Задача за изпълнимост на м-во от формули

Формулировка:

Дадено: м-во от затворени ф-ли Φ

Търси се: структура, т.е. всички ф-ли в Φ да са верни в нея.

def/ Изпълнимост на ф-ла

Нека φ е ф-ла от език $\mathcal{L}$. φ е изпълнима, ако съществува структура $\mathcal{A}$ за $\mathcal{L}$ и оценка на инд. променливи v в $\mathcal{A}$, т.е. $\mathcal{A} \models_v \varphi$

(*) Ако за всяка оценка v в $\mathcal{A}$, то $\mathcal{A} \models_v \varphi$, то пишем $\mathcal{A} \models \varphi$ и казваме, че стр. $\mathcal{A}$ е модел за φ.

(*) Ако φ е затворена, то верността на φ не зависи от оценката т.е. за затв. ф-ли е в сила, че:

За φ затворена ф-ла

φ е изпълнима в $A \Leftrightarrow A$ е модел за φ

по дефиниция

Задачата за намиране на модел на м-во от ф-ли е аналогична на задачата да напишем програма отговаряща на дадена формална спецификация

Зад. 7

$\varphi_1 \leftrightharpoons \exists x \forall y (x \neq y \Rightarrow \exists z \neg p(y,z))$ за φ_1 свидетел 0.

$\varphi_2 \leftrightharpoons \exists x \forall y\, p(x,y)$ за φ_2 свидетел 0

$\varphi_3 \leftrightharpoons \forall x \forall y (p(x,y) \Rightarrow \exists z (p(x,z) \& p(z,y)))$

$\varphi_4 \leftrightharpoons \forall x \exists y\, p(x,y)$

$\mathcal{A} = \langle A ; p^{\mathcal{A}} \rangle$, $A = ?$, $p^{\mathcal{A}} = ?$

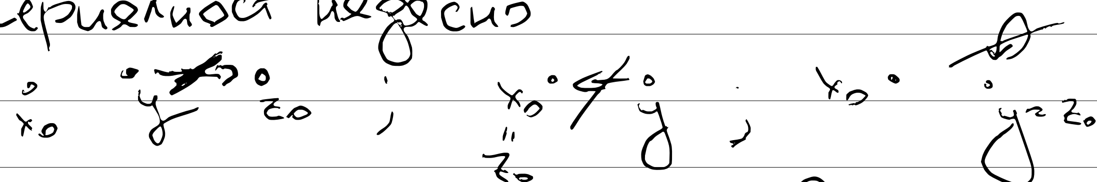
Анализ:

φ_2 - същ. най-малкия елемент

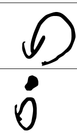
φ_3 - гъстота

φ_4 - сериалност надясно

φ_1 -



$\mathcal{A}_1 = \langle \mathbb{R}^+ \cup \{0\} ; p^{\mathcal{A}_1} \rangle$, $p^{\mathcal{A}_1} \leftrightharpoons \leq_{\mathbb{R}}$



$\mathcal{A}_2 = \langle \{0\}, p^{\mathcal{A}_2} \rangle$,

$p^{\mathcal{A}_2} = \{\langle 0, 0 \rangle\}$

3.2.2

$\varphi_1 \Leftrightarrow \neg \exists x\, p(x,x)$

$\varphi_2 \Leftrightarrow \forall x \exists y (p(x,y) \& \neg \exists z (p(x,z) \& p(z,y)))$

$\varphi_3 \Leftrightarrow \exists x \neg \exists y\, p(y,x)$

$\varphi_4 \Leftrightarrow \forall x (\exists y\, p(y,x) \Rightarrow \neg \exists y (p(y,x) \&$
$\neg \exists z (p(x,z) \& p(z,y))))$

3a2

$\varphi_1 \leftrightharpoons \neg \forall x \forall y \forall z (p(x,y) \& p(y,z) \Rightarrow p(x,z))$

$\varphi_2 \leftrightharpoons \forall y \exists x (p(x,y) \& \neg p(y,x) \& p(x,x))$

