Записване отърваване на с $\Rightarrow$ и $\Leftrightarrow$

$\varphi_1 \leftrightharpoons \forall x \forall y (\neg p(x,y) \Rightarrow \forall z (p(x,z) \Rightarrow \neg p(z,y)))$

$\varphi_2 \leftrightharpoons \forall x \neg \exists y \forall z ((p(x,z) \& p(y,z)) \Rightarrow \underbrace{(p(z,x) \& \neg p(z,x))}_{\text{лъжа}})$

$\varphi_3 \leftrightharpoons \forall x \forall y (\underbrace{\forall x \exists y (p(x,y) \lor \neg p(x,y))}_{\text{истина}} \Rightarrow \forall z \exists t (p(y,t) \& p(x,t) \& p(z,t)))$

? $\varphi_1, \varphi_2 \models \varphi_3$?

За записване.

$\psi \leftrightharpoons \neg \varphi_3$ (Да не забравя) Опа...

1 "$\Rightarrow$" и "$\Leftrightarrow$" се разкриват и "$\neg$" се бутат пред атомарни ф-ли.

$\varphi_1 \models\!\!\dashv \forall x \forall y (\neg p(x,y) \Rightarrow \forall z (\neg p(x,z) \lor \neg p(z,y))) \models\!\!\dashv$

$\models\!\!\dashv \forall x \forall y (p(x,y) \lor \forall z (\neg p(x,z) \lor \neg p(z,y)))$

$\varphi_1' \leftrightharpoons$

$\varphi_2 \models\!\!\dashv \forall x \forall y \exists z \neg (\neg (p(x,z) \& p(y,z))) \models\!\!\dashv$

$\forall x \forall y \exists z (p(x,z) \& p(y,z))$ ✓

$\psi \Leftrightarrow \neg\forall x\forall y(\cancel{\forall x\exists y(p(x,y)\vee\neg p(x,y))} \Rightarrow$

винаги истина $\quad \forall z\exists t(p(y,t)\,\&\,p(x,t)\,\&\,p(z,t)))$

$\psi \;\Leftrightarrow\; \neg\forall x\forall y\forall z\exists t\,(p(y,t)\,\&\,p(x,t)\,\&\,p(z,t)) \;\Leftrightarrow$

$\exists x\exists y\exists z\forall t\,(\neg p(y,t)\vee\neg p(x,t)\vee\neg p(z,t))$

$\psi' \Leftrightarrow$

② ПНФ

$\varphi_1'' \Leftrightarrow \forall x\forall y\forall z(p(x,y)\vee\neg p(x,z)\vee\neg p(z,y))$

$\varphi_2'' \Leftrightarrow \forall x\forall y\exists z\,(p(x,z)\,\&\,p(y,z))$

$\psi'' \Leftrightarrow \exists x\exists y\exists z\forall t\,(\neg p(y,t)\vee\neg p(x,t)\vee\neg p(z,t))$

③ СНФ

φ_1^S си е също

$\varphi_2^S \Leftrightarrow \forall x\forall y\,((p(x,f(x,y))\,\&\,p(y,f(x,y)))$

$\psi^S \Leftrightarrow \forall t\,(\neg p(b,t)\vee\neg p(a,t)\vee\neg p(c,t))$

④ Няма нужда от дистрибутивни закони тук.

$D_1 \leftrightharpoons \{p(x_1,y_1), \neg p(x_1,z_1), \neg p(z_1,y_1)\}$

$D_2 \leftrightharpoons \{p(x_2, f(x_2,y_2))\}$

$D_3 \leftrightharpoons \{p(y_3, f(x_3,y_3))\}$

$D_4 \leftrightharpoons \{\neg p(a,t_4), \neg p(b,t_4), \neg p(c,t_4)\}$

Да започнем ① с ②?

Ще има разложение тук!

