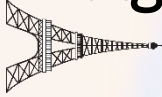
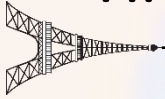


Part 2

Inter-chirp Waveform Modulation Techniques

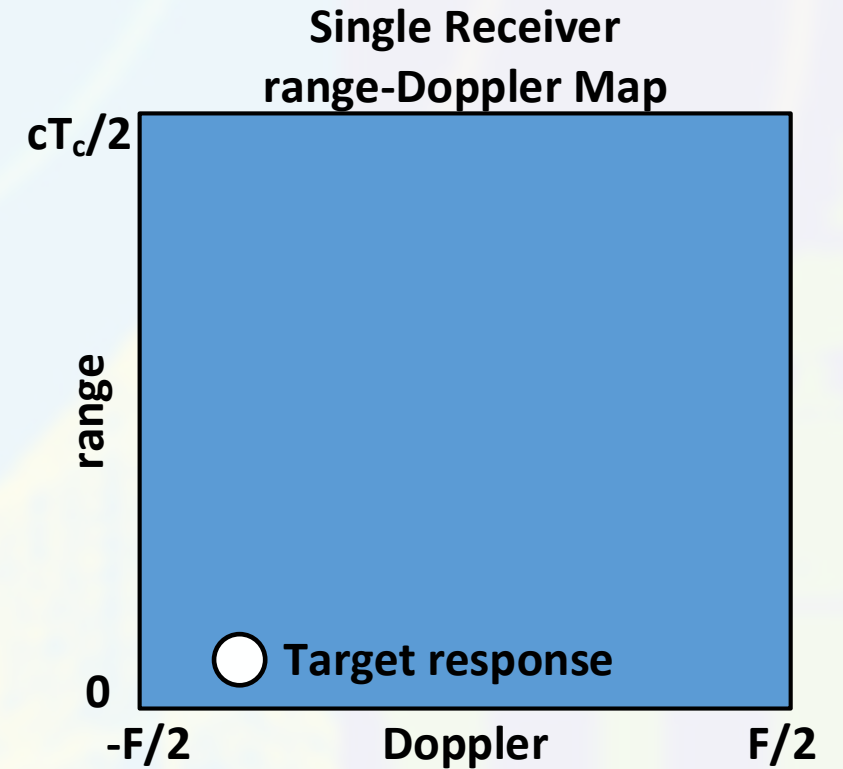
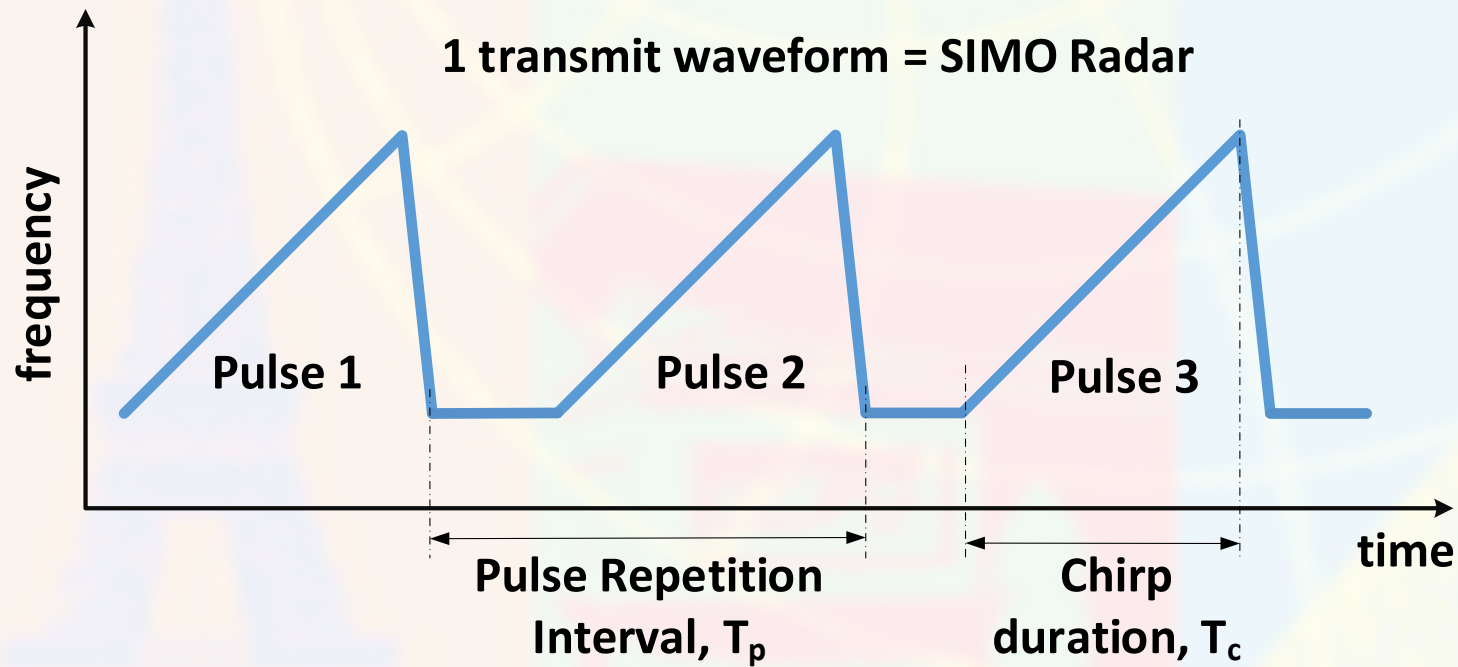


