Нека $\mathcal{L}(p)$ е език за ПС от $I^{ви}$ ред без ф.р. с двум. предик. с-л p.

Нека $\mathcal{U} = \langle U, p^{\mathcal{U}} \rangle$ е стр. за $\mathcal{L}(p)$ с универсум м-вото U от всички точки и всички затворени кръгове в една фиксирана евклидова равнина π и за произв. $a, b \in U$:

$\langle a, b \rangle \in p^{\mathcal{U}} \Leftrightarrow a$ е точка от π, b е затворен кръг в π и $a \in b$.

Да се док., че следните м-ва са определими в $\mathcal{U}$:

(i) $\{\langle b_1, b_2 \rangle \mid b_1$ и b_2 са затворени кръгове от π и $b_1 \subseteq b_2\}$

(ii) $\{\langle b_1, b_2 \rangle \mid b_1$ и b_2 са затворени кръгове от и контурите им се допират$\}$

(iii) $\{\langle a, b \rangle \mid a$ е точка от контура на затворения кръг $b\}$

(iv) $\{\langle a, a_1, a_2 \rangle \mid a, a_1, a_2$ са точки от π, $a_1 \neq a_2$ и a лежи на правата $a_1 a_2\}$

(v) $\{\langle a_1, a_2, b \rangle \mid$ отсечката $a_1 a_2$ е диаметър на затворения кръг $b\}$.

$A = \langle \mathbb{N}, p^A \rangle, \quad \#(p) = 2$

$\langle a, b \rangle \in p^A \iff a - b \geq 3$

$\neg p^A(a,b) \iff a - b < 3$

Всички синглетони са опр.

$\varphi_{\{0,1,2\}}(x) \leftrightharpoons \neg \exists y\, p(x,y).$

$\varphi_{\langle 0,3 \rangle}(x,y) \leftrightharpoons \varphi_{\{0,1,2\}}(x) \;\&\; \neg\varphi_{\{0,1,2\}}(y) \;\&$

$\forall z\, (p(y,z) \Rightarrow \varphi_=(x,z)).$

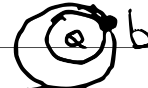
$\varphi_=(x,y) \leftrightharpoons \forall z\, (p(z,x) \iff p(z,y)).$

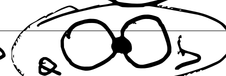
$\varphi_0(x) \leftrightharpoons \exists y\, \varphi_{\langle 0,3 \rangle}(y).$

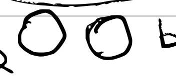
Зад. $L = \langle p \rangle$, $\#p = 2$.

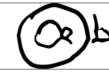
кръгове в $\mathbb{R}^2$

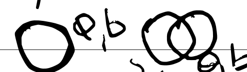
$\mathcal{A} = \langle$ окр. с ненулев радиус в $\mathbb{R}^2$, $p^{\mathcal{A}} \rangle$, където

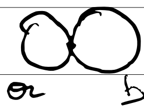
a) $p^{\mathcal{A}}(a,b) \iff$ 

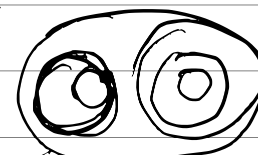
b) $p^{\mathcal{A}}(a,b) \iff$ 

c) $p^{\mathcal{A}}(a,b) \iff$ 

d) $p^{\mathcal{A}}(a,b) \iff$ 

Опр.  и останалите.

b) $\langle a,b \rangle \in p^{\mathcal{A}} \iff$ 

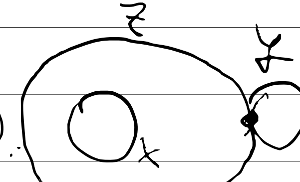


подкрег $\subseteq$

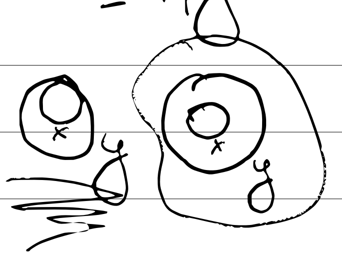
$\varphi_=(x,y) \leftrightharpoons \forall z(p(x,z) \iff p(y,z))$.