① Разгл. D_1 и D_2

$D_5 \leftrightharpoons Res(D_1\{x_1/x_2, z_1/f(x_2,y_2)\}, D_2) =$

$= \{p(x_2,y_1), \neg p(f(x_2,y_2),y_1)\}$

Сега ③ и ⑤

$\left| \begin{array}{l} y_3 = f(x_2,y_2) \\ y_1 = f(x_3,y_3) \end{array} \right. \to \left| \begin{array}{l} y_3 = f(x_2,y_2) \\ y_1 = f(x_3, f(x_2,y_2)) \end{array} \right.$

② Разгл. D_3 и D_5

$D_6 \leftrightharpoons Res(D_3\{y_3/f(x_2,y_2)\}, D_5\{y_1/f(x_3, f(x_2,y_2))\}) =$

$= \{p(x_2, f(x_3, f(x_2,y_2)))\}$

Искаме сега да направим сечение с ④

D4 ще влезне в играта, някак всичките дизюнкти ще ни трябват.

Искаме правилно вложение на термовете

$$p(x_2, f(x_3, f(x_2, y_2)))$$

Трябва стратегия за това.

③ Нека D_4 и D_2

$$\begin{cases} x_2 = a \\ f(x_2, y_2) = t_4 \end{cases} \Leftrightarrow \begin{cases} x_2 = a \\ t_4 = f(a, y_2) \end{cases}$$

$$D_7 = Res(D_2\{x_2/a\}\ D_4\{t_4/f(a,y_2)\}) =$$

$$= \{\neg p(b, f(a, y_2)), \neg p(c, f(a, y_2))\}$$

Трябва транзитивност. D_1 е транзитивност.

Нека покажем, че от коя, от която извличаме D_1 е транзитивност, защото не си личи.

Окей?

φ_1 даме

$$\forall x \forall y (\neg p(x,y) \Rightarrow \forall z (p(x,z) \Rightarrow \neg p(z,y))) \models\!\!|$$

$$\varphi \Rightarrow \forall x \psi \text{ и } x \notin Var_{free}[\varphi]$$

$$\models\!\!| \ \forall x (\varphi \Rightarrow \psi)$$

$$\models\!\!| \ \forall x \forall y \forall z (\neg p(x,y) \Rightarrow (p(x,z) \Rightarrow \neg p(z,y)))$$

разерихме $\Rightarrow$

$$\models\!\!| \ \forall x \forall y \forall z (p(x,y) \vee \underbrace{\neg p(x,z) \vee \neg p(z,y)}_{\neg(p(x,z) \,\&\, p(z,y))})$$

$$\models\!\!| \ \forall x \forall y \forall z (p(x,z) \,\&\, p(z,y) \Rightarrow p(x,z))$$

Транзитивност.

Добре, ще се боря с him.

$\forall x \exists y\, p(x,y) \,\&\, \exists z \underset{\forall t}{\cancel{\forall x}}\, q(z,\underset{t}{\cancel{x}})$

$\exists z \forall x \exists y \forall t$

$\varphi_3 \leftrightharpoons \forall x \forall y (\forall x \exists y (p(x,y) \vee \neg p(x,y)) \Rightarrow \forall z \exists t (p(y,t) \,\&\, p(x,t) \,\&\, p(z,t))))$

