Нека $\mathcal{L}(p)$ е език за ПС от $I^{ви}$ ред без ф.р. с двум. предик. с-л p.

Нека $\mathcal{U} = \langle U, p^{\mathcal{U}} \rangle$ е стр. за $\mathcal{L}(p)$ с универсум мн-вото U от всички точки и всички затворени кръгове в една фиксирана евклидова равнина π и за произ. $a, b \in U$:

$\langle a, b \rangle \in p^{\mathcal{U}} \iff a$ е точка от π, b е затворен кръг в π и $a \in b$.

Да се док., че следните мн-ва са определими в $\mathcal{U}$:

(i) $\{\langle b_1, b_2 \rangle \mid b_1$ и b_2 са затворени кръгове от π и $b_1 \subseteq b_2\}$

(ii) $\{\langle b_1, b_2 \rangle \mid b_1$ и b_2 са затворени кръгове от и контурите им се допират$\}$

(iii) $\{\langle a, b \rangle \mid a$ е точка от контура на затворения кръг $b\}$

(iv) $\{\langle a, a_1, a_2 \rangle \mid a, a_1, a_2$ са точки от π, $a_1 \neq a_2$ и a лежи на правата $a_1 a_2\}$

(v) $\{\langle a_1, a_2, b \rangle \mid$ отсечката $a_1 a_2$ е диаметър на затворения кръг $b\}$.

$\mathcal{A} = \langle \mathbb{N}, p^{\mathcal{A}} \rangle, \ \#(p) = 2$

$\langle a, b \rangle \in p^{\mathcal{A}} \Leftrightarrow a - b \geq 3$

$\neg p^{\mathcal{A}}(a, b) \Leftrightarrow a - b < 3$

Всички синглетони са опр.

$\varphi_0(x) \leftrightharpoons \forall y \neg p(x, y).$ } 0 выполнит но и 1 и 2 го изп.

$\varphi_{\{0,1,2\}} \leftrightharpoons \forall y \neg p(x, y).$

$\rightarrow x \in \{3, 4, \ldots\}$ $\& p(x, y)$

$\varphi_{\langle 3, 0 \rangle}(x, y) \leftrightharpoons \neg \varphi_{\{0,1,2\}}(x) \ \&$ $(\varphi_{\{0,1,2\}}(y) \ \& \forall z (p(x, z) \Rightarrow \varphi_{=}(z, y)))$

$x \notin \{0, 1, 2\}$

$\varphi_0(x) \leftrightharpoons \exists y \varphi_{\langle 3, 0 \rangle}(y, x).$ $a = 3 \rightsquigarrow b = 0 \quad a - b = 3 \geq 3$

$\varphi_3(x) \leftrightharpoons \exists y \varphi_{\langle 3, 0 \rangle}(x, y).$

~~$\varphi_{=}(x, y) \leftrightharpoons \forall z (p(x, z) \Leftrightarrow p(y, z)).$~~

0 и 1 са неразличими

$\varphi_=(x,y) \leftrightharpoons \forall z\,(p(z,x) \Leftrightarrow p(z,y))$.

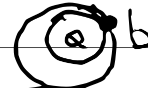
$\langle 1,4\rangle$, $\langle 2,5\rangle$,
$\langle 4,1\rangle$, $\langle 5,2\rangle$, $\langle 6,3\rangle$

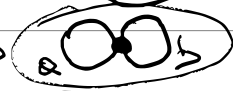
$\hookrightarrow \varphi_{\langle 4,1\rangle}(x,y) \leftrightharpoons$

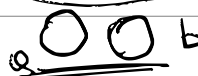
Зад. $\mathcal{L} = \langle p \rangle$, $\#p = 2$.

Кръгове в $\mathbb{R}^2$

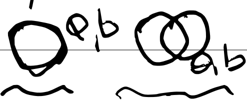
$\mathcal{A} = \langle$ ~~окр. с ненулев радиус в $\mathbb{R}^2$~~, $p^{\mathcal{A}} \rangle$, където