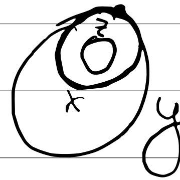
$\varphi_{\circledcirc}(x,y) \leftrightharpoons \boxed{\forall z(p(z,y) \Rightarrow \neg p(z,x))}$.

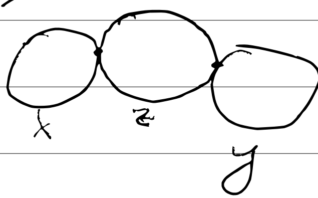
$\varphi_{\circ\circ}(x,y) \leftrightharpoons \exists z(\varphi_{\circledcirc}(x,z) \,\&\, p(z,y))$.



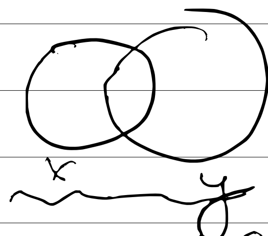
$$\varphi_{\subseteq}(x,y) \rightleftharpoons \forall z\,(\varphi_{\odot}(z,x) \Rightarrow \varphi_{\odot}(z,y))$$







$$\varphi_{\odot}(x,y) \rightleftharpoons \varphi_{\subseteq}(x,y) \,\&\, \neg\varphi_{\circledcirc}(x,y).$$

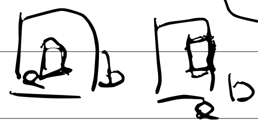


partial overlap

$$\varphi_{\circ\circ}(x,y) \rightleftharpoons \exists z\,(\varphi_{\subseteq}(z,x) \,\&\, \varphi_{\subseteq}(z,y) \,\&\, \neg\varphi_{\subseteq}(x,y) \,\&\, \neg\varphi_{\subseteq}(y,x)).$$

Зад 6/ $A < \text{всички квадрати}, p^R >$: с ненулеви страни в $\mathbb{R}^2$ плътни фигури

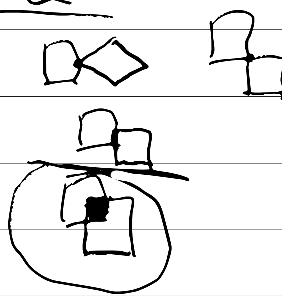
$\boxed{p^R(a, b) \Leftrightarrow a \text{ и } b \text{ имат поне една обща точка}}$

a)  $a \subseteq b \rightarrow a \cap b = a$

b) $a \cap b = \text{точка} \rightarrow \varphi_{\bullet} \longrightarrow$

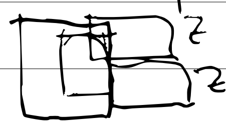
c) $a \cap b = \text{отсечка} \longrightarrow \varphi_{|} \longrightarrow$

d) $a \cap b = \text{квадрат} \longrightarrow \varphi_{\square} \longrightarrow$



$\varphi_{=}(x, y) \leftrightharpoons \varphi_{\subseteq}(x, y) \,\&\, \varphi_{\subseteq}(y, x).$

