3зад. Нека $\mathcal{L} = \langle p \rangle$ FOL, $p \in Pred$, $\#p = 3$.

Нека $\mathcal{A} = \langle \mathcal{P}(\mathbb{N}), p^{\mathcal{A}} \rangle$ е структура за $\mathcal{L}$, като:

- $\mathcal{P}(\mathbb{N})$ - м-вото от всички подмн-ва на $\mathbb{N}$ т.е. $\mathcal{P}(\mathbb{N}) = \{B \mid B \subseteq \mathbb{N}\}$
- $\langle \tilde{a}, \tilde{b}, \tilde{c} \rangle \in p^{\mathcal{A}} \Longleftrightarrow \underbrace{a \cup b} = c$

$a, b, c \in \mathcal{P}(\mathbb{N})$

Да се докаже, че:

a) $\{\emptyset\}$ е определимо

b) $\{\mathbb{N}\}$ е определимо

c) $\underset{\sim}{\subseteq} \leftrightharpoons \{\langle x, y \rangle \mid x, y \in \mathcal{P}(\mathbb{N}) \,\&\, x \subseteq y\}$ е определимо

d) $\cap \leftrightharpoons \{\langle x, y, z\rangle \mid x, y, z \in \mathcal{P}(\mathbb{N})\ \&\ x \cap y = z\}$

е определимо

e) $^{-} \leftrightharpoons \{\langle x, y\rangle \mid x, y \in \mathcal{P}(\mathbb{N})\ \&\ y = \mathbb{N} \setminus x\}$

т.е. y е <u>допълнението на x с точност до $\mathbb{N}$</u>

е определимо.

f) Докажете, че за <u>$a \in \mathcal{P}(\mathbb{N}) \setminus \{\emptyset, \mathbb{N}\}$</u>, то $\{a\}$ е <u>неопределимо</u>.

a) $\{\emptyset\}$ е определимо

b) $\{\mathbb{N}\}$ е определимо

e) $\subseteq \leftrightharpoons \{\langle x, y\rangle \mid x, y \in \mathcal{P}(\mathbb{N}) \,\&\, x \subseteq y\}$
е определимо

$\langle a, b, c\rangle \in p^{t} \iff a \cup b = c$

$\varphi_\emptyset(x) \leftrightharpoons \forall y\, p(x, y, y)$.

за оценка v в $\mathcal{A}$, то:

$\mathcal{A} \models_v \varphi_\emptyset(x) \iff v(x) = \emptyset$

$(\Leftarrow)$ Нека $v(x) = \emptyset$

Тогава $\underbrace{\mathcal{A} \models_v \forall y\, p(x, y, y)}$ $\qquad v_a^y(z) = \begin{cases} a & z = y \\ v(z), & z \neq y \end{cases}$

$\underline{\text{за вс. } a \in \mathcal{P}(\mathbb{N})}: \underbrace{\underset{\emptyset}{v(x)} \cup v_a^y(y) = v_a^y(y)}$

$\underline{\text{за вс. } a \in \mathcal{P}(\mathbb{N}) : \emptyset \cup \underset{a}{v_a^y(y)} = v_a^y(y) = a}$

за вс. $a \in \mathcal{P}(\mathbb{N})$: $\emptyset \cup a = a$ $\quad$ V

(→) Нека $v(x) \neq \emptyset$.

• $\varphi \to \psi$

• $\neg\psi \to \neg\varphi$

$$\|\forall y\, p(x,y,y)\|^{t}[v] = ?$$

Нека $v^{y}_{\emptyset}$ вземем.

Тогава $\underbrace{\|p(x,y,y)\|^{t}[v^{y}_{\emptyset}] = \mathbb{T}}$

$$\langle \underset{\neq \emptyset}{v(x)}, \underset{= \emptyset}{v^{y}_{\emptyset}(y)}, \underset{= \emptyset}{v^{y}_{\emptyset}(y)} \rangle \in p^{t}$$

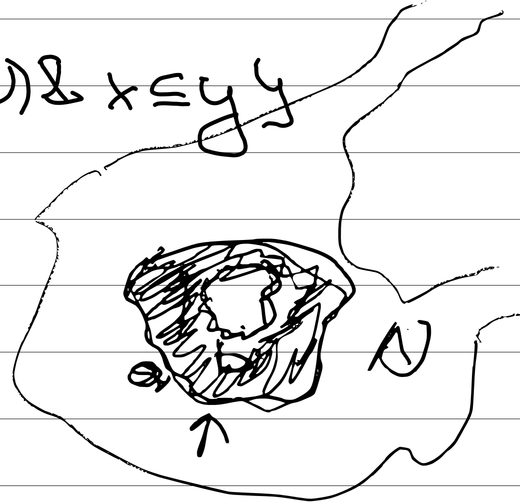
$$\underset{\neq \emptyset}{v(x)} \cup \emptyset = \emptyset \quad ↯$$