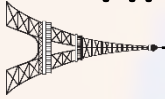
- **Inter-Pulse Modulation Techniques**
 - **Time Division Multiplexing (TDM)**
 - **Frequency Division Multiplexing (FDM)**
 - **Doppler Division Multiplexing (DDM)**
 - **Binary Phase Modulation (BPM)**
 - **Code Division Multiplexing (CDM)**



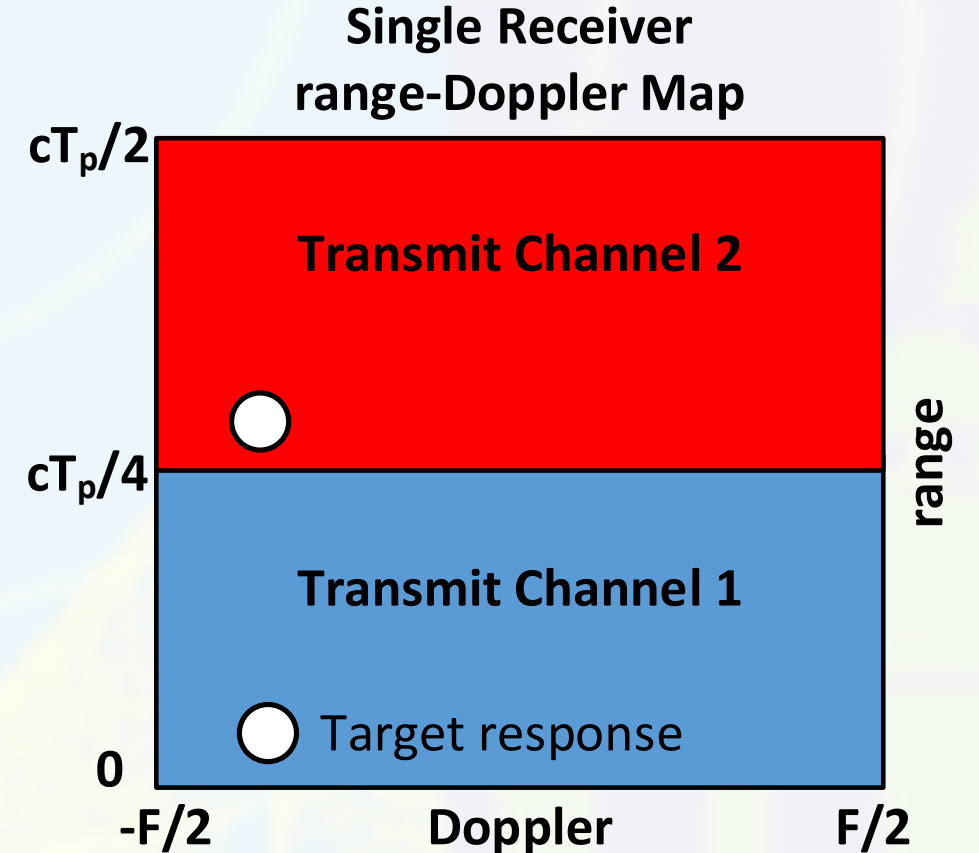
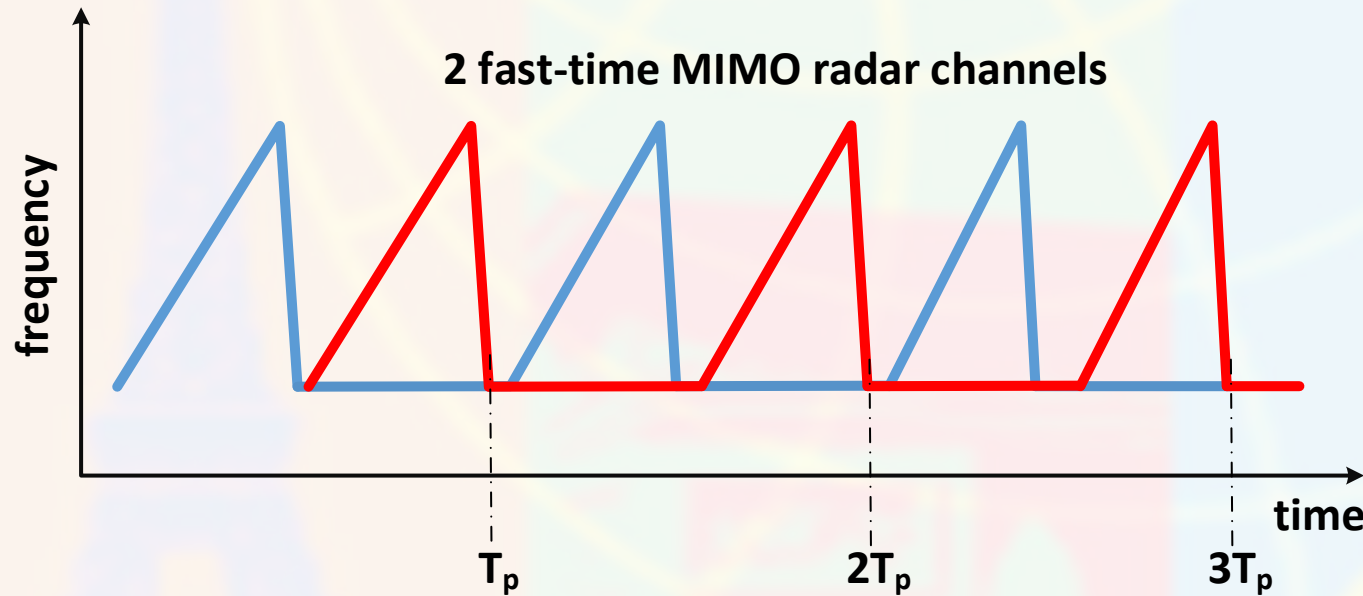
Time Division Multiplexing (TDM)