Зад

$$\varphi_1 \leftrightharpoons \forall x \exists y \exists z (y \not\doteq z \;\&\; q(x,z) \;\&\; q(x,y))$$

$$\varphi_2 \leftrightharpoons \forall x \forall z ((q(x,z) \Rightarrow p(x,z)) \;\&\; (p(x,z) \Rightarrow \neg p(z,x)))$$

$$\varphi_3 \leftrightharpoons \forall x \forall y \forall z (p(x,y) \;\&\; p(y,z) \Rightarrow p(x,z))$$

$$\varphi_4 \leftrightharpoons \forall x \forall y (h(x) \doteq h(y) \Rightarrow \neg q(x,y))$$

$$\varphi_5 \leftrightharpoons \forall x \exists y (h(x) \doteq h(y) \;\&\; p(x,y))$$

Зад

$\varphi_1 \leftrightharpoons \forall x \forall y (p(x,y) \Rightarrow \neg \exists z (q(y,z) \,\&\, q(z,x)))$

$\varphi_2 \leftrightharpoons \exists x \exists y (\neg q(x,y) \,\&\, \neg q(y,x))$

$\varphi_3 \leftrightharpoons \forall z \exists x (q(x,z) \vee p(z,x))$

Зад. $\varphi_1 \leftrightharpoons \forall x\, (x \neq f(x) \,\&\, x \neq g(x))$.

$\varphi_2 \leftrightharpoons \exists x\, (x \doteq f(g(x)) \,\&\, x \doteq g(f(x)))$.

$\varphi_3 \leftrightharpoons \exists x\, (f(x) \doteq g(x))$.

$\varphi_4 \leftrightharpoons \exists x\, (g(f(x)) \neq f(g(x)))$.

$\varphi_5 \leftrightharpoons \forall y \exists x\, (y \doteq g(x))$.

f, g – нелоги.

φ_1 – $f^A \neq id_A$ и $g^A \neq id_A$

φ_2 – има точка, т.е. f^A и g^A са обратни една на др.

φ_3 – има точка, т.ч. f^A съвп. с g^A

φ_4 – има точка т.ч. $g \circ f \neq f \circ g$ $\to$ $f \neq g$

φ_5 – g^A е сюрекция

$\cdot$ 0 $\quad \cdot$ 1 $\quad \cdot$ 2	$f^A(0) = 1$ φ_3	$g^A(0) = 1$ φ_3
	$f^A(1) = 2$	$g^A(1) = 2$
	$f^A(2) = 1$	$g^A(2) = 0$

φ_4 φ_5 φ_2: $f(g(1)) = \; = g(f(2))$

φ_3:

$$\mathcal{A} = \langle \mathbb{N};\ f^{\mathcal{A}},\ g^{\mathcal{A}} \rangle$$

$$f^{\mathcal{A}}(x) \leftrightharpoons x+1$$

$$g^{\mathcal{A}}(x) \leftrightharpoons \begin{cases} x-1, & x>0 \\ 1, & x=0 \end{cases}$$

e_1: V

e_3: $f^{\mathcal{A}}(0) = 1 = 1 = g^{\mathcal{A}}(0)$

e_2: $\underbrace{g^{\mathcal{A}}(\underbrace{f^{\mathcal{A}}(1)}_{2})} = 1 = f^{\mathcal{A}}(\underbrace{g^{\mathcal{A}}(1)}) $ with 1 below: $\underbrace{f^{\mathcal{A}}(g^{\mathcal{A}}(1))}_{1}$

$\underline{e_4}$: $\underbrace{g^{\mathcal{A}}(\overbrace{\underbrace{f^{\mathcal{A}}(0)}}^{1})}_{\underline{0}} = 0 \neq 2 = \underbrace{f^{\mathcal{A}}(\underbrace{g^{\mathcal{A}}(0)}_{1})}_{2}$

$\underline{e_5}$: $g^{\mathcal{A}}$ е сюрекция? Нека $\underline{y \in \mathbb{N}}$ произв.