Значи $\|\varphi_{\emptyset}(x)\|^{t}[v] = \mathbb{F}$

b) $\exists N y$ е определимо

c) $\subseteq \rightleftharpoons \{\langle x,y\rangle \mid x,y \in \mathcal{P}(N) \ \& \ x \subseteq y\}$
е определимо

$\varphi_N(x) \rightleftharpoons \forall y\, \varphi_{\subseteq}(y,x).$
$\varphi_N(x) \rightleftharpoons \forall y\, p(x,y,x)$



$\varphi_{\subseteq}(x,y) \rightleftharpoons \exists z\, p(x,z,y)$

$\varphi_{\subseteq}(x,y) \rightleftharpoons p(x,y,y)$ // $A \cup B = B \Leftrightarrow A \subseteq B$

$\models_v \varphi_N(x) \Longleftrightarrow v(x) = N$

$(\rightarrow)$ Нека $\models_v \varphi_N(x)$ т.е. $\|\varphi_N(x)\|^{A}[v] = T$

т.е. за вс. $Q \in \mathcal{P}(N)$: $v(x) \cup v^{y}_{Q}(y) = v(x)$

Нека $Q = N$, тогава $v(x) \cup N = v(x)$

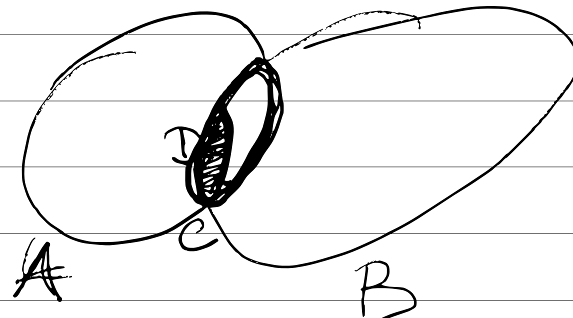
$N \subseteq v(x)$, от ус. $v(x) \subseteq N$ $\rightarrow$ $v(x) = N$

d) $\cap \leq \{\langle x, y, z\rangle \mid x, y, z \in \mathcal{P}(\mathbb{N})$ &
$x \cap y = z\}$

е определимо

e) $^{-} \leq \{\langle x, y\rangle \mid x, y \in \mathcal{P}(\mathbb{N})$ &
$y = \mathbb{N} \setminus x\}$

т.е. y е допълнението на x с точност до $\mathbb{N}$
е определимо.



$C \subseteq A$ и $C \subseteq B$ и
зоще
дее
$D \subseteq A$ и $D \subseteq B$, то $D \subseteq C$

$\varphi_\cap(x, y, z) \leq \varphi_\subseteq(z, x)$ & $\varphi_\subseteq(z, y)$ &
$\forall t(\varphi_\subseteq(t, x)$ & $\varphi_\subseteq(t, y) \Rightarrow \varphi_\subseteq(t, z))$

e) $\overline{} \leftrightharpoons \{\langle x, y\rangle \mid x, y \in \mathcal{P}(\mathbb{N})\ \&$
$y = \mathbb{N} \setminus x\}$

т.е. y е допълнението на x с точност до $\mathbb{N}$
е определимо.

$$x \overset{\cap}{\cup} \overline{x}$$

$$\varphi_c(x, y) \leftrightharpoons \exists z \exists t(\varphi_\emptyset(z)\ \&\ \varphi_{\mathbb{N}}(t)\ \&$$
$$P(x, y, t)\ \&\ \varphi_\cap(x, y, z)).$$

f) Докажете, че за $a \in \mathcal{P}(\mathbb{N}) \setminus \{\emptyset, \mathbb{N}\}$, то $\{a\}$ е неопределимо.

Нека $a \subseteq \mathbb{N}$, т.е. $a \neq \emptyset$ и $a \neq \mathbb{N}$.

