<(x,y).$$

300/ Нека $\mathcal{L} = \langle \underline{ll} \rangle$, $\underline{ll} \in Preds$, $\#(\underline{ll}) = 3$.
Нека $\mathcal{A} = \langle \mathbb{R}^2, \underline{ll}^{\mathcal{A}} \rangle$, където

$$\underline{ll}^{\mathcal{A}}(A, B, C) \leftrightharpoons A \neq B \text{ и } A \neq C \text{ и } \angle BAC = 30^\circ$$

Да се док., че в стр. $\mathcal{A}$ са определими:

- $Eq \leftrightharpoons \{\langle A, A \rangle \mid A \in \mathbb{R}^2\}$
- $Col \leftrightharpoons \{\langle A, B, C \rangle \mid$ лежат на една права$\}$
- $Circ \leftrightharpoons \{\langle A, B, C \rangle \mid C$ лежи на окръжност с диаметър $AB\}$

Вярно ли е, че в $\mathcal{A}$ са опр. м-вата и защо?

- $Mid \leftrightharpoons \{\langle A, B, C \rangle \mid C$ е среда на отс. $AB\}$
- $Seg \leftrightharpoons \{\langle A, B, C \rangle \mid C$ лежи на отс. $AB\}$

Зад | $\mathcal{L} = \langle p \rangle$, $\#p = 3$, $p \in \mathrm{Pred}$

$\mathcal{A} = \langle \mathbb{N}, p^{\mathcal{A}} \rangle$

$$p^{\mathcal{A}}(a,b,c) \iff \underline{a \cdot b + 1 = c^2}$$

Докажете, че:

$M = \{\langle a,b\rangle \mid a,b \in \mathbb{N} \ \& \ a = b\}$ е определимо м-во и

$(\forall n \in \mathbb{N})[\{n\}$ е определимо м-во$]$

$$\varphi = (x,y) \leftrightharpoons \forall z \forall t\,(p(x,z,t) \iff p(y,z,t))$$

$$x \cdot z + 1 = t^2 \iff y \cdot z + 1 = t^2$$

$$x \cdot z + \cancel{1} = y \cdot z + \cancel{1}$$

$$x \cdot z = y \cdot z \quad :z$$

$\underline{z = 0}$ $\qquad$ $\underline{z > 0}$

$\underline{0 = 0}$ $\qquad$ $x = y$

$$p^{A}(a,b,c) \iff \underline{a\cdot b}+1=c^2 \qquad \mathbb{N}$$

$$\varphi_0(x) \leftrightharpoons \forall y \forall z \neg p(y,z,x) \qquad y\cdot z + 1 \neq 0$$

$$c^2 = n^2 > 0 \to y = (n^2-1), \; z = 1$$

$$\varphi_1(x) \leftrightharpoons \exists y\,(\varphi_0(y) \,\&\, p(y,y,x))$$

$$\psi_1(x) \leftrightharpoons \exists y\, \cancel{\forall} \exists z\,(\varphi_0(y) \,\&\, p(y,z,x))$$

$$\psi_0(x) \leftrightharpoons \exists y \forall z\,(p(x,z,y))$$

$$\zeta_{\langle 0,1\rangle}(x,y) \leftrightharpoons \forall z\, p(x,z,y).$$

$$\varphi_0'(x) \leftrightharpoons \exists y\, \zeta_{\langle 0,1\rangle}(x,y).$$

$$\varphi_1'(x) \leftrightharpoons \exists y\, \zeta_{\langle 0,1\rangle}(y,x).$$

$p(a,a,c)$

$a^2+1=c^2 \Leftrightarrow a=0,\ c=1$

$p(a,b,a)$

$a.b+1=a^2 \Leftrightarrow a^2-a.b=1$

$\Leftrightarrow a.(a-b)=1$

$\Leftrightarrow a=1,\ b=0$

$p(a,b,b)$

$a.b+1=b^2 \Leftrightarrow b^2-a.b=1$

$\Leftrightarrow b(b-a)=1$

$\Leftrightarrow b=1,\ a=0.$

$\{2\}$?

$p^t(a,b,c) \Leftrightarrow a.b+1=c^2 \Leftrightarrow$

$c^2-1=a.b$

$(c-1).(c+1)=a.b$

$$\underbrace{(c-1).(c+1)}_{\text{четно}} = 2.\underbrace{b}_{\text{четно}}$$

c - нечетно, b - четно

твърде голяма свобода

$$\underbrace{\underbrace{(2-1)}_{1}.\underbrace{(2+1)}_{3}} = a.b$$

3 е просто число

$3 = 3.1 = 1.3 \to$ или $a = 1$,

или $b = 1$

$$\varphi_2(x) \leftrightharpoons \forall y \forall z\, (p(y,z,x) \Rightarrow \varphi_1(y) \vee \varphi_1(z))$$

$\& \neg \underline{\varphi_1(x)}\ \& \underline{\varphi_1(x)}$

$$\underline{(c-1).(c+1) = \text{просто число}} \neq 3 = a.b = \begin{cases} 1.b \\ 1.a \end{cases}$$

т.е. $\underbrace{c-1=1}$ или $\underbrace{c+1=1}$

$\underline{c=2}$ ✓ $\qquad c=0$ ↯ $\to c-1 = 0-1 = 1$ ↯

$\{3\}, \{4\}, \{5\}$

$\varphi_3(x) \Leftrightarrow \exists y \exists z(\varphi_1(y) \,\&\, \varphi_2(z) \,\&\, p(x,y,z))$

$p^A(a,b,c) \Longleftrightarrow a \cdot b + 1 = c^2$ $\quad x \cdot 1 + 1 = 2^2 = 4$