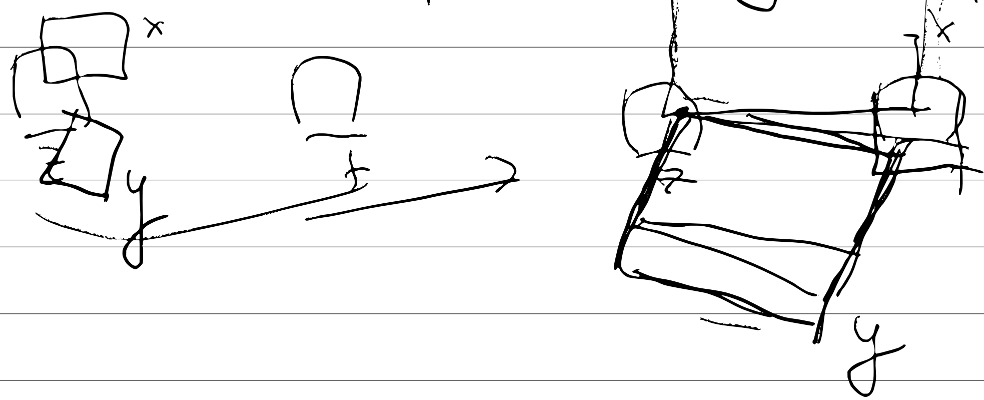
$\varphi_{\subseteq}(x, y) \leftrightharpoons \forall z\, (p(x, z) \Rightarrow p(y, z)).$



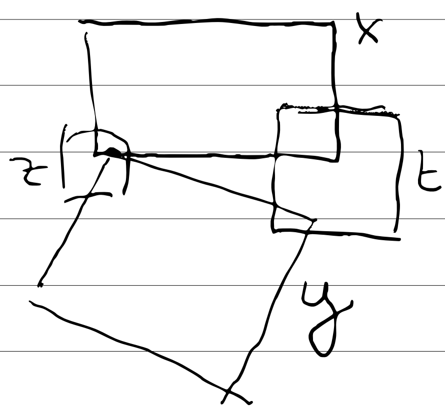
$\varphi_{\bullet, |}(x, y) \leftrightharpoons p(x, y) \,\&\, \neg\exists z\, (\varphi_{\subseteq}(z, x) \,\&\, \varphi_{\subseteq}(z, y)).$

$\varphi_{\square\,\square}(x, y) \leftrightharpoons \neg p(x, y).$

$$\mathcal{C}_1(x,y) \Leftrightarrow \exists z \exists t \, (\mathcal{C}_{\sqcap\sqcap}(z,t) \,\&\, p(x,z) \,\&\, p(y,z) \,\&\, p(x,t) \,\&\, p(y,t) \,\&\, \mathcal{C}_{2\bullet,18}(x,y)).$$



$\cdot \quad | \quad \sqcap$



Зад | $\mathcal{L} = \langle p \rangle$, $\Rightarrow$

$\mathcal{A} = \langle \{a,b\}^*, p^{\mathcal{A}} \rangle$

$p^{\mathcal{A}}(u,v) \Longleftrightarrow ||u| - |v|| = 1$

a) $\{\varepsilon\}$

b) $\{w \mid |w| = 2\}$

c) Док-те $\{b\}$ е неопределимо.

Зад | Нека $\Sigma = \{0,1\}$ и $P(\Sigma^*)$. Сигн. $\mathcal{L} = \langle cat, sub \rangle$, за $cat \in Func$, $sub \in Pred$, $\#cat = 2$, $\#sub = 2$.

$\mathcal{A} = \langle P(\Sigma^*), cat^{\mathcal{A}}, sub^{\mathcal{A}} \rangle$

$cat^{\mathcal{A}}(L_1, L_2) \leftrightharpoons L_1 \circ L_2$

$sub^{\mathcal{A}}(L_1, L_2) \Leftrightarrow L_1 \subseteq L_2$

Опр: $\{\emptyset\}$, $\{\Sigma^*\}$, $\{\{\varepsilon\}\}$

$single = \{\{w\} \mid w \in \Sigma^*\}$ всички едноезд. езици

$union = \{\langle L_1, L_2, L_3 \rangle \mid L_3 = L_1 \cup L_2\}$

$star = \{\langle L, L^* \rangle \mid L \subseteq \Sigma^*\}$

Никой непразно дума от Σ^* не е определима т.е. $w \in \Sigma^*$, то $\{\{w\}\}$ е неопр.