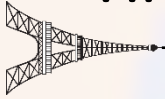
$$F = \frac{1}{T_p}$$



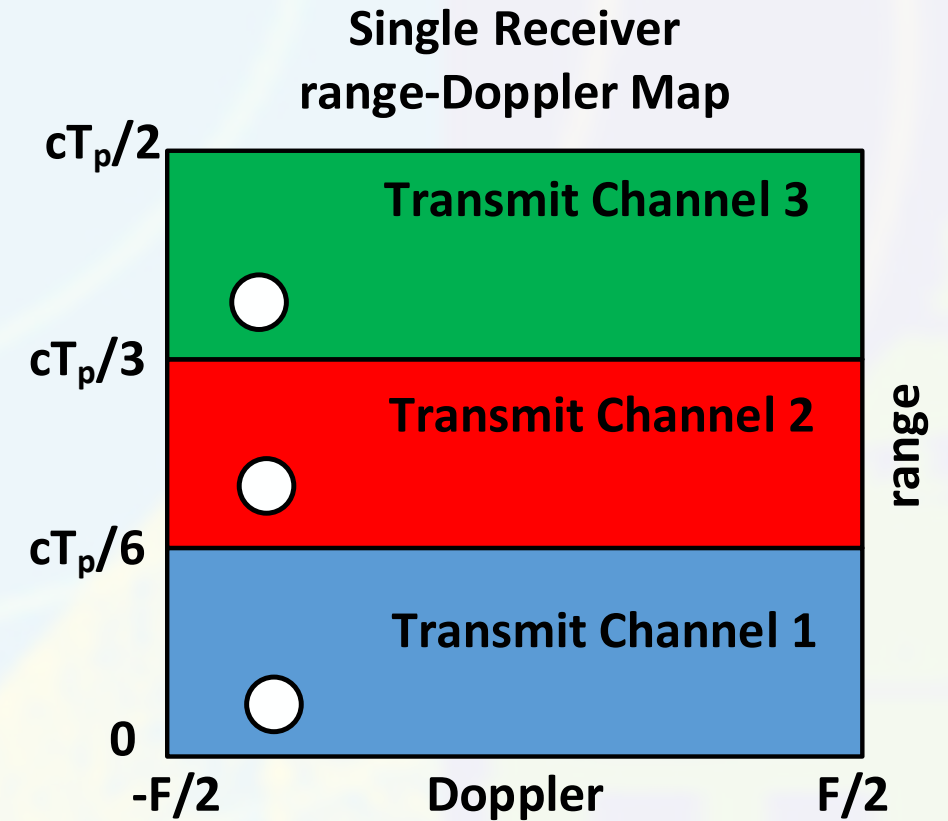
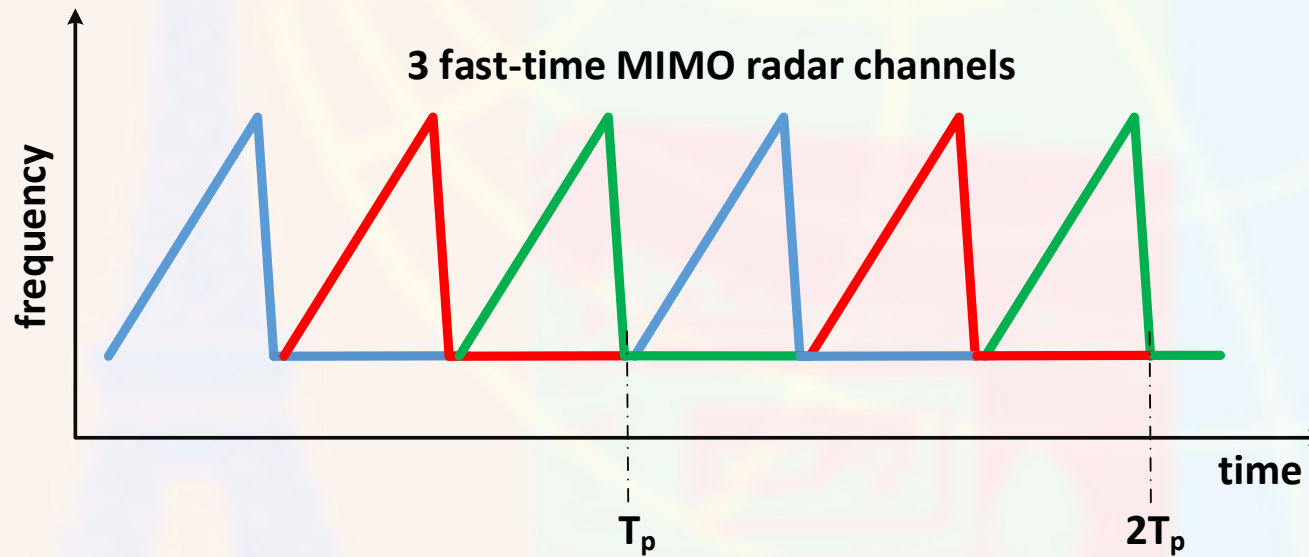