$D_1 = \{ p(x_1, y_1), \neg p(x_1, z_1), \neg p(z_1, y_1) \}$

$D_2 = \{ p(x_2, f(x_2, y_2)) \}$

$D_3 = \{ p(y_3, f(x_3, y_3)) \}$

$D_4 = \{ \neg p(a, t_4), \neg p(b, t_4), \neg p(c, t_4) \}$

$D_7 = Res(D_2 \{x_2/a\}, D_4 \{t_4 / f(a, y_2)\}) =$

$= \{ \neg p(b, f(a, y_2)), \neg p(c, f(a, y_2)) \}$

④ Разг. D_1 и D_7

$D_8 = Res(D_1 \{x_1/b, y_1/f(a, y_2)\}, D_7) =$

$= \{ \neg p(b, z_1), \neg p(z_1, f(a, y_2)), \neg p(c, f(a, y_2)) \}$

$\to z_1/y_2$

⑤ Разгл. D_3 и D_8

| $z_1 = y_3$
| $f(a, y_2) = f(x_3, y_3) \to x_3/a$

$D_9 = Res(D_3 \{x_3/a\}, D_8 \{z_1/y_2\}) = \{ \neg p(b, y_2)$

$\neg p(c, f(a, y_2)) \}$

$$D_3 = \{\neg p(b, y_2),\ \neg p(c, f(a, y_2))\}$$

$$\begin{cases} y_3 = b \\ f(x_3, y_3) = y_2 \end{cases}$$

$$\{y_3/b,\ y_2/f(x_3, b)\}$$

(6) $D_3 \cup D_6$

$$D_{10} \Leftarrow Res(D_3\{y_3/b\},\ D_3\{y_2/f(x_3, b)\}) =$$

$$= \{\neg p(c, f(a, f(x_3, b)))\}$$

$$D_6 = \{p(x_2, f(x_3, f(x_2, y_2)))$$

(7) $D_{10} \cup D_6$

$$D_6' \Leftarrow D_6\{x_2/x_2',\ x_3/x_3',\ y_2/y_2'\}$$

$$= Res(D_{10}\{x_3/c\},\ D_6'\{x_2'/c,\ x_3'/a,\ y_2'/b\})$$

$\varphi_1 \leftrightharpoons \forall x \exists y ((q(x,y) \Rightarrow p(x,y)) \,\&\, \forall z (p(z,y) \Rightarrow r(x,z))).$

$\varphi_2 \leftrightharpoons \forall x (\exists y\, p(y,x) \Rightarrow \exists y (p(y,x) \,\&\, \neg \exists z (p(z,y) \,\&\, p(z,x)))).$

$\varphi_3 \leftrightharpoons \forall z (\exists x \exists y (\neg q(x,y) \,\&\, \neg p(x,y)) \Rightarrow \forall z_1 q(z_1,z)).$

$\varphi_4 \leftrightharpoons \forall x \forall y \forall z ((p(x,y) \,\&\, r(y,z)) \Rightarrow p(x,z)).$

$\varphi_5 \leftrightharpoons \exists x\, p(x,x)$

Док, че $\{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5\}$ е неизпълнимо.

1)

$\varphi_1 \vDash\!\dashv \forall x \exists y ((\neg q(x,y) \lor p(x,y)) \,\&\, \forall z (\neg p(z,y) \lor r(x,z)))$

$\varphi_1' \leftrightharpoons$

$\varphi_2 \vDash\!\dashv \forall x (\forall y \neg p(y,x) \lor \exists y (p(y,x) \,\&\, \neg \exists z (p(z,y) \,\&\, p(z,x))))$

$\forall z (\neg p(z,y) \lor \neg p(z,x)))$

$\varphi_2' \leftrightharpoons$

$\varphi_3 \vDash\!\dashv \forall z (\forall x \forall y (q(x,y) \lor p(x,y)) \lor \forall z_1 q(z_1, z)).$

$\forall w \qquad w$

$\varphi_3' \leftrightharpoons$

$\varphi_4 \vDash\!\dashv \forall x \forall y \forall z (\neg p(x,y) \lor \neg r(y,z) \lor p(x,z))$

φ_4'

(2) $\varphi_1'' \leftrightharpoons \forall x \exists y \forall z ((\neg q(x,y) \vee p(x,y)) \& (\neg p(z,y) \vee r(x,z)))$

$\varphi_2'' \leftrightharpoons \forall x \exists y \forall z \forall t (\neg p(t,x) \vee (p(y,x) \& (\neg p(z,y) \vee \neg p(z,x))))$

$\varphi_3'' \leftrightharpoons \forall z \forall x \forall y \forall w (q(x,y) \vee p(x,y) \vee q(w,z))$

$\varphi_4'' \leftrightharpoons$ ✓

$\varphi_5'' \leftrightharpoons$ ✓

(3) $\varphi_1^s \leftrightharpoons \forall x \forall z ((\neg q(x,f(x)) \vee p(x,f(x))) \& (\neg p(z,f(x)) \vee r(x,z)))$

