Задача за изпълнимост на м-во от формули

Формулировка:

Дадено: м-во от затворени ф-ли Φ

Търси се: структура, т.е. всички ф-ли в Φ да са верни в нея.

def / Изпълнимост на ф-ла

Нека φ е ф-ла от език $\mathcal{L}$. φ е изпълнима, ако съществува структура $\mathcal{A}$ за $\mathcal{L}$ и оценка на инд. променливи v в $\mathcal{A}$, т.е. $\mathcal{A} \models_v \varphi$

(*) Ако за всяка оценка v в $\mathcal{A}$, то $\mathcal{A} \models_v \varphi$, то пишем $\mathcal{A} \models \varphi$ и казваме, че стр. $\mathcal{A}$ е модел за φ.

(*) Ако φ е затворена, то верността на φ не зависи от оценката т.е. за затв. ф-ли е в сила, че:

За Ч затворена ф-ла

Ч е изпълнима в A $\overset{\text{по дефиниция}}{\Longleftrightarrow}$ A е модел за Ч

Задачата за намиране на модел на м-во от ф-ли е аналогична на задачата да напишем програма отговаряща на дадена формална спецификация

3 зад

$\varphi_1 \leftrightharpoons \exists x \forall y (x \neq y \Rightarrow \exists z \neg p(y, z))$

$\varphi_2 \leftrightharpoons \exists x \forall y \, p(x, y)$

$\varphi_3 \leftrightharpoons \forall x \forall y (p(x, y) \Rightarrow \exists z (p(x, z) \& p(z, y)))$

$\varphi_4 \leftrightharpoons \forall x \exists y \, p(x, y)$

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$\varphi_1 \Leftrightarrow \neg \exists x\, p(x,x)$

$\varphi_2 \Leftrightarrow \forall x \exists y (p(x,y) \,\&\, \neg \exists z (p(x,z) \,\&\, p(z,y)))$

$\varphi_3 \Leftrightarrow \exists x \neg \exists y\, p(y,x)$

$\varphi_4 \Leftrightarrow \forall x (\exists y\, p(y,x) \rightarrow \neg \exists y (p(y,x) \,\&\,$
$\neg \exists z (p(x,z) \,\&\, p(z,y))))$

3a2)

$$\varphi_1 \leftrightharpoons \neg\forall x \forall y \forall z\,(p(x,y) \,\&\, p(y,z) \Rightarrow p(x,z))$$

$$\varphi_2 \leftrightharpoons \forall y \exists x\,(p(x,y) \,\&\, \neg p(y,x) \,\&\, p(x,x))$$

зад

HW

$\varphi_1 \leftrightharpoons \forall x \exists y \exists z\,(y \not\doteq z \;\&\; q(x,z) \;\&\; q(x,y))$

$\varphi_2 \leftrightharpoons \forall x \forall z\,((q(x,z) \Rightarrow p(x,z)) \;\&\; (p(x,z) \Rightarrow \neg p(z,x)))$

$\varphi_3 \leftrightharpoons \forall x \forall y \forall z\,(p(x,y) \;\&\; p(y,z) \Rightarrow p(x,z))$

$\varphi_4 \leftrightharpoons \forall x \forall y\,(h(x) \doteq h(y) \Rightarrow \neg q(x,y))$

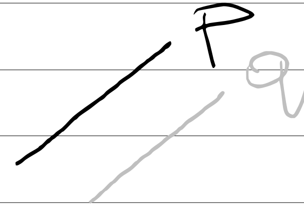
$\varphi_5 \leftrightharpoons \forall x \exists y\,(h(x) \doteq h(y) \;\&\; p(x,y))$

Зад. 6 $\varphi_1 \leftrightharpoons \forall x \forall y (p(x,y) \Rightarrow \exists z (q(y,z) \,\&\, q(z,x)))$

$p^{-1} \longrightarrow p(z,y) \longrightarrow p(x,z)$

$\varphi_2 \leftrightharpoons \exists x \exists y (\neg q(x,y) \,\&\, \neg q(y,x))$