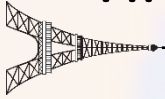
Time Division Multiplexing (TDM)





Time Division Multiplexing (TDM)





Time Division Multiplexing (TDM)

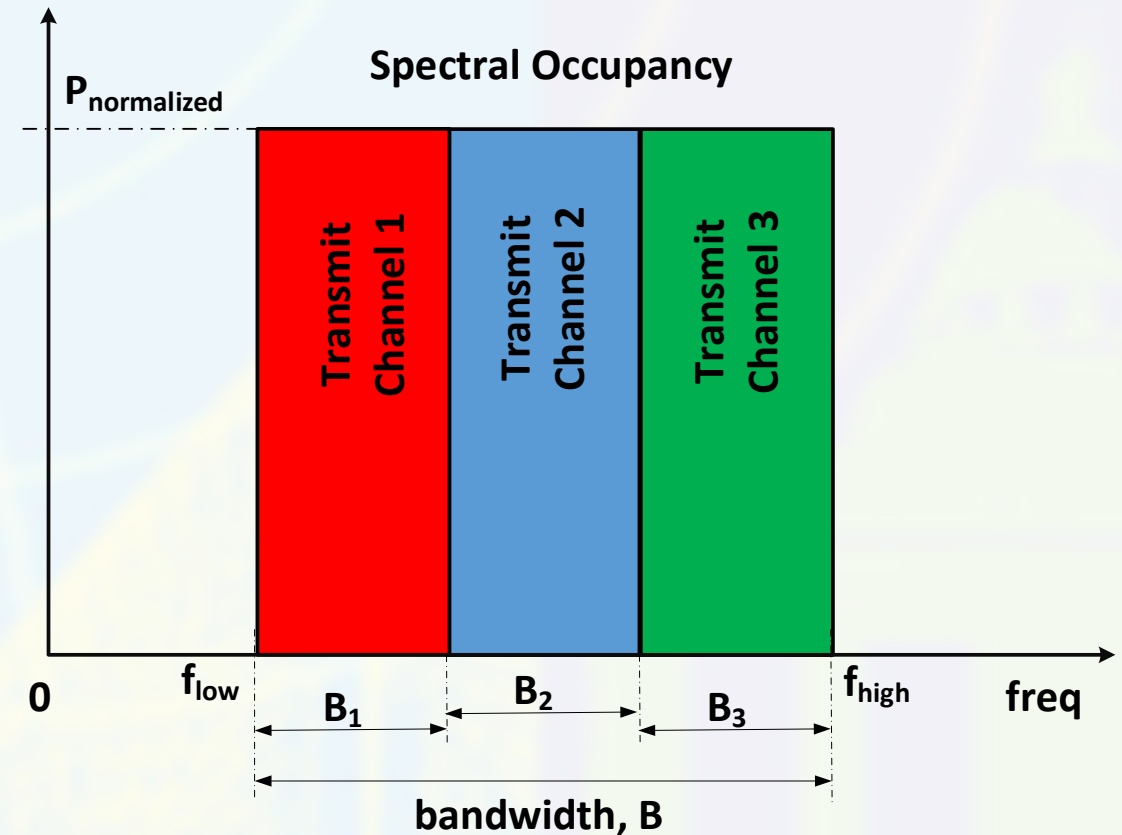
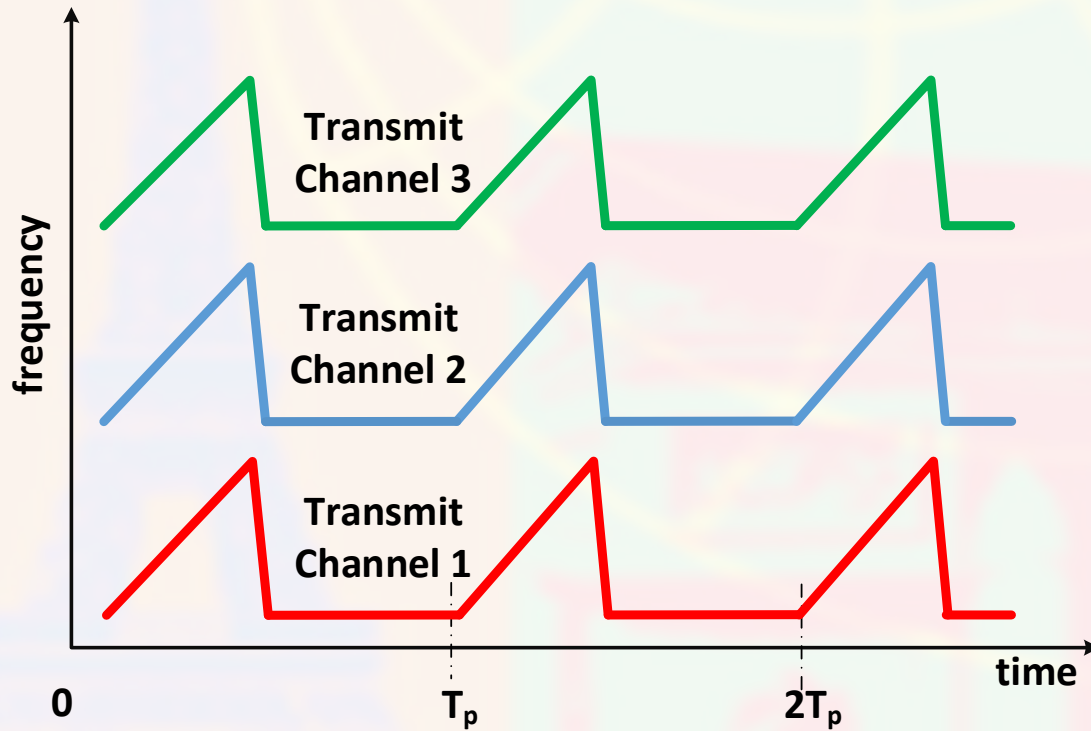
- TDM scheme requires least demanding hardware
- Transmission switches from one antenna to another sequentially within a pulse
- Simple radars with mass manufacturing requirement need to be cost efficient and majorly apply TDM scheme