Ако $y=1 \to \underline{x=0}$

Ако $y \neq 1 \to x=y+1$

Зад. $\varphi_1 \leftrightharpoons \exists x \exists y (g(x) \doteq y \,\&\, f(x) \doteq y)$

$\varphi_2 \leftrightharpoons \forall x \forall y \forall z (f(x) \doteq y \,\&\, f(y) \doteq z \Rightarrow g(z) \doteq x)$

$\varphi_3 \leftrightharpoons \exists x \exists y \exists z (\neg(x \doteq y) \,\&\, \neg(y \doteq z) \,\&\, \neg(z \doteq x))$

$\varphi_4 \leftrightharpoons \forall x \neg(f(x) \doteq x)$

Док., че $\{\varphi_1, \varphi_2, \varphi_3\}$ и $\{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$ са изпълними м-ва от ф-ли.

Зад. 8

$\varphi_1 \leftrightharpoons \exists x (p(x,x) \,\&\, q(x,x))$

$\varphi_2 \leftrightharpoons \forall x (p(x,x) \Rightarrow p(a,x))$

$\varphi_3 \leftrightharpoons \forall x \exists y (q(x,y) \,\&\, q(y,b))$.

$\varphi_4 \leftrightharpoons \exists x (p(b,x) \,\&\, q(c,x))$.

$\varphi_5 \leftrightharpoons q(b,b) \,\&\, \neg p(c,c) \,\&\, \neg q(c,c)$.

1) a, b и c са индивидни константи.

$\mathcal{A} = \langle A; p^{\mathcal{A}}, q^{\mathcal{A}}; a^{\mathcal{A}}, b^{\mathcal{A}}, c^{\mathcal{A}} \rangle$

φ_1 - $p^{\mathcal{A}}$, $q^{\mathcal{A}}$ не са антирефлексивни

φ_2 - всички рефл. по $p^{\mathcal{A}}$ трябва да бъде сочен от $a^{\mathcal{A}}$

φ_3 - сериалност надясно по $q^{\mathcal{A}}$ като поне един от сочените върхове трябва да сочи $b^{\mathcal{A}}$ по $q^{\mathcal{A}}$

φ_4 - има точка сочена от $b^{\mathcal{A}}$ по $p^{\mathcal{A}}$ и от $c^{\mathcal{A}}$ от $q^{\mathcal{A}}$

φ_5 - $b^{\mathcal{A}}$ е рефл. по $q^{\mathcal{A}}$; $c^{\mathcal{A}}$ не е рефл. нито по $p^{\mathcal{A}}$, нито по $q^{\mathcal{A}}$

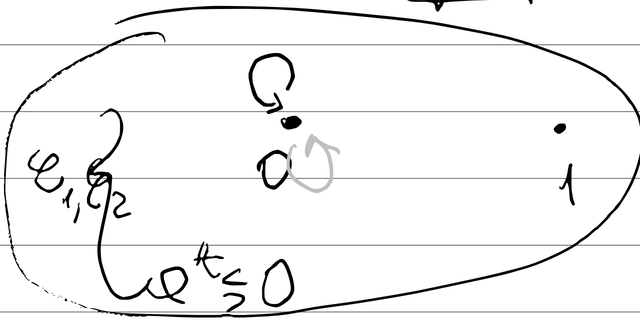
$\varphi_1 \leftrightharpoons \exists x(p(x,x) \,\&\, q(x,x))$ / свид. 0

$\varphi_2 \leftrightharpoons \forall x(p(x,x) \Rightarrow p(a,x))$

$\varphi_3 \leftrightharpoons \forall x \exists y(q(x,y) \,\&\, q(y,b))$.

$\varphi_4 \leftrightharpoons \exists x(p(b,x) \,\&\, q(c,x))$ свид. 0

$\varphi_5 \leftrightharpoons q(b,b) \,\&\, \neg p(c,c) \,\&\, \neg q(c,c)$



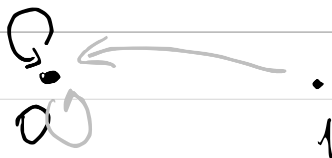
$\mathcal{A} = \langle \{0,1\}; p^{\mathcal{A}}, q^{\mathcal{A}}; a^{\mathcal{A}}, b^{\mathcal{A}}, c^{\mathcal{A}} \rangle$

