

$$\mathcal{C}_1 \equiv \forall x \forall y (\neg p(x, y) \Rightarrow \forall z (p(x, z) \Rightarrow \neg p(z, y)))$$

$$\mathcal{C}_2 \equiv \forall x \neg \exists y \forall z ((p(x, z) \& p(y, z)) \Rightarrow (p(z, x) \& \neg p(z, x)))$$

$$\mathcal{C}_3 \equiv \forall x \forall y (\forall x \exists y (p(x, y) \vee \neg p(x, y)) \Rightarrow \forall z \exists t (p(y, t) \& (p(x, t) \& p(z, t))))$$

Ποκωμε, να $\mathcal{C}_1, \mathcal{C}_2 \models \mathcal{C}_3$.

$$\mathcal{C}_2' \equiv \forall x \neg \exists y \forall z (\neg p(x, z) \& p(y, z)).$$

$$\mathcal{C}_3' \equiv \forall x \forall y \forall z \exists t (p(y, t) \& p(x, t) \& p(z, t)).$$

$$\mathcal{C}_1, \mathcal{C}_2' \models \mathcal{C}_3' \iff \{\mathcal{C}_1, \mathcal{C}_2', \neg \mathcal{C}_3'\} \text{ e non satisfatto}$$

$$\mathcal{C}_1 \models \forall x \forall y (p(x, y) \vee \forall z (\neg p(x, z) \vee \neg p(z, y))) \models$$

$$\mathcal{C}_1' \equiv \forall x \forall z \forall y (\neg p(x, z) \vee p(z, y) \vee p(x, y)).$$

$$\mathcal{C}_2' \models \forall x \forall y \exists z (p(x, z) \& p(y, z)).$$

$$\mathcal{C}_2' \models \mathcal{C}_2^S \models \forall x \forall y (p(x, f(x, y)) \& p(y, f(x, y))).$$

$$\mathcal{C}_3' \models \mathcal{C}_3^S \models \forall x \exists y \exists z \exists t (p(y, t) \vee \neg p(x, t) \vee \neg p(z, t))$$

$$\mathcal{C}_3^S \models \forall t (\neg p(b, t) \vee \neg p(a, t) \vee \neg p(c, t))$$

$$\mathcal{C}_1^f \equiv \forall x \forall z \forall y (\neg p(x, z) \vee \neg p(z, y) \vee p(x, y)).$$

$$\mathcal{C}_2^f \equiv \forall x \forall y (p(x, f(x, y)) \& p(y, f(x, y))).$$

$$\mathcal{C}_3^f \equiv \forall t (\neg p(b, t) \vee \neg p(a, t) \vee \neg p(c, t)).$$

$$\underline{D_1 = \{ \neg p(x_1, z_1), \neg p(z_1, y_1), p(x_1, y_1) \}}$$

$$\underline{D_2 = \{ p(x_2, f(x_2, y_2)) \}}$$

$$\underline{D_3 = \{ p(y_3, f(x_3, y_3)) \}}$$

$$\underline{D_4 = \{ \neg p(b, t), \neg p(a, t), \neg p(c, t) \}}.$$

$$\textcircled{1} \text{ Res}(D_1 \{ x_1/x_2, z_1/f(x_2, y_1) \}, D_2) = \{ \neg p(f(x_2, y_2), y_1), p(x_2, y_1) \}$$

$$\textcircled{2} \text{ Res}(D_1 \{ z_1/x_2, y_1/f(x_2, y_2) \}, D_2) = \{ \neg p(x_1, x_2), p(x_1, f(x_2, y_2)) \}$$

$$\textcircled{3} \text{ Res}(D_1 \{ z_1/y_3, y_1/f(x_3, y_3) \}, D_3) = \{ \neg p(x_1, y_3), p(x_1, f(x_3, y_3)) \}$$

$$\textcircled{3} \text{ Res}(D_4 \{ t/f(x_2, y_2) \}, D_6 \{ x_1/c \}) = \{ \neg p(b, f(x_2, y_2)), \neg p(a, f(x_2, y_2)), \neg p(c, x_2) \} = D_7$$

$$\textcircled{4} \text{ Res}(D_4 \{ y_2/b \}, D_3 \{ y_3/b, x_2/x_3 \}) = \{ \neg p(a, f(x_3, b)), \neg p(c, x_3) \}$$

D_8

$$(5) \text{Res}(D_1 \{x_1/a, y/f(x_3, b)\}, D_8) = \{ \neg p(c, x_3), \neg p(a, \underline{z_1}), \neg p(\underline{z_1}, f(x_3, b)) \} y = D_3$$

$$(6) \text{Res}(D_9 \{x_3/f(c, y_2)\}, D_2) = \{ \neg p(a, z_1), \neg p(z_1, f(f(c, y_2), b)) \}$$

$$(7) \text{Res}(D_{10} \{z_1/f(a, y_2')\}, D_2) = \{ \neg p(\underline{f(a, y_2')}, \underline{f(f(c, y_2), b)}) \}$$

$$(8) \text{Res}(D_1 \{x_1/f(a, y_2'), y/f(f(c, y_2), b)\}, D_{11}) = \{ \neg p(\underline{f(a, y_2')}, \underline{z_1}), \neg p(\underline{z_1}, f(f(c, y_2), b)) \} y = D_{12}$$

$$(9) \text{Res}(D_3 \{y_3/f(x_2, y_2)\}, D_5 \{y_1/f(x_3, f(x_2, y_2))\}) = \{ p(x_2, f(x_3, \underline{f(x_2, y_2)})) \} y = D_{13}$$