$\varphi_2^s \leftrightharpoons \forall x \forall z \forall t (\neg p(t,x) \vee (p(g(x),x) \& (\neg p(z,g(x)) \vee \neg p(z,x))))$

$\varphi_5^s \leftrightharpoons p(a,a)$

(4) φ_1^s ✓

φ χ

$\varphi \vee (\psi \& \chi) \dashv\vdash (\varphi \vee \psi) \& (\varphi \vee \chi)$

$\varphi_2^s \dashv\vdash \forall x \forall z \forall t ((\neg p(t,x) \vee p(g(x),x)) \& (\neg p(t,x) \vee \neg p(z,g(x)) \vee \neg p(z,x)))$

$D_1 = \{\neg q(x_1, f(x_1)), p(x_1, f(x_1))\}$

$D_2 = \{\neg p(z_2, f(x_2)), r(x_2, z_2)\}$

$D_3 = \{\neg p(t_3, x_3), p(g(x_3), x_3)\}$

$D_4 = \{\neg p(t_4, x_4), \neg p(z_4, g(x_4)), \neg p(z_4, x_4)\}$.

$D_5 = \{q(x_5, y_5), p(x_5, y_5), q(w, z_5)\}$

$D_6 = \{\neg p(x_6, y_6), \neg r(y_6, z_6), p(x_6, z_6)\}$

$D_7 = \{p(a, a)\}$

1) Розгл. D_5

$D_8 = \text{Collapse}(D_5\{w/x_5, z_5/y_5\}) = \{q(x_5, y_5), p(x_5, y_5)\}$

2) Розгл. D_8, D_1

$D_9 = \text{Res}(D_1\{x_1/x_5\}, D_8\{y_5/f(x_5)\}) = \{p(x_5, f(x_5))\}$

3) Розгл. D_2 и D_6

$D_{10} = \text{Res}(D_2\{x_2/y_6, z_2/z_6\}, D_6) = \{\neg p(z_6, f(y_6)), \neg p(x_6, y_6), p(x_6, z_6)\}$

④ Розгл. D_4, D_{10}

$D_{11} = Res(D_4\{{}^{t_4}/x_6, {}^{x_4}/z_6\}, D_{10}) =$

$\{\neg p(z_6, f(y_6)), \neg p(x_6, y_6)\},$

$\cancel{\neg p(z_4, g(z_6))}, \neg p(z_4, z_6)\}$

$z_4 = g(x_3) \Rightarrow z_4 = g(g(z_6))$

$g(z_6) = x_3 \rightsquigarrow$

⑤ Розгл. D_3 и D_{10}

$z_6 = g(x_3) \rightarrow z_6 = g(f(y_6))$

$x_3 = f(y_6)$

$D_{12} = Res(D_3, D_{10}\{ \quad \}) =$

$= \{\neg p(t_3, f(y_6)), \neg p(x_6, y_6), \underline{p(x_6, g(f(y_6)))}\}$

⑥ D_4 и D_{12}

$D_{13} = Res(D_4\{{}^{z_4}/x_6, {}^{x_4}/f(y_6)\}, D_{12}) =$

$= \{\neg p(t_3, f(y_6)), \neg p(x_6, y_6), \neg p(t_4, f(y_6)), \neg p(x_6, f(y_6))\}$

(7) D_{13}

$D_{14} = \text{Collapse}(D_{13}\{x/a\}) = \{\neg p(a, f(a)), \neg p(a,a)\}$

(8) D_{14} u D_7

$D_{15} = \text{Res}(D_7, D_{14}) = \{\neg p(a, f(a))\}$

(9) D_8 u D_{15}

~~$D_{15} = \text{Res}(D_8\{x_5/a\}, D_{15})$~~

(1) D_3 u D_2

$D_{16} = \text{Res}(D_2\{x_2/x_5, z_2/x_5\}, D_3) =$

$= \{r(x_5, x_5)\}$

(2) D_{16} u D_6

~~$D_{17} = \text{Res}(D_6\{y_6/x_5, z_6/x_5\}) = \{\neg p(x_6, x_5), p(x_6, x_5)\}$~~