$p^{\mathcal{A}} \leftrightharpoons \{\langle 0,0\rangle\}$

$q^{\mathcal{A}} \leftrightharpoons \{\langle 0,0\rangle, \langle 1,0\rangle\}$

$a^{\mathcal{A}} \leftrightharpoons 0, \quad c^{\mathcal{A}} \leftrightharpoons 1$

$b^{\mathcal{A}} \leftrightharpoons 0$



$$\underbrace{a^{\mathcal{A}} \leftrightharpoons 0 \quad b^{\mathcal{A}} \leftrightharpoons 0 \quad c^{\mathcal{A}} \leftrightharpoons 1}_{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5}$$

$\forall x \forall y \forall z$

Зад. 8

$\varphi_1 \leftrightharpoons f(f(x,y,a),z,a) \doteq f(x,f(y,z,a),a)$

$\varphi_2 \leftrightharpoons f(f(x,y,b),z,b) \doteq f(a,f(y,z,b),b)$

$\varphi_3 \leftrightharpoons f(f(x,y,a),z,b) \doteq f(f(x,z,b),f(y,z,b),a)$

$\varphi_4 \leftrightharpoons a \doteq b$

$\varphi_5 \leftrightharpoons \neg(a \doteq b)$

$\varphi_6 \leftrightharpoons \forall x \forall y \exists z \neg(f(x,y,z) \doteq y)$.

f, a, b

Да се докаже кои от м-вата:

a) $\{\varphi_1, \varphi_2, \varphi_3\}$

b) $\{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$

c) $\{\varphi_1, \varphi_2, \varphi_3, \varphi_5\}$

d) $\{\varphi_1, \varphi_2, \varphi_3, \varphi_5, \varphi_6\}$

са изпълними!

a) $a_0 \in A_2$

Нека $A_2 \neq \emptyset$

$\mathcal{A}_2 = \langle A_2; f^{\mathcal{A}_2}, a^{\mathcal{A}_2}, b^{\mathcal{A}_2} \rangle$

$f^{\mathcal{A}}(x,y,z) \leftrightharpoons y$

$a^{\mathcal{A}_2} \leftrightharpoons a_0$, $b^{\mathcal{A}_2} \leftrightharpoons a_0$

a) $\mathcal{A}_1 = \langle \{0\}; f^{\mathcal{A}}, a^{\mathcal{A}}, b^{\mathcal{A}} \rangle$

$Dom(f^{\mathcal{A}}) = \{\langle 0,0,0 \rangle\}$

$f^{\mathcal{A}}(0,0,0) \leftrightharpoons 0$

$a^{\mathcal{A}} \leftrightharpoons 0$

$b^{\mathcal{A}} \leftrightharpoons 0$

b) : t_1, t_2 (V)

c) $t_2' = \langle A; f^t; a^t, b^t \rangle$

$A \neq \emptyset, \ |A| > 1$ $\qquad a^t \leq 0$

$0, 1 \in A$ $\qquad b^t \leq 1$

$0 \neq 1$ $\qquad f^A(x, y, z) \leq y$

$t_1' = \langle \{0, 1\}; f^t, a^t, b^t \rangle$

$a^t \leq 0, \ b^t \leq 1$

$f^A(x, y, z) \leq y$

d) $\varphi_6 \leq \forall x \forall y \exists z \neg (f(x, y, z) \doteq y)$.

$f(x, y, z) \neq y$

$t_3 = \langle \mathbb{N}; f^t; a^t, b^t \rangle$

$f^t(x, y, z) \leq \max\{x, y, z\}$

$a^t \leq \cancel{0} / n$ $\wedge \ \cancel{\#} \to \varphi_5$

$b^t \leq \cancel{1} / m$

Нека $x, y \in \mathbb{N}$. Нека $z \leftrightharpoons \max\{x, y\} + 1$.
Тогава $\max\{x, y, z\} = f^A(x, y, z) = z \neq y$

$A_4 \leftrightharpoons \langle \mathbb{Z}; f^A; a^A; b^A \rangle$

$f^A(x, y, z) \leftrightharpoons \min\{x, y, z\}$

$a^A \leftrightharpoons 10$ и $\forall \to e_5$

$b^A \leftrightharpoons 5$ и