Зад/ $\mathcal{L} = \langle p \rangle$, $p \in Pred_{\mathcal{L}}$, $\#(p) = 3$

$\mathcal{A} = \langle \mathbb{N}, p^{\mathcal{A}} \rangle$, където

$p^{\mathcal{A}}(k, n, m) \Longleftrightarrow k + n = m + 2$

a) Док. че всеки синглетон е определим

b) Да се определят $=$ и $<$.

??? $\varphi(x) \leftrightharpoons p(x, x, x)$. // $x + \cancel{x} = \cancel{x} + 2$

$x \stackrel{?}{=} 2$

$\mathcal{A} \models_v \varphi(x) \Longleftrightarrow v(x) = 2$

$\varphi_=(x, y) \leftrightharpoons \exists z(\varphi_2(z) \,\&\, p(x, z, y))$ // $x + \cancel{2} = y + \cancel{2}$

$x \stackrel{?}{=} y$

$\varphi_0(x) \leftrightharpoons \neg \exists t\, p(x, x, t)$. // $0 + 0 = 0 \neq t + 2$

$t \geq 0$

$\varphi_1(x) \leftrightharpoons \exists t(p(x, x, t) \,\&\, \varphi_0(t))$

$1 + 1 = 0 + 2 = 2$

(база) $\varphi_0, \varphi_1, \varphi_2$

(i.h) Нека за $n \geq 3$ то $\{n\}$ се опр. от ф-лата φ_n

(стъпка) $\varphi_{n+1}(x) \leftrightharpoons \exists y \exists z \, (\varphi_1(y) \,\&\, \varphi_n(z) \,\&\, p(x, y, z))$.

$(n+1)+1 = n+2$

$\varphi_2(x, y) \leftrightharpoons \exists z \, (p(x, z, y) \,\&\, \neg\varphi_0(z) \,\&\, \neg\varphi_1(z) \,\&\, \neg\varphi_2(z))$

$x + z = y + 2 \to x + \cancel{2} + z_1 = y + \cancel{2}$

$z > 2 \to z = 2 + \underbrace{z_1}_{\geq 1}$ $\quad x + \underbrace{z_1}_{z_1 > 0} = y$

300/ Нека $\mathcal{L} = \langle \underline{\amalg} \rangle$, $\underline{\amalg} \in Preds$, $\#(\underline{\amalg}) = 3$.

Нека $\mathcal{A} = \langle \mathbb{R}^2, \underline{\amalg}^{\mathcal{A}} \rangle$, където

$$\underline{\amalg}^{\mathcal{A}}(A, B, C) \leftrightharpoons A \neq B \text{ и } A \neq C \text{ и } \angle BAC = 30^\circ$$

Да се док., че в стр. $\mathcal{A}$ са определими:

- $Eq \leftrightharpoons \{ \langle A, A \rangle \mid A \in \mathbb{R}^2 \}$
- $Col \leftrightharpoons \{ \langle A, B, C \rangle \mid$ лежат на една права $\}$
- $Circ \leftrightharpoons \{ \langle A, B, C \rangle \mid C$ лежи на окръжност с диаметър $AB \}$

Вярно ли е, че в $\mathcal{A}$ са опр. м-вата и защо?

- $Mid \leftrightharpoons \{ \langle A, B, C \rangle \mid C$ е среда на отс. $AB \}$
- $Seg \leftrightharpoons \{ \langle A, B, C \rangle \mid C$ лежи на отс. $AB \}$

Зад | $\mathcal{L} = \langle p \rangle$, $\#p = 3$, $p \in Pred$

$\mathcal{A} = \langle \mathbb{N}, p^{\mathcal{A}} \rangle$

$p^{\mathcal{A}}(a, b, c) \leftrightarrow \underline{a.b + 1 = c^2}$

Докажете, че:

$M = \{\langle a, b \rangle \mid a, b \in \mathbb{N} \ \& \ a = b\}$ е

определимо м-во и

$(\forall n \in \mathbb{N})[\{n\}$ е определимо м-во$]$

- $\varphi_0(x) \leftrightarrows \exists y(\varphi_1(y) \ \& \ p(x, x, y))$.
- $\varphi_1(x) \leftrightarrows \exists y(p(x, y, x))$ // $x.y + 1 = x^2 \leftrightarrow$

$\leftrightarrow 1 = x^2 - x.y$

$\leftrightarrow 1 = x.(x - y)$

$\leftrightarrow \underline{x = 1}$ и $\underline{y = 0}$

- $p(x, y, x)$
- $\varphi_{\langle 0, 1 \rangle}(y, x) \leftrightarrows p(x, y, x)$

- $p(x,x,y)$

$$x^2+1=y^2 \iff x=0,\ y=1$$

- $p(y,x,x)$

$$y\cdot x+1=x^2 \iff x=1,\ y=0$$

- $\varphi_0(x) \leftrightharpoons \neg\exists y \exists z\ p(y,z,x).$
- $\varphi_{\{0,1\}}(x,y) \leftrightharpoons \forall z\,(p(x,z,y))$ за вс. z

$$x\cdot z+1=y^2$$

$$x=0,\ y=1$$

{2,3}

$$p^A(a,b,c) \iff a\cdot b+1=c^2 \iff$$

$$a\cdot b=c^2-1=(c-1)\cdot(c+1) \iff$$

$\varphi_2(x) \leftrightharpoons \forall y \forall z\,(p(y,z,x) \Rightarrow$

$(\varphi_1(y) \vee \varphi_1(z))\,\&$

$\neg\varphi_0(x)\,\&\,\neg\varphi_1(x)).$

$$a\cdot b=\underbrace{\underbrace{(c-1)}_{1}\cdot\underbrace{(c+1)}_{3}}_{}$$

$c=2$

3 просто число

$c=3$

$2\cdot 4=8=1\cdot 8=$

защ $3=1.3=3.1$

(=) 2n3 за $n > 2$

$p^A(a, b, c)$

2з3: $3 = 1 \cdot x + 1 = 2^2 \iff$

$x = 2^2 - 1 = 3$

$\varphi_3(x) \leftrightharpoons \exists y \exists z (\varphi_1(y) \,\&\, \varphi_2(z) \,\&\, p(y, x, z)).$

з4з: $\overset{y}{2} \cdot \underbrace{4}_{x} + 1 = \overset{z}{3}^2$

$\varphi_4(x) \leftrightharpoons \exists y \exists z (\varphi_2(y) \,\&\, \varphi_3(z) \,\&\, p(y, x, z)).$

(база): $\varphi_0, \varphi_1, \varphi_2$

(и.х) за вс. $2 < k \le n+1$, то φ_k опр. 2k3

(step): $\varphi_{n+2}(x) \leftrightharpoons \exists y \exists z (\varphi_n(y) \,\&\, \varphi_{n+1}(z) \,\&\, p(y, x, z)).$

$(n+1)^2 = n^2 + 2n + 1 = n \cdot (n+2) + 1$

1. $\varphi_=(x,y) \leftrightharpoons \forall z \forall t\,(\underbrace{p(\overset{1}{z},\overset{1}{t},x)}_{\Lambda \quad 0} \Longleftrightarrow \underbrace{p(z,t,y)}_{\text{и } 1\ 1\ 1}).$

$p^A(a,b,c) \Longleftrightarrow a.b+1=c^2$ и за ве. n лну се опр.

φ

$\varphi_=(x,y) \leftrightharpoons \forall z \forall t\,(p(x,z,t) \Longleftrightarrow p(y,z,t))$

2. Извод: ф-лата си е окей!
Няма контрапример, с който да я счупим!