има $n \in \mathbb{N}$, т.е. $n \in a$ има $m \in \mathbb{N}$, т.е. $m \notin a$

$h_0 : \mathbb{N} \to \mathbb{N}$

$$h_0(x) = \begin{cases} n, & x = m \\ m, & x = n \\ x, & \text{else} \end{cases}$$

$h_0 = h_0^{-1}$

биекция

$h : \mathcal{P}(\mathbb{N}) \to \mathcal{P}(\mathbb{N})$

$$h(X) = \{h_0(y) \mid y \in X\}, \quad X \subseteq \mathbb{N}.$$

$$h(h(X)) = \{h_0(y) \mid y \in h(X)\} =$$
$$= \{h_0(y) \mid y \in \{h_0(z) \mid z \in X\}\} =$$
$$= \{h_0(h_0(z)) \mid z \in X\} = \{z \mid z \in X\} = X.$$

$\langle a, b, c \rangle \in p^{\mathcal{A}} \iff \langle h(a), h(b), h(c) \rangle \in p^{\mathcal{A}}$

- $n, m \in a,\ n, m \in b$ $\quad$ $n \notin a,\ m \notin a,\ n \notin b,\ m \notin b$)

$$\emptyset \subsetneq \underbrace{\{2, 5, 6, 8\}}_{A} \subsetneq \mathbb{N}$$

$5 \in$ $\qquad$ $0 \notin$

$$h_{0,A}(x) = \begin{cases} 0, & x = 5 \\ 5, & x = 0 \\ x, & \text{else} \end{cases}$$

$h(X) = \{h_0(y) \mid y \in X\}$

$h(\{2, 5, 6, 8\}) = \{0, 2, 6, 8\} \notin \{\{2, 5, 6, 8\}\}$

т.е. $\{\{2, 5, 6, 8\}\}$ не е определим.

Зад | $\mathcal{L} = \langle p \rangle$, $\#p = 2$

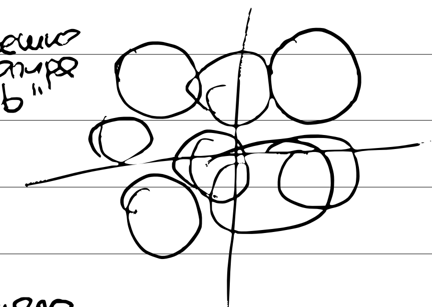
$\mathcal{A} = \langle$ кръгове с ненулев радиус в $\mathbb{R}^2$, $p^{\mathcal{A}} \rangle$, където

a) $p^{\mathcal{A}}(a, b) \iff$ a b "a вътрешно се допира до b"

b) $p^{\mathcal{A}}(a, b) \iff$ a b

c) $p^{\mathcal{A}}(a, b) \iff$ a b

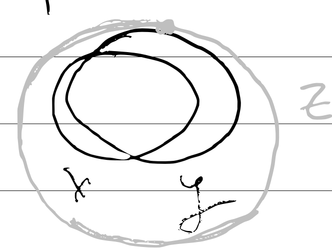
d) $p^{\mathcal{A}}(a, b) \iff$ a b "a е изцяло вътре без допир в b"

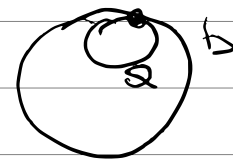


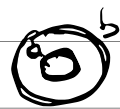
Опр. a,b ; a b и останалите.

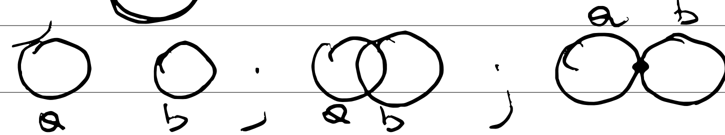
= ; partial overlap) PO

$\varphi_= (x, y) \leftrightharpoons \forall z (p(x, z) \iff p(y, z))$

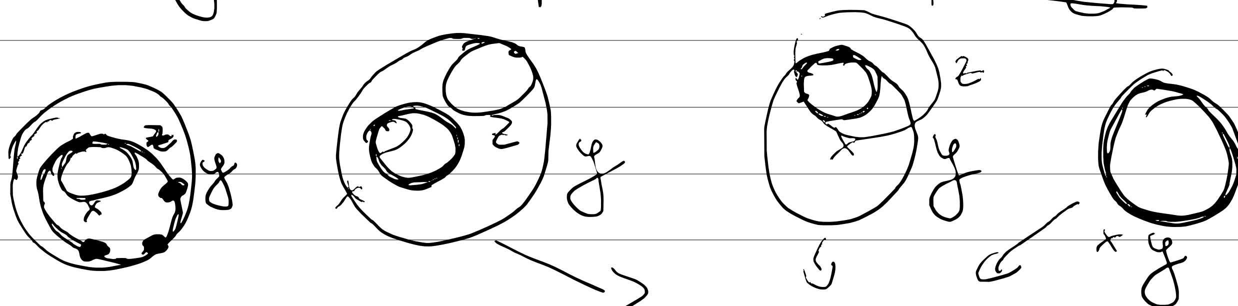


$p^{A}(a,b) \leftrightarrow$ 

Done: = 

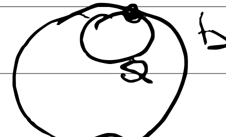
TODO: 