$\varphi_3 \leftrightharpoons \forall z \exists x (q(x,z) \vee p(z,x))$



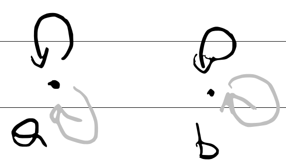
$\mathcal{A} = \langle A;\ p^{\mathcal{A}}, q^{\mathcal{A}} \rangle$

$A = ?,\ p^{\mathcal{A}} = ?,\ q^{\mathcal{A}} = ?$

φ_2 – има два върха несравними по $q^{\mathcal{A}}$ $\to$ $\overline{\overline{A}} \geq 2$

φ_3 – за всяка точка: има сравн. наляво по $q^{\mathcal{A}}$ или съседни надясно по $p^{\mathcal{A}}$

φ_1 – т.е. с $q^{\mathcal{A}} = (p^{-1})^{\mathcal{A}}$, то получаваме гъстота



$\mathcal{A} = \langle \{a, b\};\ p^{\mathcal{A}}, q^{\mathcal{A}} \rangle$

$p^{\mathcal{A}} = q^{\mathcal{A}} = \{\langle a,a \rangle, \langle b,b \rangle\}$

$$\mathfrak{B} = \langle \mathbb{Q}, p^{\mathfrak{B}}, q^{\mathfrak{B}} \rangle$$

$$p^{\mathfrak{B}} \subseteq \leq_{\mathbb{Q}}, \quad q^{\mathfrak{B}} \subseteq \geq_{\mathbb{Q}} \setminus \underline{\{\langle 1, 0 \rangle\}}$$

(зад) $\varphi_1 \leftrightharpoons \forall x\, (x \neq f(x) \,\&\, x \neq g(x))$.

$\varphi_2 \leftrightharpoons \exists x\, (x \doteq f(g(x)) \,\&\, x \doteq g(f(x)))$.

$\varphi_3 \leftrightharpoons \exists x\, (f(x) \doteq g(x))$.

$\varphi_4 \leftrightharpoons \exists x\, (g(f(x)) \neq f(g(x)))$

$\varphi_5 \leftrightharpoons \forall y \exists x\, (y \doteq g(x))$

$\mathcal{A} = \langle \underline{A}, \underline{f}^{\mathcal{A}}, \underline{g}^{\mathcal{A}}, \ldots \rangle$

$Dom(f^{\mathcal{A}}) = Dom(g^{\mathcal{A}}) = \underline{A}$

φ_1 – $f^{\mathcal{A}}, g^{\mathcal{A}}$ не са id_A

φ_5 – сюрективност на $g^{\mathcal{A}}$

φ_2 – същ. x, за което $(f \circ g)(x) = (g \circ f)(x) = id_A(x)$

обратни за $\underline{f}$ такова x

φ_3 – същ. x, за което функц. ст-ти на f^A и g^A съвпадат

φ_4 – същ. x, за което $(f \circ g)(x) \neq (g \circ f)(x)$

$\mathcal{A} = \langle \mathbb{N}; f^{\mathcal{A}}, g^{\mathcal{A}} \rangle$

$f^{\mathcal{A}} : \mathbb{N} \to \mathbb{N}$ $\quad Dom(f^{\mathcal{A}}) = \mathbb{N}$

$g^{\mathcal{A}} : \mathbb{N} \to \mathbb{N}$ $\quad Dom(g^{\mathcal{A}}) = \mathbb{N}$

$$g^{\mathcal{A}}(x) \leftrightharpoons \begin{cases} x-1, & x>0 \\ 1, & x=0, \end{cases} \qquad f^{\mathcal{A}}(x) \leftrightharpoons x+1$$

0 ← Никог φ_3: $f^{\mathcal{A}}(0) = 1 = g^{\mathcal{A}}(0)$

← φ_2: $f^{\mathcal{A}}(g^{\mathcal{A}}(1)) = 1 = g^{\mathcal{A}}(f^{\mathcal{A}}(1))$