(зад) Числа на Фибоначи

Редицата $\{F_n\}_{n=0}^{\infty}$ се дефинира така:

$$\begin{cases} F_0 = 0 \\ F_1 = 1 \\ F_{n+2} = F_n + F_{n+1} \quad \text{за } n \geq 0 \end{cases}$$

Разгл. език $\mathcal{L}$ без ф.р. и единст. нелог. символи: $Func_{\mathcal{L}} = \{f\}$, $Pred_{\mathcal{L}} = \{p\}$, $Const_{\mathcal{L}} = \emptyset$, като $\#(f) = 2$, $\#(p) = 1$.

Нека $S = \langle \mathbb{N}, f^S, p^S \rangle$ е стр. за $\mathcal{L}$ като:

$$f^S(x, y) = z \leftrightarrows x + F_{y+1} = z$$

$$p^S(x) \leftrightarrows x \in \{F_n\}_{n=0}^{\infty}$$

Да се док., че в S са определени:

a) $\{>\}$

b) $\{1\}$

c) $Eq = \{\langle a, a\rangle \mid a \in \mathbb{N}\}$ равенство

d) Вярно ли е, че в S е опр. д-во $Prev = \{\langle F_n, F_{n+1}\rangle \mid n \in \mathbb{N}\}$

e) Да се намерят всички автоморф. на S с д-во.

$$f^S(x,y) = z \leftrightharpoons x + F_{y+1} = z$$

$$p^S(x) \leftrightharpoons x \in \{F_n\}_{n=0}$$

$\{0\}, \{1\}, =$ Def. ф.р

$F_0 = 0$

$F_1 = 1$

$$\varphi_=(x,y) \leftrightharpoons \forall z\,(p(f(z,x)) \iff p(f(z,y))).$$

$$\varphi_0(x) \leftrightharpoons \forall y\, p(f(x,y)) \quad // \quad \overset{0}{x} + F_{y+1} \in \{F_n\}_{n=0}^{\infty}$$

$$\varphi_1(x) \leftrightharpoons \exists y\,(\varphi_0(y) \;\&\; \varphi_=(f(y,y),x)).$$

$$\underbrace{0 + F_1}_{1} = x \qquad x = 1$$

$$z + F_{y+1} \in \{F_n\}_{n=0}^{\infty} \iff z + F_{x+1} \in \{F_n\}_{n=0}^{\infty}$$

~~за~~ z: $14 + F_{z+1} \in \{F_n\}_{n=0}^{\infty} \iff 15 + F_{z+1} \in \{F_n\}_{n=0}^{\infty}$

$Prev = \{ \langle F_n, F_{n+1} \rangle \mid n \in \mathbb{N} \}$

$\langle F_0, F_1 \rangle, \langle F_1, F_2 \rangle, \ldots$

$f^S(x, y) = z \Leftrightarrow x + F_{y+1} = z$

$\{0\}, \{1\}, \ldots = P^S$

$P^S(a) \Leftrightarrow \exists z \in \mathbb{N} : a = F_z$

$x + F_1 = y$

$\varphi_{+1}(x, y) \leftrightharpoons \exists z (\varphi_0(z) \& \varphi_=(f(x, z), y)).$

$\varphi_{getIndex}(x, ind) \leftrightharpoons P(x) \& (\exists z (\varphi_0(z) \& \exists t\ \varphi_=(f(z, t), x) \& \varphi_{+1}(t, ind))$

$\vee (\varphi_0(ind) \& \varphi_0(x)).$

$x = F_z$

$\exists t \exists z (\varphi_0(z) \& 0 + F_{t+1} = x \& \varphi_{+1}(t, ind)).$

F_z

$t+1 = z$

$Prev(x, y) \Leftrightarrow \varphi_{getIndex}(\underline{x}, \underline{ind})\ \&$

$\exists z(\varphi_{+1}(ind, \underline{z}) \ \&\ \varphi_{get}(y, z))$

$\underline{ind+1}$ Index