$\cancel{\mathcal{C}_{\subseteq}(x,y) \lesssim \forall z(p(z,x) \Rightarrow p(z,y))}$



$\mathcal{C}_{\subseteq}(x,y) \lesssim \exists z(p(x,z) \,\&\, p(z,y))$

$\mathcal{C}_{\odot}(x,y) \lesssim \mathcal{C}_{\subseteq}(x,y) \,\&\, \neg\mathcal{C}_{=}(x,y) \,\&\, \neg p(x,y)$

$p(a,b) \leftrightarrow$ 

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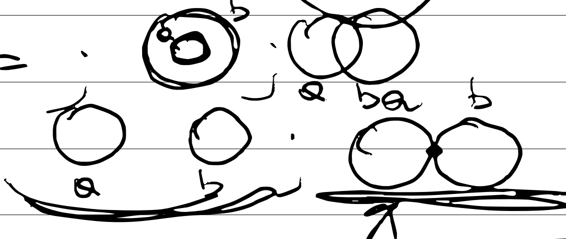
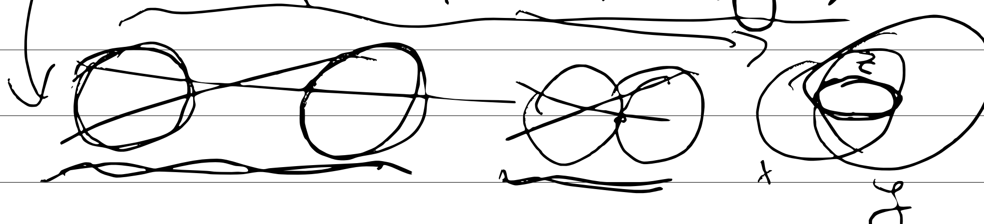
TODO:

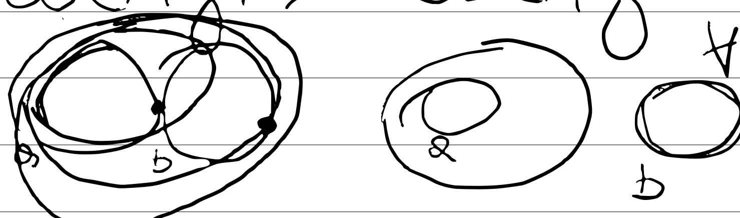


$\varphi_{OO}(x,y) \leftrightharpoons$ "отриц. всички останали"

$\varphi_{PO}(x,y) \leftrightharpoons \neg\varphi_{\le}(x,y) \,\&\, \neg\varphi_{\le}(y,x) \,\&\, \exists z(p(z,x) \,\&\, p(z,y))$



$\varphi_{OO}(x,y) \leftrightharpoons \neg\varphi_{\le}(x,y) \,\&\, \neg\varphi_{\le}(y,x) \,\&\, \neg\varphi_{PO}(x,y) \,\&\, \forall z(\varphi_{\le}(x,z) \Rightarrow \varphi_{\le}(y,z) \vee \varphi_{PO}(y,z))$



Зад/ $\mathcal{L} = \langle p \rangle$, $p \in Pred_{\mathcal{L}}$, $\underline{\#(p) = 3}$

$\mathcal{A} = \langle \underline{\mathbb{N}}, p^{\mathcal{A}} \rangle$, където

$p^{\mathcal{A}}(k, n, m) \Longleftrightarrow \underline{k + n = m + 2}$

__a)__ Док., че всеки синглетон е определим

__b)__ Да се определят $=$ и $<$.

3зад. Нека $\mathcal{L} = \langle Ш \rangle$, $Ш \in Preds$, $\#(Ш) = 3$.
Нека $\mathcal{A} = \langle \mathbb{R}^2, Ш^{\mathcal{A}} \rangle$, където:

$$Ш^{\mathcal{A}}(A, B, C) \leftrightharpoons A \neq B \text{ и } A \neq C \text{ и } \angle BAC = 30^\circ$$

Да се док., че в стр. $\mathcal{A}$ са определими:

- $Eq \leftrightharpoons \{\langle A, A \rangle \mid A \in \mathbb{R}^2\}$
- $Col \leftrightharpoons \{\langle A, B, C \rangle \mid$ лежат на една права$\}$
- $Circ \leftrightharpoons \{\langle A, B, C \rangle \mid C$ лежи на окръжност с диаметър $AB\}$

Вярно ли е, че в $\mathcal{A}$ са опр. м-вата и зад.?

- $Mid \leftrightharpoons \{\langle A, B, C \rangle \mid C$ е среда на отс. $AB\}$
- $Seg \leftrightharpoons \{\langle A, B, C \rangle \mid C$ лежи на отс. $AB\}$