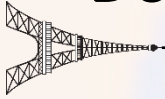
Frequency Division Multiplexing (FDM)

- Easy to implement with minimal hardware complexity
- range resolution compromised for more channels

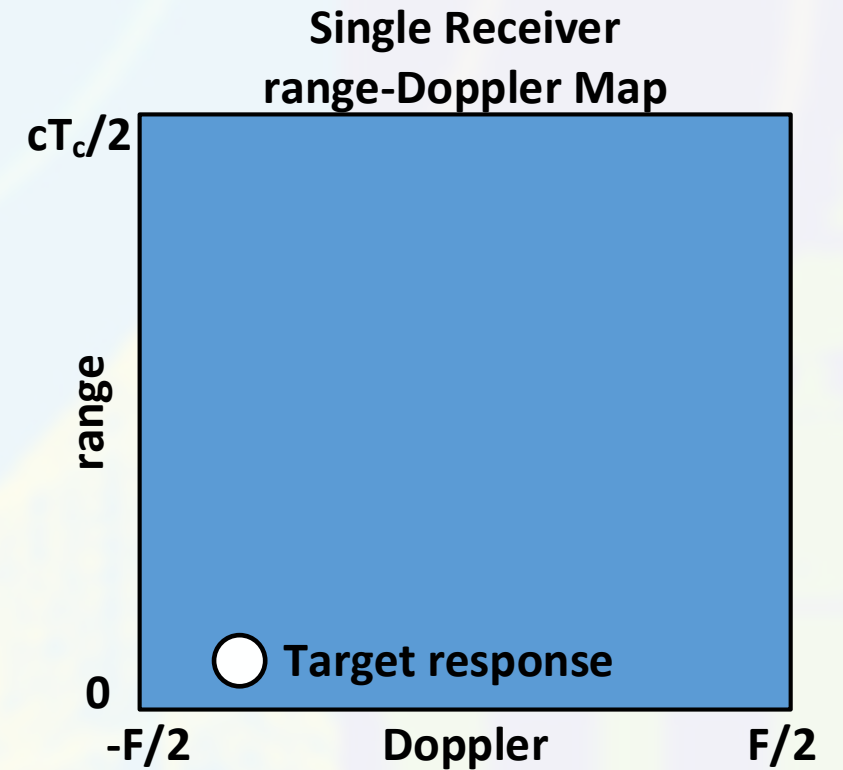
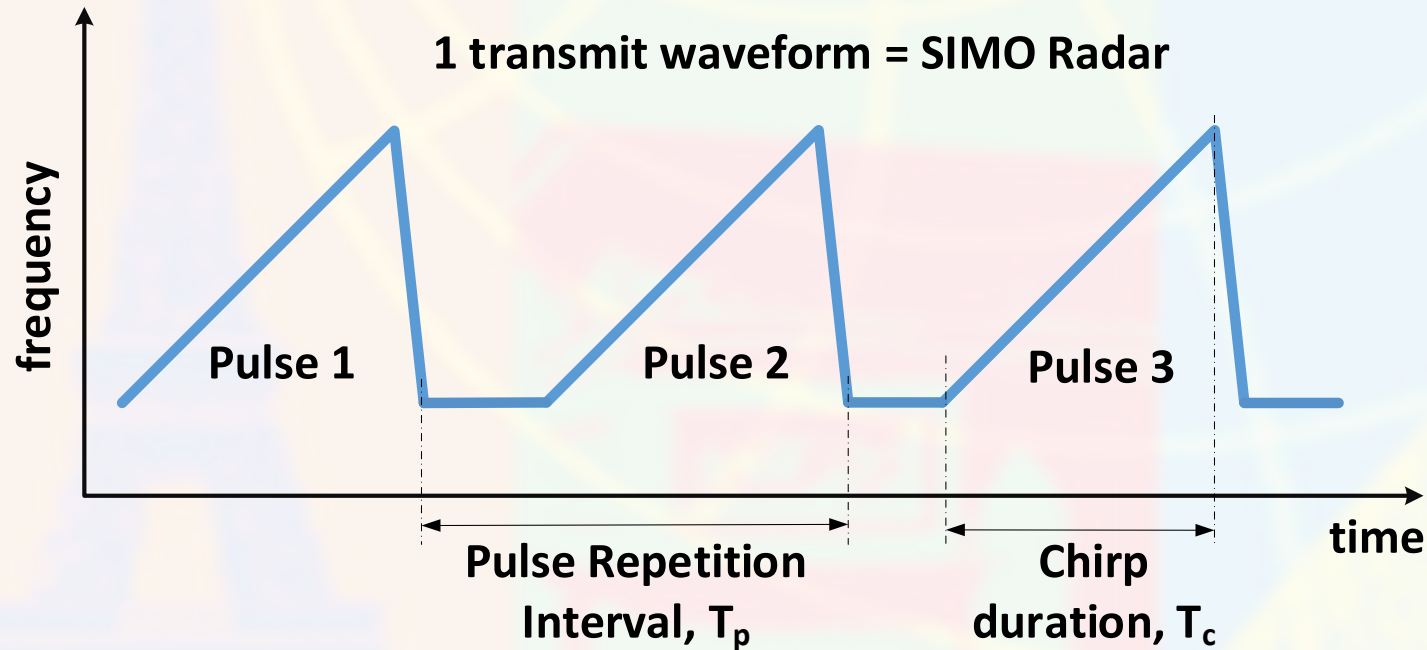
$$\text{range resolution} = \frac{c}{2B},$$

