SPCOM homework 3

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In this report, we will show our results in channel and symbol estimation.

I. Pilot-based channel estimation

We first decompose $\mathbf{X}$ using this equation:

$$\mathbf{X} = \mathbf{H}\mathcal{S}_L + \mathbf{N} \tag{1}$$

After that, based on the information of $\mathcal{S}_L$, which is constructed by data sequence $\mathbf{s}$ and signal $\mathbf{X}$ from antennas, we can estimates the channel by pseudo inverse operation. The results are shown in Fig 2. As we can see, both the real and imaginary part of the channel estimation are close to the true one.

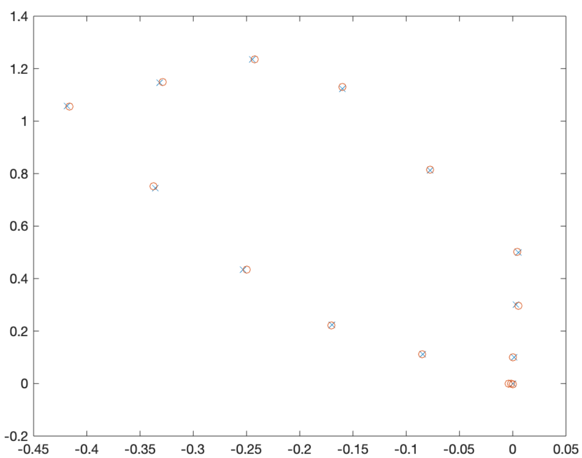

Fig. 1: True channel (x) versus Estimated channel (o)

II. Blind channel estimation

In order to estimate the channel $\mathbf{h}$ from the column null-space of $\mathbf{X}$, we can first establish this noiseless data model:

$$\mathbf{X} = \mathbf{H}\mathcal{S}_L \tag{2}$$

And the singular value decomposition of $\mathbf{X}$ is:

$$\mathbf{X} = \mathbf{U}\Sigma\mathbf{V}^H \tag{3}$$

where $\mathbf{U}$ can be written as $[\mathbf{U}_s\mathbf{U}_n]$ and $\mathbf{U}_n$ is the columns of $\mathbf{U}$ corresponding to the 0 singular values. Therefore, we have:

$$\mathbf{U}_n^H\mathbf{X} = 0 \tag{4}$$

Since $\mathbf{H}$ and $\mathbf{X}$ have the same column space, then we can rewrite Equation 4 as:

$$\mathbf{U}_n^H\mathbf{H} = 0 \tag{5}$$

After this, we will look into the structure of $\mathbf{H}$ matrix. $\mathbf{H} = [\mathbf{h}_0\mathbf{h}_1...\mathbf{h}_{L-1}]$. We can make all of the $\mathbf{h}$ vectors into one column and correspondingly re-arrange the Equation 5, which is shown in Equation 6.

$$\begin{bmatrix} \mathbf{U}_n^H & 0 & 0 \\ 0 & \mathbf{U}_n^H & 0 \\ 0 & 0 & \mathbf{U}_n^H \end{bmatrix} \begin{bmatrix} \mathbf{h}_0 \\ \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix} = \mathbf{0} \tag{6}$$

We denote the first matrix is $\mathbf{U}_{n,T}$.Therefore, the re-arranged $\mathbf{h}$ vector is in the null space of the $\mathbf{U}_{n,T}$. Then we can do singular value decomposition of this matrix and find the null-space.

Here, the basis of the null space has 9 vectors, thus there are infinity solutions of $\mathbf{h}$, we cannot estimate channel using blind estimation.

III. Blind symbol estimation

Here we can use our data model again shown in Equation 2. But now we focus on matrix $\mathcal{S}_L$.

We can first do singular value decomposition of $\mathbf{X}$, after that, we divide $\mathbf{V}$ into two parts instead of $\mathbf{U}$. The $\mathbf{X}$ has rank = 3, and the division is $[\mathbf{V}_s\mathbf{V}_n]$ where $\mathbf{V}_s$ corresponds to the non-zero singular values. So we will have:

$$\mathbf{X}\mathbf{V}_n = 0 \tag{7}$$

Since $\mathbf{X}$ and $\mathcal{S}_L$ have the same row space, thus we have:

$$\mathcal{S}_L\mathbf{V}_n = 0 \tag{8}$$

By investigating the structure of $\mathcal{S}_L$, in our case we can re-arrange Equation 8 into the following one: We do singular

$$[s_{-2}\ \ s_{-1}\ \ s_0 \cdots s_{N-1}] \cdot \begin{bmatrix} V_n & 0 & 0 \\ 0 & V_n & 0 \\ 0 & 0 & V_n \end{bmatrix} = 0$$

value decomposition of the second matrix. We found that there is a singular value is equal to 0 (actually it's not 0 but a value that smaller than any other singular values). Then we just pick up this column as the estimation of symbols.

