Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

ET4 147: Signal Processing for Communications

Spring 2020

Homework (deadline 8 May)

This homework consists of two parts: (a) generating and studying an instantaneous MIMO model and (b) deriving a convolutive model with a single source and a single receiver. Make a short report containing the required Matlab files, plots, explanations, and answers, and turn it in on the deadline (after the class) or by e-mail. In case something is unclear, assistance is given by Pim van der Meulen, room HB17.130 (during office hours), e-mail: `P.Q.vanderMeulen@tudelft.nl`.

You may work in groups of two, unless you prefer to work alone.

Instantaneous model

1. Make Matlab subroutines to

   (a) generate the array response $\mathbf{a}(\theta)$ of a uniform linear array with M elements and spacing Δ wavelengths to a source coming from direction θ degrees;

   ```
   function a = gen_a(M,Delta,theta)
   ```

   (b) plot the spatial response of a given beamformer $\mathbf{w}$ as a function of the direction θ of a source with array response $\mathbf{a}(\theta)$;

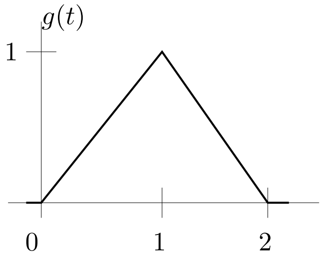
   ```
   function y = spat_response(w,Delta,theta_range)
   ```

   (c) generate a data matrix $\mathbf{X} = \mathbf{A}_\theta \mathbf{S} + \mathbf{N}$ as function of the directions $\boldsymbol{\theta} = [\theta_1 \ \cdots \ \theta_d]^{\mathrm{T}}$, number of antennas M, number of samples N, and signal-to-noise ratio (SNR) in dB (the SNR is defined as the ratio of the source power of a single user over the noise power). $\mathbf{S}$ and $\mathbf{N}$ are respectively a $d \times N$ and $M \times N$ random zero-mean complex Gaussian matrix;

   ```
   function X = gen_data(M,N,Delta,theta,SNR)
   ```

2. Test your routines on a few scenarios (1 or 2 sources, varying directions, varying number of antennas, varying number of samples) and make plots of the spatial responses (try to reproduce the graphs in chapter 1).

3. Plot the singular values of $\mathbf{X}$. Investigate the behavior of the singular values for varying DOA separation, number of antennas, number of samples, SNR.



Convolutive model

1. Make Matlab subroutines to

   (a) generate a rate-$\frac{1}{P}$ sampled version of the pulse $g(t)$, as shown in the figure, but delayed over an arbitrary delay $\tau \in [0, 1)$, i.e., generate $\mathbf{g}(\tau) = [g(0-\tau)\ g(\frac{1}{P}-\tau)\ \cdots\ g(L-\frac{1}{P}-\tau)]^{\mathrm{T}}$, where L is chosen such that the complete pulse is contained in $\mathbf{g}(\tau)$ for any $\tau \in [0, 1)$;

   ```
   function g = pulse(tau,L,P)
   ```

   (b) construct a rate-$\frac{1}{P}$ sampled version of the channel response $h(t)$ resulting from the sum of r paths with delays $\boldsymbol{\tau} = [\tau_1\ \cdots \tau_r]^{\mathrm{T}}$ ($\tau_i \in [0, 1)$) and gains $\boldsymbol{\beta} = [\beta_1\ \cdots\ \beta_r]^{\mathrm{T}}$, i.e., construct $\mathbf{h} = [h(0)\ h(\frac{1}{P})\ \cdots\ h(L-\frac{1}{P})]^{\mathrm{T}}$;

   ```
   function h = channel(tau,beta,L,P)
   ```

   (c) construct a source sequence $\mathbf{s} = [s_1\ s_2\ \cdots\ s_N]^{\mathrm{T}}$, where every entry is a random QPSK symbol. The corresponding analog sequence is $s_\delta(t) = \sum_k s_k \delta(t-k)$;

   ```
   function s = source(N)
   ```

   (d) construct a rate-$\frac{1}{P}$ sampled version of the output $x(t) = h(t) * s_\delta(t)$, i.e, construct $\mathbf{x} = [x(0)\ x(\frac{1}{P})\ \cdots\ x(N-\frac{1}{P})]^{\mathrm{T}}$;

   ```
   function x = gen_data1(h,s,P,N)
   ```

   Hint: You can use the Matlab `conv` command. You will need to extend $\mathbf{s}$ to $\mathbf{s}_{ext} = [s_1\ 0\ \cdots 0\ s_2\ 0 \cdots\ 0\ \cdots\ s_N\ 0 \cdots\ 0]^{\mathrm{T}}$. This is done with the command `kron(s,[1;zeros(P-1,1)])`.

2. Test your routines for $\boldsymbol{\tau} = [0.1\ 0.6]^{\mathrm{T}}$ and $\boldsymbol{\beta} = [1e^{j\phi_1}\ 0.7e^{j\phi_2}]^{\mathrm{T}}$ with random ϕ_1 and ϕ_2. Take an oversampling factor of $P = 5$ and a burst length of $N = 50$. Make a plot of the real and imaginary part of $\mathbf{h}$ and $\mathbf{x}$. Can you detect the source $\mathbf{s}$ from $\mathbf{x}$ by visual inspection?

3. Construct the corresponding data matrix

$$\mathbf{X} = \begin{bmatrix} x(0) & x(1) & \cdots & x(N-1) \\ x(\frac{1}{P}) & x(1+\frac{1}{P}) & \cdots & x(N-1+\frac{1}{P}) \\ \vdots & \vdots & & \vdots \\ x(\frac{P-1}{P}) & x(1+\frac{P-1}{P}) & \cdots & x(N-1+\frac{P-1}{P}) \end{bmatrix} : P \times N$$

   What is the rank of $\mathbf{X}$? Explain.