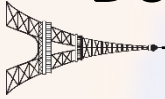
where c = speed of light, B = bandwidth.





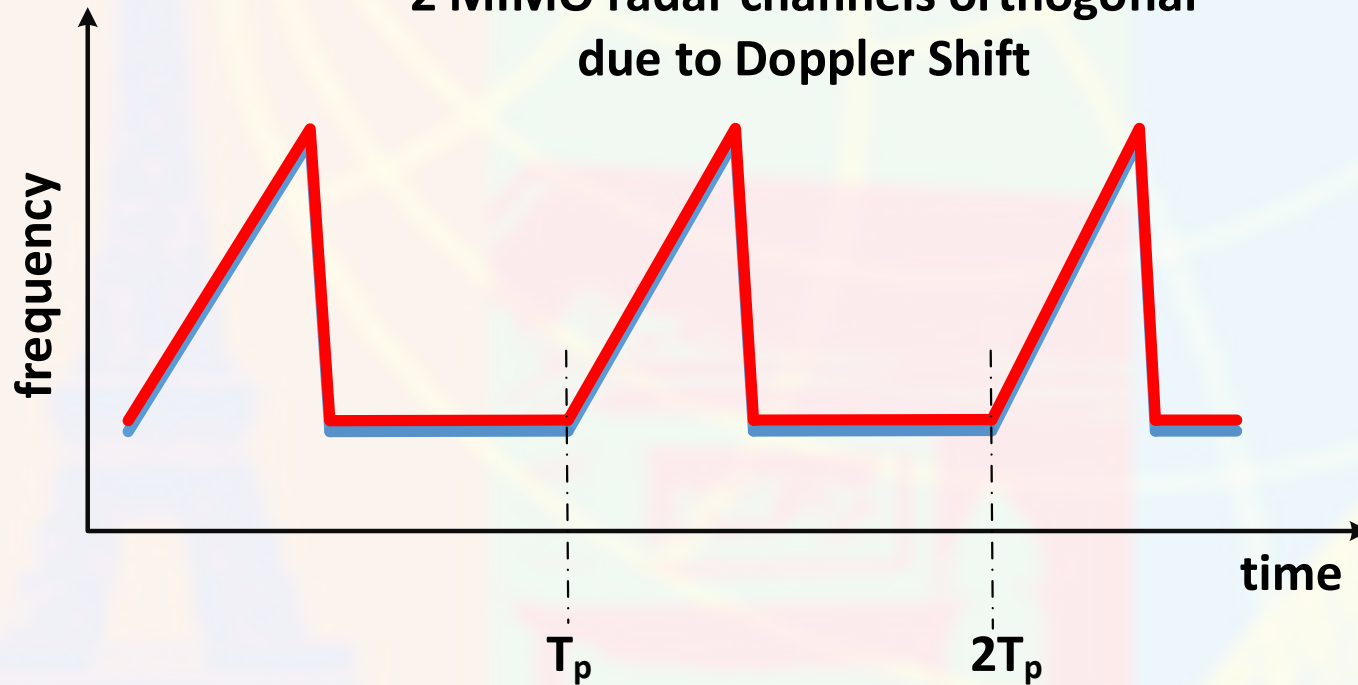
Doppler Division Multiplexing (DDM)