However, there is still a scalar between the estimated symbol vector and the true symbol vector. We address this problem by moving the cluster in the first quadrant to (1+0j), and use the ratio to map all the other values. The result is shown in Fig 2. They do match.

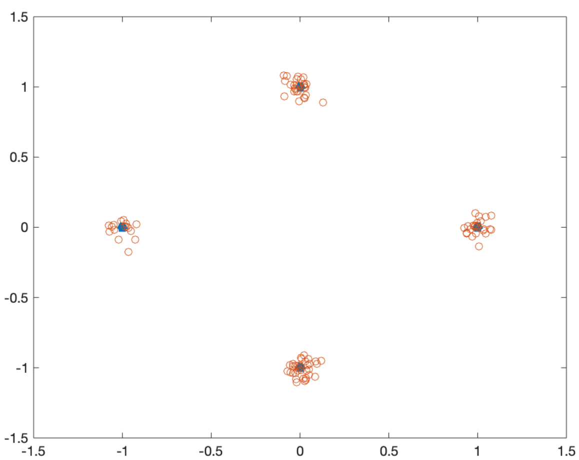

Fig. 2: The estimated symbols (red) and the true symbols (blue)

Appendix A
Pilot

```
function h = pilot(X,s)
% just compute the pinv?
[P,N] = size(X);
L = 3;
S_L = zeros(L,N);
for i = 1:L
    for j = 1:N
        S_L(i,j+i-1) = s(j);
    end
end
S_L = S_L(([1:L]),([1:N]));
H = X*pinv(S_L);
[a1,a2] = size(H);
h = zeros(L*P,1);
ll = 1;
for i = 1:a2
    for j = 1:a1
        h(ll) = H(j,i);
        ll = ll + 1;
    end
end
end
```

Appendix B
Test 1

```
%% test 1
%% question 1
tau = [0.1,0.6]';
% generate beta vector
ph1 = rand()*2*pi;
ph2 = rand()*2*pi;
beta = [exp(1i*ph1),0.7*exp(1i*ph2)]';
%% generate x and X
P = 5;
N = 100;
L = 3;
SNR = 30;
s = source(N);
h = channel(tau,beta,L,P);
X = gen_data1(h,s,P,N,SNR);
h_ = pilot(X,s);
figure(1)
title('True versus Estimated')
plot(h,'x');
hold on
plot(h_,'o');
```

Appendix C
Blind channel

```
function h = blind_channel(X)
%
[S,V,D] = svd(X);
U_n = S(:,([4:5]));
% Construct U_T
[a1,a2] = size(U_n');
```

```
U_T = zeros(6,3*a2);
U_T(([1:2]),([1:5]))=U_n';
U_T(([3:4]),([6:10]))=U_n';
U_T(([5:6]),([11:15]))=U_n';
[S_,V_,D_] = svd(U_T);
end
```

Appendix D
Blind Symbol

```
function s_ = blind_symbol(X)
%
[S,V,D] = svd(X);
V_n = D(:,[4:100]);
V_t = zeros(102,97*3);
V_t([1:100],[1:97]) = V_n;
V_t([2:101],[98:2*97]) = V_n;
V_t([3:102],[2*97+1:3*97]) = V_n;
[s,v,d]=svd(V_t);
s_ = s([3:102],102);
end
```

Appendix E
Test 2

```
%% test 2
%% question 2
tau = [0.1,0.6]';
% generate beta vector
ph1 = rand()*2*pi;
ph2 = rand()*2*pi;
beta = [exp(1i*ph1),0.7*exp(1i*ph2)]';
%% generate x and X
P = 5;
N = 100;
L = 3;
SNR = 30;
s = source(N);
h = channel(tau,beta,L,P);
X = gen_data1(h,s,P,N,SNR);
% h_ = blind_channel(X);
s_ = blind_symbol(X);
cluster = zeros();
k = 1;
for i = 1:N
    if real(s_(i))>0 && imag(s_(i))>0
        cluster(k) = s_(i);
        k = k + 1;
    end
end
s_est = mean(cluster);
Ratio = 1/s_est;
figure(1)
title('True versus Estimated')
plot(s,'^','markersize',5,'linewidth',10);
hold on
plot(s_*Ratio,'o');
```