0 ← φ_4: $f^{\mathcal{A}}(\underbrace{g^{\mathcal{A}}(0)}_{1}) = 2 \neq 0 = \underbrace{g^{\mathcal{A}}(\underbrace{f^{\mathcal{A}}(0)}_{1})}_{0}$

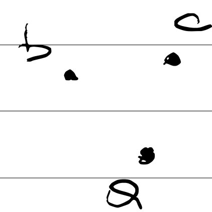
$\varphi_1 \leftrightharpoons \forall x\, (x \neq f(x) \,\&\, x \neq g(x))$.

$\varphi_2 \leftrightharpoons \exists x\, (x \doteq f(g(x)) \,\&\, x \doteq g(f(x)))$.

$\varphi_3 \leftrightharpoons \exists x\, (f(x) \doteq g(x))$.

$\varphi_4 \leftrightharpoons \exists x\, (g(f(x)) \neq f(g(x)))$

$\varphi_5 \leftrightharpoons \forall y \exists x\, (y \doteq g(x))$



Зад. 1

$\varphi_1 \leftrightharpoons \exists x \exists y (g(x) \doteq y \ \& \ f(x) \doteq y)$

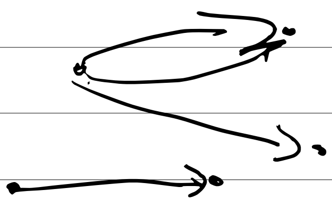
$\varphi_2 \leftrightharpoons \forall x \forall y \forall z (f(x) \doteq y \ \& \ f(y) \doteq z \Rightarrow g(z) \doteq x)$

$\varphi_3 \leftrightharpoons \exists x \exists y \exists z (\neg(x \doteq y) \ \& \ \neg(y \doteq z) \ \& \ \neg(z \doteq x))$

$\varphi_4 \leftrightharpoons \forall x \neg(f(x) \doteq x)$

Док., че $\overbrace{\{\varphi_1, \varphi_2, \varphi_3\}}^{T_1}$ и $\overbrace{\{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}}^{T_2}$ са изпълними м-ва от ф-ли.

f^A е частен случай на релация



от тоталност: от всеки връх излиза поне едно ребро

от функц.: от всеки връх излиза най-много едно ребро

→ точно едно

$\varphi_1 \leftrightharpoons \exists x \exists y\,(g(x) \doteq y \;\&\; f(x) \doteq y)$

$\varphi_2 \leftrightharpoons \forall x \forall y \forall z\,(f(x) \doteq y \;\&\; f(y) \doteq z \Rightarrow \overset{f}{g}(z) \doteq x)$

$\varphi_3 \leftrightharpoons \exists x \exists y \exists z\,(\neg(x \doteq y) \;\&\; \neg(y \doteq z) \;\&\; \neg(z \doteq x))$

$\varphi_4 \leftrightharpoons \forall x \neg(f(x) \doteq x)$ → няма рефл. точки по f^A

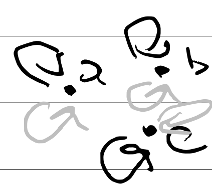
φ_1 – има два върха т.е. $g^A(x) = y = f^A(x)$

$$x_0 \xrightarrow{f} y_0 \qquad x_0 \xrightarrow{g} y_0$$

φ_3 – $\overline{\overline{A}} \geq 3$

φ_2 – подобие на транзитивност (при $g^A = f^A$)

$\Pi_1 : A \leftrightharpoons \langle \{a, b, c\}, f^A, g^A \rangle$, $g^A \leftrightharpoons f^A$



$f^A(x) = x$

$\varphi_1 \leftrightharpoons \exists x \exists y (g(x) \doteq y \& f(x) \doteq y)$

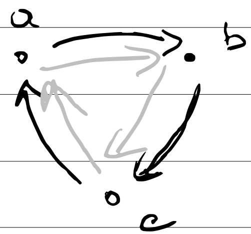
$\varphi_2 \leftrightharpoons \forall x \forall y \forall z (f(x) \doteq y \& f(y) \doteq z \Rightarrow g(z) \doteq x)$