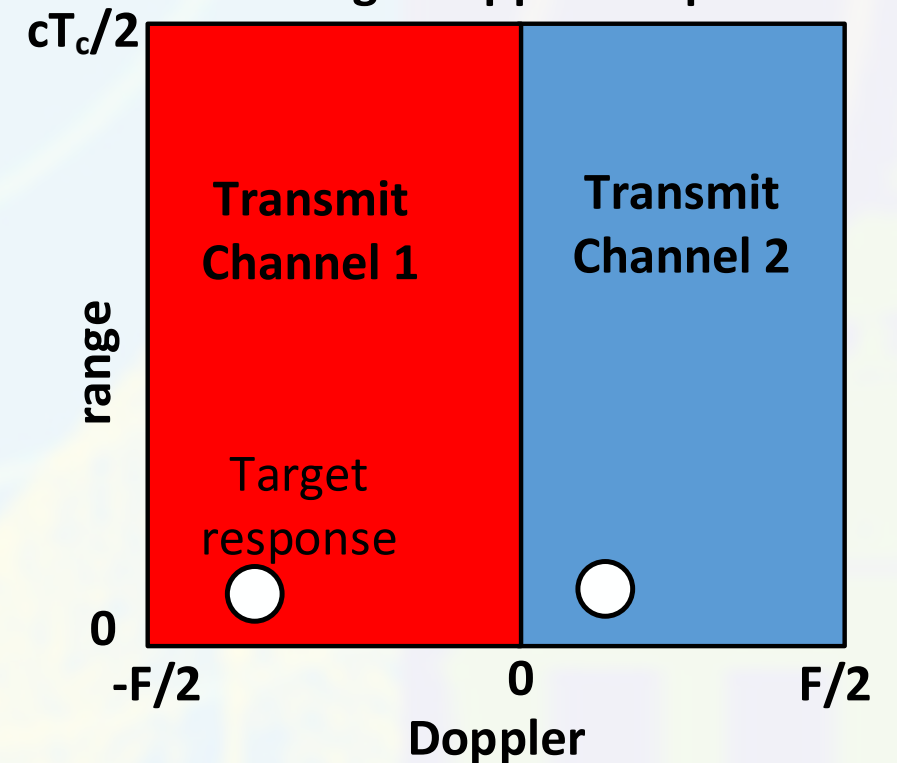


Doppler Division Multiplexing (DDM)

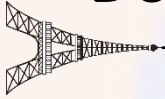
2 MIMO radar channels orthogonal
due to Doppler Shift