$\varphi_0, \varphi_1, \varphi_2, \varphi_=$ $\quad x \cdot 1 = 3$

$x = 3$

$\varphi_4(x) \Leftrightarrow \exists y \exists z(\varphi_2(y) \,\&\, \varphi_3(z) \,\&\, p(x,y,z))$

$x \cdot 2 + 1 = 3^2 = 9$

$x \cdot 2 = 8$

$x = 4$

(base) $0, 1, 2$

(i.h.) Нека за $2 \leq k \leq n+1$, φ_k опр. $\{k\}$

(step): $\varphi_{n+2}(x) \Leftrightarrow \exists y \exists z(\varphi_n(y) \,\&\, \varphi_{n+1}(z) \,\&\, \underbrace{p(x,y,z)}_{n})$

$x \cdot n + \cancel{1} = (n+1)^2 = n^2 + 2n + \cancel{1}$

$x = \dfrac{n^2 + 2n}{n}, \quad n > 0$

$x = n + 2$

Зад

Числа на Фибоначи

Редицата $\{F_n\}_{n=0}^{\infty}$ се дефинира така:

$$\begin{cases} F_0 = 0 \\ F_1 = 1 \\ F_{n+2} = F_n + F_{n+1} \quad \text{за } n \geq 0 \end{cases}$$

Разгл. език $\mathcal{L}$ без ф.р. и единст. нелог. символи: $Func_{\mathcal{L}} = \{f\}$, $Pred_{\mathcal{L}} = \{p\}$, $Const_{\mathcal{L}} = \emptyset$ като $\#(f) = 2$, $\#(p) = 1$.

Нека $S = \langle \mathbb{N}, f^S, p^S \rangle$ е стр. за $\mathcal{L}$

като: $f^S(x, y) = z \iff x + F_{y+1} = z$

$p^S(x) \iff x \in \{F_n\}_{n=0}^{\infty}$

Да се док., че в S са определени:

a) $\{>\}$

b) $\{1\}$

c) $eq = \{\langle a, a\rangle \mid a \in \mathbb{N}\}$ равенство

d) Вярно ли е, че в S е опр. ф-во

$Fib = \{\langle F_n, F_{n+1}\rangle \mid n \in \mathbb{N}\}$

e) Да се намерят всички автоморф. на S с д-во.

$f^S(x,y) = z \leftrightharpoons x + F_{y+1} = z$

$p^S(x) \leftrightharpoons x \in \{F_n\}_{n=0}^{\infty}$

$\varphi_0(x) \leftrightharpoons \forall y\, p(f(x,y))$

Нека $v(x) \in \mathbb{N} \setminus \{0\}$.

$S \not\models_v \varphi_0(x)$?

0, 1, 1, 2, 3, 5, 8, 13, 21, ...

Доп., че $S \models_v \varphi_0(x)$

$S \models_v \forall y\, p(f(x,y))$

за вс. $n \in \mathbb{N}$: $S \models_{v^y_n} p(f(x,y))$

$f^S(v^y_n(x), v^y_n(y)) \in p^S$

$m > 0$, $m + F_{n+1} \in \{F_k\}_{k=0}^{\infty}$

$n \geq 0$

Нека $n = 10.m > 0$: $m + F_{10.m+1} \notin \{F_k\}_{k=0}^{\infty}$

$\boxed{HW}$ За $m \in \mathbb{N}_{\geq 1}$ същ. $n \in \mathbb{N}$, т.е.

$$F_{n+1} - F_n > m$$

$$21Y: \varphi_1(x) \leftrightharpoons \exists y(\varphi_0(y) \ \& \ \varphi_=(\underbrace{f(y,y)}_{0 + \underbrace{F_{0+1}}_{F_1} = 1}, \underbrace{x}_{=x}))$$

$$=: \varphi_=(x,y) \leftrightharpoons \forall z(\underbrace{p(f(x,z))}_{x + F_{z+1} \in \{F_k\}_{k=0}^{\infty}} \Leftrightarrow p(f(y,z)))$$

$$x + F_{z+1} \in \{F_k\}_{k=0}^{\infty} \Leftrightarrow y + F_{z+1} \in \{F_k\}_{k=0}^{\infty}$$

$$Prev = \{ \langle F_n, F_{n+1} \rangle \mid n \in \mathbb{N} \}$$

$$f^S(x,y) = z \Leftrightarrow x + F_{y+1} = z$$

$$p^S(x) \Leftrightarrow x \in \{F_k\}_{k=0}^{\infty}$$

$$\varphi_{getIndex}(x, ind_x) \leftrightharpoons p(x) \,\&\, \big((\varphi_0(x) \,\&\, \varphi_0(ind_x)) \vee (\neg\varphi_0(x) \,\&\, \exists z \exists t(\varphi_0(z) \,\&\, \varphi_=(f(z,t), x) \,\&\, \varphi_{+1}(t, ind_x)))\big).$$

$$f(x,y) = x + F_{y+1}$$

$$ind_x > 0$$

$$t = ind_x - 1 \geq 0$$

$$\varphi_{+1}(x,y) \leftrightharpoons \exists z(\varphi_0(z) \,\&\, \varphi_=(y, f(x,z)))$$

$$x + F_{1} = 0+1 = 1$$

$$y = x+1$$

$$U \leftrightharpoons \{\langle F_n, n\rangle \mid n \in \mathbb{N}\}$$

$$Prev(x,y) \leftrightharpoons \exists z \exists t(\varphi_{getIndex}(x,z) \,\&\, \varphi_{+1}(z,t) \,\&\, \varphi_{getIndex}(y,t)).$$

u от $\{n\}$

$\forall n \in \mathbb{N}$ $\{n\}$ е опр. $\rightarrow Aut(S) = \{Id_S\}$