Док, че $h \in Aut(\mathcal{A})$, то h запазва регулярните езици.

Зад. $\mathcal{A} = \langle \mathbb{Q}, p^{\mathcal{A}} \rangle$: $a, b, c \in \mathbb{Q}$

$\langle a, b, c \rangle \in p^{\mathcal{A}} \iff a = b^3 \cdot c$

Да се док., че в $\mathcal{A}$ са опр. м-вата:

$\{0\}$, $\{1\}$, $\{-1\}$, $\{\langle x, x \rangle \mid x \in \mathbb{Q}\}$, $\{\langle x, y \rangle \mid x \cdot y = 1\}$,

$\{\langle x, y, z \rangle \mid x = y \cdot z\}$.

Да се док., че в $\mathcal{A}$ м-вото $\{3\}$ не е опр.

зад! $A = \langle \mathbb{N}; p^A \rangle$, $\#(p)=3$, $p \in Pred_{\mathcal{L}}$

$a, b, c \in \mathbb{N}$:

$$\langle a,b,c\rangle \in p^A \Leftrightarrow a.b+1=c^2$$

Определете всички синглетони.

$\cancel{\varphi_0(x) \leftrightharpoons \neg \exists y \exists z\, p(y,z,x).}$

$\psi_0(x) \leftrightharpoons \exists y \forall z\, p(x,z,y)$

$\varphi_{\langle 0,1\rangle}(x,y) \leftrightharpoons \underline{\forall z}\, p(x,z,y).$

$\varphi_1(x)\,(x) \leftrightharpoons \exists y\, \varphi_{\langle 0,1\rangle}(y,x).$ $\to$ $\underline{0^2+1=1^2}$

$\psi_{\langle 0,1\rangle}(x,y) \leftrightharpoons p(x,x,y).$

$\varphi_=(x,y) \leftrightharpoons \forall z \forall t\,(p(x,z,t) \Leftrightarrow p(y,z,t)).$

$\psi_=(x,y) \leftrightharpoons \forall z \forall t\,(p(z,t,x) \Leftrightarrow p(z,t,y)) \vee$
$\varphi_0(x) \,\&\, \varphi_0(y).$

$\langle 0,1\rangle$: $a.b+1=a^2 \Leftrightarrow a^2-ab=1 \Leftrightarrow \underbrace{a}_{1}.\underbrace{(a-b)}_{1}=1 \to \underline{\mathbb{N}}$

$$\varphi_2(x) \leftrightharpoons \forall y \forall z \left(p(y, z, x) \Rightarrow \varphi_1(y) \vee \varphi_1(z) \right) \& \neg \varphi_0(x).$$

$$a \cdot b + 1 = c^2 \Leftrightarrow a \cdot b = (c-1) \cdot (c+1)$$

2

3 – просто число

$$4 = 1 + a \cdot b$$

$c = 2$

1 . 3

3 . 1

8 – не е просто

$$9 = 1 + a \cdot b$$

$c = 3$

2 . 4

4 . 2

1 . 8

8 . 1

$\varphi_3(x) \leftrightharpoons \exists y \exists z (\varphi_1(y) \& \varphi_2(z) \& p(x,y,z))$.

$x.y+1=z^2$

$x.4+1=4$

$x=3$

$p^{\mathcal{A}}(a,b,c) \leftrightarrow a.b+1=c^2$

$0, 1, 2$

$1.3+1=2^2$

$\hookrightarrow n-1, n, n+1$

$(n+1).(n-1)+1=n^2$

$\boxed{(n+1).(n-1)=n^2-1}$

Всеки синглетон от $\mathbb{N}$ число е опр. тук.

Защо не опр. $\{2\}$ по този схема?

$x.0+1=1^2$

характеризуме на $\langle 0, 1 \rangle$.