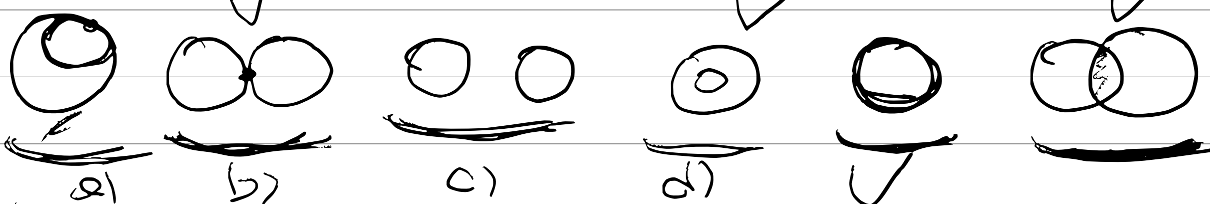
a) $p^{\mathcal{A}}(a, b) \Leftrightarrow$ 

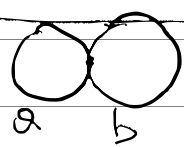
b) $p^{\mathcal{A}}(a, b) \Leftrightarrow$ 

c) $p^{\mathcal{A}}(a, b) \Leftrightarrow$ 

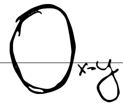
d) $p^{\mathcal{A}}(a, b) \Leftrightarrow$ 

Опр.  и останалите.

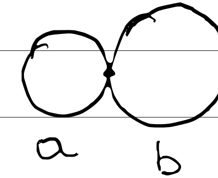


b) $p^{\mathcal{A}}(a, b) \Leftrightarrow$ 

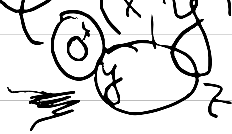
$$\varphi_{=}(x,y) \leftrightharpoons \forall z\,(p(x,z) \Leftrightarrow p(y,z)).$$



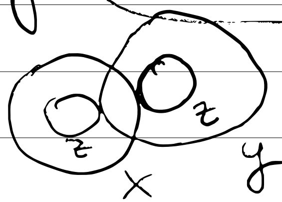
$$\langle a,b\rangle \in p^{\mathfrak{A}} \Leftrightarrow$$

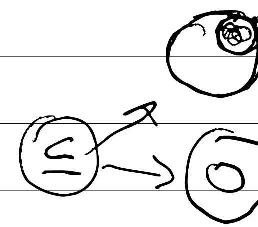


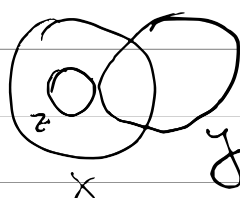
$$\varphi_{\circledcirc}(x,y) \leftrightharpoons \forall z\,(p(y,z) \Rightarrow \neg p(x,z)).$$

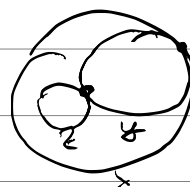


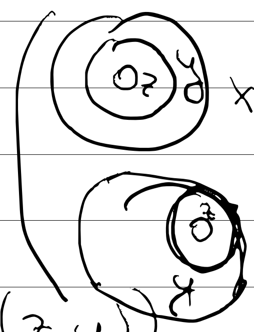
$$\varphi_{\bigcirc\!\bigcirc}(x,y) \leftrightharpoons \exists z\,(\varphi_{\circledcirc}(z,x) \,\&\, p(z,y)) \,\&\, \exists z\,(\varphi_{\circledcirc}(z,y) \,\&\, p(z,x)).$$

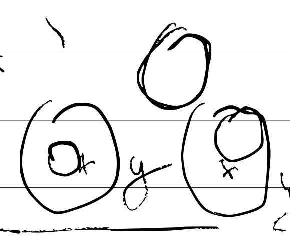












$$\varphi_{\subseteq}(x,y) \leftrightharpoons \forall z\,(\varphi_{\circledcirc}(z,x) \Rightarrow \varphi_{\circledcirc}(z,y))$$

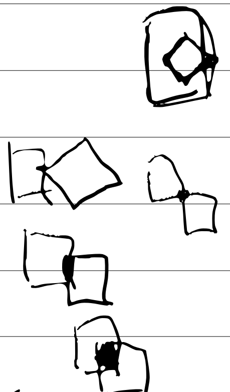
$$\varphi_{\bigcirc\!\!\bigcirc}(x,y) \leftrightharpoons \exists z \big(\varphi_{\odot}(z,x) \,\&\, \varphi_{\odot}(z,y) \big) \,\&\, \neg\varphi_{\subseteq}(x,y) \,\&\, \neg\varphi_{\subseteq}(y,x) \big).$$