$\varphi_3 \leftrightharpoons \exists x \exists y \exists z (\neg(x \doteq y) \& \neg(y \doteq z) \& \neg(z \doteq x))$

$\varphi_4 \leftrightharpoons \forall x \neg (f(x) \doteq x)$

B_2

$\mathcal{B} = \langle \{a, b, c\}, f^{\mathcal{A}}, g^{\mathcal{A}} \rangle$



$g^{\mathcal{A}} \leftrightharpoons f^{\mathcal{A}}$

$f^{\mathcal{A}}(a) = b$

$f^{\mathcal{A}}(b) = c$

$f^{\mathcal{A}}(c) = a$

Зад 8 $\varphi_1 \leftrightharpoons \exists x (p(x,x) \,\&\, q(x,x))$

$\varphi_2 \leftrightharpoons \forall x (p(x,x) \Rightarrow p(y,x))$

$\varphi_3 \leftrightharpoons \forall x \exists y (q(x,y) \,\&\, q(y,b))$.

$\varphi_4 \leftrightharpoons \exists x (p(b,x) \,\&\, q(c,x))$.

$\varphi_5 \leftrightharpoons q(b,b) \,\&\, \neg p(c,c) \,\&\, \neg q(c,c)$

(!) a, b и c са индивидни константи.

P корени

Зад. 8

$\varphi_1 \leftrightharpoons f(f(x,y,a),z,a) \doteq f(x,f(y,z,a),a)$

$\varphi_2 \leftrightharpoons f(f(x,y,b),z,b) \doteq f(x,f(y,z,b),b)$

$\varphi_3 \leftrightharpoons f(f(x,y,a),z,b) \doteq f(f(x,z,b),f(y,z,b),a)$

$\varphi_4 \leftrightharpoons a = b$

$\varphi_5 \leftrightharpoons \neg(a \doteq b)$ → ако поне два обекта в света

$\mathcal{B} \leftrightharpoons \langle \{0,1\}, f^{\mathcal{B}}, a^{\mathcal{B}}, b^{\mathcal{B}} \rangle$

$a^{\mathcal{B}} \leftrightharpoons 0 \quad b^{\mathcal{B}} \leftrightharpoons 1,$

$c^{\mathcal{B}} \leftrightharpoons 1$

$f^{\mathcal{B}}(x,y,z) \leftrightharpoons y$

$\varphi_6 \leftrightharpoons \forall x \forall y \exists z \neg(f(x,y,z) \doteq y)$

Да се докаже кои от м-вата

a) $\{\varphi_1, \varphi_2, \varphi_3\}$

b) $\{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$

c) $\{\varphi_1, \varphi_2, \varphi_3, \varphi_5\}$

d) $\{\varphi_1, \varphi_2, \varphi_3, \varphi_5, \varphi_6\}$

са изпълними

→ $\mathcal{A} = \langle \{0\}, f^{\mathcal{A}}, a^{\mathcal{A}}, b^{\mathcal{A}} \rangle$

$f^{\mathcal{A}}(x,y,z) \leftrightharpoons 0$

$a^{\mathcal{A}} \leftrightharpoons 0; \; b^{\mathcal{A}} \leftrightharpoons 0$

$\mathcal{C} = \langle \mathbb{N}; f^{\mathcal{C}}; a^{\mathcal{C}}; b^{\mathcal{C}} \rangle$

$f^{\mathcal{C}}(x, y, z) = \max\{x, y, z\}$

$a^{\mathcal{C}} = 0$

$b^{\mathcal{C}} = 1$

Нека $x, y \in \mathbb{N}$. Нека $z = \max\{x, y\} + 1$

Тогава $\max\{x, y, z\} =$

$\max\{x, y, \max\{x, y\} + 1\} =$

$= \max\{x, y\} + 1 \geq y$

$\neq y$