$$(10) \text{Res}(D_2 \{x_2'/f(x_2, y_2)\}, D_5 \{y_1/f(f(x_2, y_2), y_2')\}) = \{ p(\underline{x_2}, f(\underline{f(x_2, y_2)}, y_2')) \} y = D_{14}$$

$$(11) \text{Res}(D_{14} \{x_2'/f(a, y_2'), x_2/c, y_2'/b\}, D_{11}) =$$

$$\varphi_1 \equiv \forall x \exists y (q(y, x) \wedge \forall z (q(y, z) \Rightarrow r(z, x)))$$

$$\varphi_2 \equiv \forall x (\exists y q(x, y) \Rightarrow \exists y (q(x, y) \wedge \neg \exists z (q(y, z) \wedge q(x, z))))$$

$$\varphi_3 \equiv \forall x \forall y \forall z (q(y, x) \wedge r(z, y) \rightarrow q(z, x))$$

$$\varphi_4 \equiv \neg \forall x \forall z (q(x, x) \Rightarrow (r(x, z) \wedge \neg r(x, z)))$$

Докажите, что $\{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$ — независимый
м-во от ф-л.

$$\textcircled{1} \varphi_1 \not\models \forall x \exists y (q(y, x) \wedge \forall z (\neg q(y, z) \vee r(z, x)))$$

$$\varphi_2 \not\models \forall x (\exists y q(x, y) \vee \exists y (q(x, y) \wedge \forall z (\neg q(y, z) \vee \neg q(x, z))))$$

$$\varphi_3 \not\models \forall x \forall y \forall z (\neg q(y, x) \vee r(z, y) \vee q(z, x))$$

$$\varphi_4 \not\models \exists x (\exists z (q(x, x) \wedge (\neg r(x, z) \vee r(x, z))))$$

$$\models \exists x q(x, x)$$

$$(2) \varphi_1' \models \forall x \exists y \forall z (q(y, x) \& (\neg q(y, z) \vee r(z, x)))$$

$$\varphi_1'' \models \forall x \exists y \forall z (q(x, y) \& (\neg q(y, z) \vee \neg q(x, z)))$$

$$(3) \varphi_1^s \models \forall x \forall z (q(f(x), x) \& (\neg q(x, z) \vee r(z, x)))$$

$$\varphi_2^s \models \forall x \forall t \forall z (q(x, t) \vee (q(x, g(x)) \& (\neg q(g(x), z) \vee \neg q(x, z))))$$

$$\varphi_3^s \models \varphi_1' \wedge \forall x \forall y \forall z (\neg q(y, x) \vee r(z, y) \vee q(z, x))$$

$$\varphi_4^s \models q(a, a)$$

$$(4) \varphi_2^f \models \forall x \forall t \forall z ((q(x, t) \vee q(x, g(x))) \& (q(x, t) \vee \neg q(g(x), z) \vee \neg q(x, z)))$$

$$D_1 = \{q(f(x_1), x_1)\}$$

$$D_6 = \{q(a, a)\}$$

$$D_2 = \{\neg q(f(x_2), z_2), r(z_2, x_2)\}$$

$$D_3 = \{\neg q(x_3, t_3), q(x_3, g(x_3))\}$$

$$D_4 = \{q(x_4, t_4), \neg q(g(x_4), z_4), \neg q(x_4, z_4)\}$$

$$D_5 = \{\neg q(y_5, x_5), \neg r(z_5, y_5), q(z_5, x_5)\}$$

$$D_1 = \{ \neg q(f(x_1), x_1) \}$$

$$D_6 = \{ \neg q(a, a) \}$$

$$D_2 = \{ \neg q(f(x_2), z_2), r(z_2, x_2) \}$$

$$D_3 = \{ \neg q(x_3, t_3), q(x_3, g(x_3)) \}$$

$$D_4 = \{ \neg q(x_4, t_4), \neg q(g(x_4), z_4), \neg q(x_4, z_4) \}$$

$$D_5 = \{ \neg q(y_5, x_5), \neg r(z_5, y_5), q(z_5, x_5) \}$$

$$\textcircled{1} \text{Res}(D_2 \{ z_5/x_2, y_5/z_2 \}, D_5) = \{ \neg q(f(y_5), z_5), \neg q(y_5, x_5), q(z_5, x_5) \} = D_7$$

$$\textcircled{2} \text{Res}(D_7 \{ z_5/g(f(y_5)) \}, D_3 \{ x_3/y_5 \}) = \{ \neg q(y_5, t_3), \neg q(y_5, x_5), q(q(f(y_5)), x_5) \} = D_8$$

$x_3 = f(y_5)$
 $z_5 = g(f(y_5))$

$$\textcircled{3} \text{Res}(D_4 \{ x_4/f(y_5), z_4/x_5 \}, D_8) =$$

$$\{ \neg q(f(y_5), t_4), \neg q(f(y_5), x_5), \neg q(y_5, t_3), \neg q(y_5, x_5) \} = D_9$$

$$\textcircled{4} \text{Collapse}(D_9 \{ y_5/a, t_3/a, x_5/a \}) = \{ \neg q(f(a), a), \neg q(a, a) \} = D_{10}$$

$$\textcircled{5} \text{Res}(D_1 \{ x_1/a \}, D_{10}) = \{ \neg q(a, a) \} = D_6$$

$$\textcircled{6} \text{Res}(D_6, D_6) = \text{false}$$