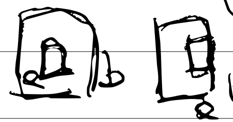
$$\varphi_{\odot}(x,y) \leftrightharpoons \varphi_{\subseteq}(x,y) \,\&\, \neg\varphi_{=}(x,y) \,\&\, \neg\varphi_{\odot}(x,y).$$

$$\varphi_{\bigcirc\,\circ}(x,y) \leftrightharpoons \ldots$$

Зад / $A < $ всички квадрати с ненулеви страни в $\mathbb{R}^2$ плътни фигури $, p^R >$:

$p^R(a, b) \Leftrightarrow$ a и b имат поне една обща точка

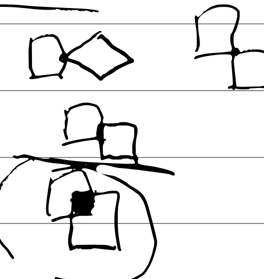


a)  $a \subseteq b \to a \cap b = a$

b) $a \cap b =$ точка $\to \varphi.$ $\to$

c) $a \cap b =$ отсечка $\to \varphi_1 \to$

d) $a \cap b =$ квадрат $\to \varphi_{\square} \to$



$\varphi_{\subseteq}(x,y) \Leftrightarrow \forall z (p(x,z) \Rightarrow p(y,z)).$

$\varphi_{=}(x,y) \Leftrightarrow \varphi_{\subseteq}(x,y) \,\&\, \varphi_{\subseteq}(y,x).$

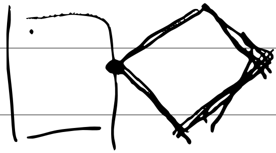
Т.е. [illegible] ги определим едновременно.

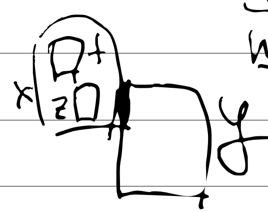
b) и c)

$\varphi_{b) \vee c)}(x,y) \Leftrightarrow p(x,y) \,\&\, \neg \exists z (\varphi_{\subseteq}(z,x) \,\&\, \varphi_{\subseteq}(z,y)).$

$\varphi_{\circ}(x,y) \rightleftharpoons \neg\exists t\,\exists z\,(\varphi_{\subseteq}(t,x)\ \&\ \varphi_{\subseteq}(z,x)\ \&\ \varphi_{\sqcap\sqcap}(t,z)$
$\&\ \underline{p(t,y)}\ \&\ p(z,y))\ \&$
$\varphi_{b)\vee c)}(x,y).$

поне една





А $\varphi_{\sqcap\sqcap}$? $\varphi_{\sqcap\sqcap}(x,y) \rightleftharpoons \neg p(x,y).$

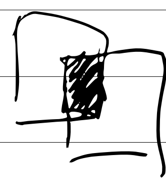
Да, но не го описахме

$\varphi_{1}(x,y) \rightleftharpoons \varphi_{b)\vee c)}(x,y)\ \&\ \neg\varphi_{\circ}(x,y).$

Другото следващия път.

$a \cap b$

екстремалност



зад | $\mathcal{L} = \langle p \rangle$, $\Rightarrow$

$\mathcal{A} = \langle \{a,b\}^*, p^{\mathcal{A}} \rangle$

$p^{\mathcal{A}}(u,v) \Leftrightarrow \big||u| - |v|\big| = 1$

a) $\{\varepsilon\}$

b) $\{w \mid |w| = 2\}$

c) Док-те $\{b\}$ е неопределимо.

Зад 1 Нека $\Sigma = \{a, b\}$ и $P(\Sigma^*)$. Сигн. $\mathcal{L} = \langle cat, sub \rangle$,

за $cat \in Func$, $sub \in Pred$, $\#cat = 2$, $\#sub = 2$.

$\mathcal{A} = \langle P(\Sigma^*), cat^{\mathcal{A}}, sub^{\mathcal{A}} \rangle$

$cat^{\mathcal{A}}(L_1, L_2) \leftrightharpoons L_1 \circ L_2$

$sub^{\mathcal{A}}(L_1, L_2) \Leftrightarrow L_1 \subseteq L_2$

Опр: $\{\emptyset\}$, $\{\Sigma^*\}$, $\{\{\varepsilon\}\}$

$single = \{\{w\} \mid w \in \Sigma^*\}$ всички еднодум. езици

$union = \{\langle L_1, L_2, L_3 \rangle \mid L_3 = L_1 \cup L_2\}$

$star = \{\langle L, L^* \rangle \mid L \subseteq \Sigma^*\}$

Никой непразен език от Σ^* не е

определим т.е. $w \in \Sigma^*$, то $\{\{w\}\}$ е неопр.

Да, че $h \in Aut(\mathcal{A})$, то h запазва

регулярните езици.

Зад $\mathcal{A} = \langle \mathbb{Q}, p^{\mathcal{A}} \rangle : a, b, c \in \mathbb{Q}$

$\langle a, b, c \rangle \in p^{\mathcal{A}} \longleftrightarrow a = b^3 . c$

Да се док, че в $\mathcal{A}$ са опр. м-вата:

$\{0\}$, ~~{1}~~ $\{-1\}$, $\{\langle x, x \rangle \mid x \in \mathbb{Q}\}$, $\{\langle x, y \rangle \mid x . y = 1\}$, $\{\langle x, y, z \rangle \mid x = y . z\}$.

Да се док, че в $\mathcal{A}$ м-вото $\{3\}$ не е опр.

зад! $\mathcal{A} = \langle \mathbb{N}; p^{\mathcal{A}} \rangle$, $\#(p)=3$, $p \in Pred_{\mathcal{L}}$

$a, b, c \in \mathbb{N}$:

$$\langle a, b, c \rangle \in p^{\mathcal{A}} \iff a.b+1 = c^2$$

Определете всички синглетони.

$a = 0$

$a \in \mathbb{N}$

$$a.0+1 = c^2$$
$$1 = c^2$$
$$x^2+1 = y^2 \iff \frac{x=0}{y=1}$$

$$a.b+1 = 0^2$$
$$a.b = -1$$
No way in $\mathbb{N}$

$\varphi_0(x) \leftrightharpoons \exists y\, p(x,x,y)$.

$\varphi_{\langle 0,1 \rangle}(x,y) \leftrightharpoons p(x,x,y)$.

$\psi_0(x) \leftrightharpoons \neg \exists y \exists z\, p(y,z,x)$

$\psi_0(x) \leftrightharpoons \exists y \forall z\, p(x,z,y)$.

$\psi_{\langle 0,1 \rangle}(x,y) \leftrightharpoons \forall z\, p(x,z,y)$.

забивя всичко в една точка от $*$ в $\mathbb{N}$.

$$a.b+1 = a^2 \iff a^2 - a.b = 1 \iff \overset{1}{a}.\overset{1}{(a-b)} = 1$$
$b = 0$

$\theta_0(x) \leftrightharpoons \exists y\, p(x,y,x)$

$\theta_{\langle 0,1 \rangle}(x,y) \leftrightharpoons p(x,y,x)$

$\varphi_2(x) \leftrightharpoons \forall y \forall z (p(y,z,x) \Rightarrow \varphi_1(y) \vee \varphi_1(z)) \& \neg\varphi_0(x)$.

$a \cdot b + 1 = c^2 \Leftrightarrow$ кога е просто число $\Leftrightarrow c = 2$

$a \cdot b = c^2 - 1 = (c-1) \cdot (c+1)$

Ако $c = 2 \longrightarrow (2-1) \cdot (2+1) = 1 \cdot 3 = \boxed{3 = a \cdot b}$

$1 \cdot 3 + 1 = 2^2$ 3 е просто.

$\varphi_3(x) \leftrightharpoons \exists y \exists z (\varphi_1(y) \& \varphi_2(z) \& p(y,x,z))$.

$n+1 \rightsquigarrow n, n-1$

$(n-1) \cdot (n+1) + 1 = n^2 \Leftrightarrow (n-1) \cdot (n+1) = n^2 - 1$

x

$\varphi_{n+1}(x) \leftrightharpoons \exists y \exists z (\varphi_{n-1}(y) \& \varphi_n(z) \& p(x,y,z))$.

Защо за 2 не става $2 \rightsquigarrow 0, 1$

$0 \cdot 2 + 1 = 1^2$

$\varphi_=(x,y) \leftrightharpoons \forall z \forall m\left(\underline{p(z,x,m)} \Leftrightarrow \underline{p(z,y,m)}\right).$

$\underline{a \cdot b + 1 = c^2}$

$\underline{a \cdot b + 1 = 0}$