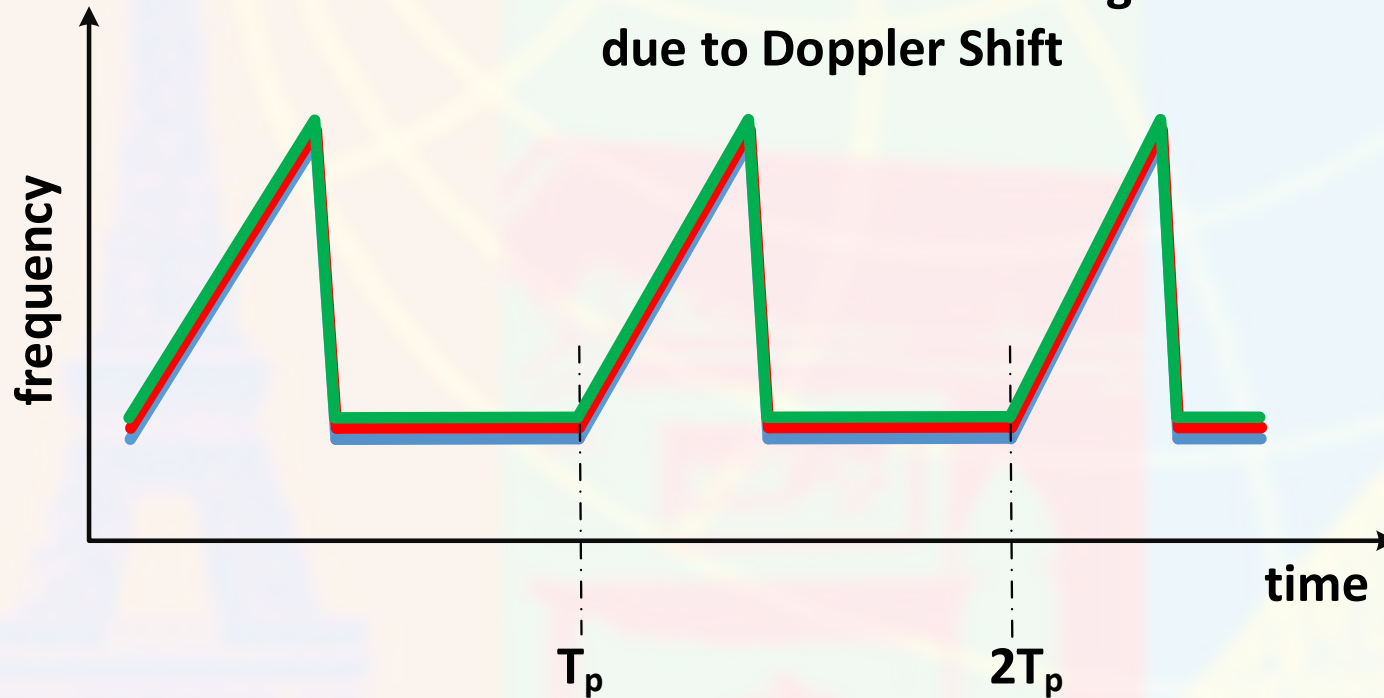
Single Receiver
range-Doppler Map



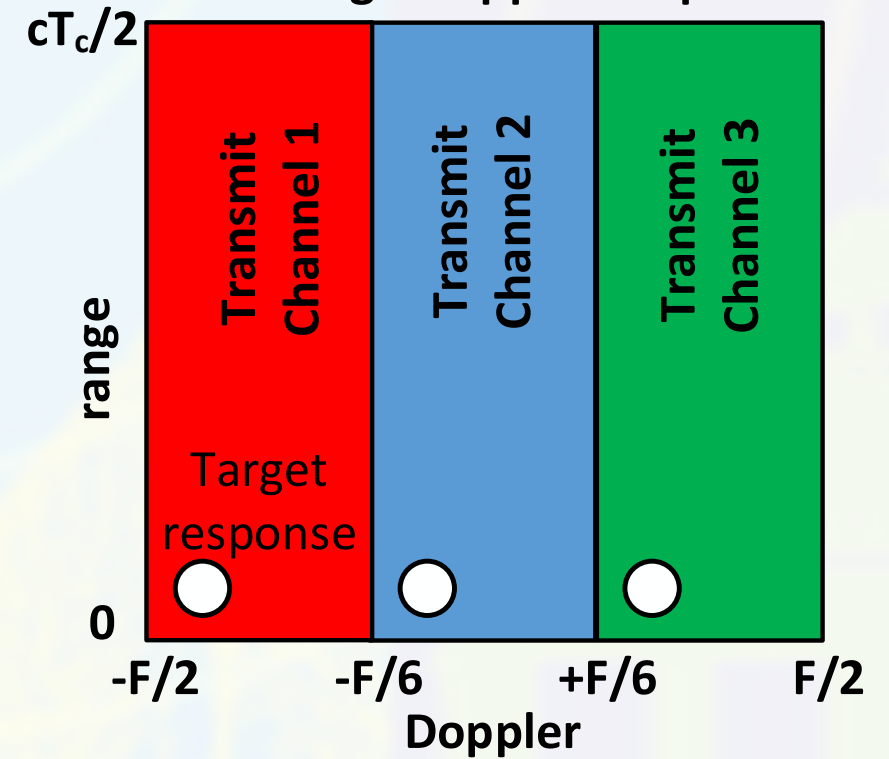
Doppler Division Multiplexing (DDM)

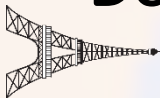


3 MIMO radar channels orthogonal
due to Doppler Shift

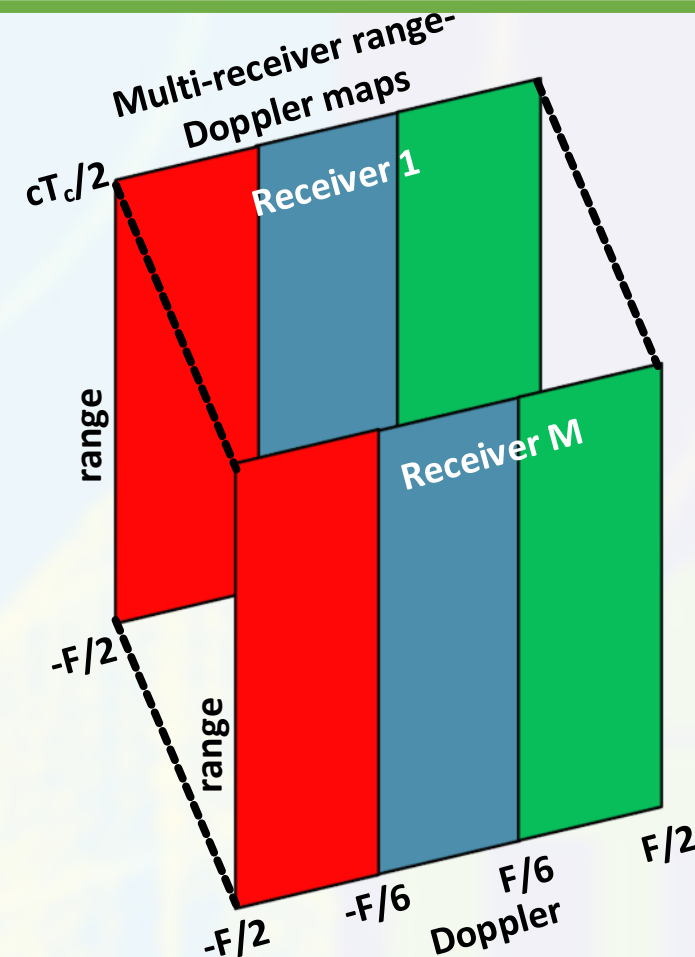
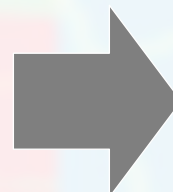
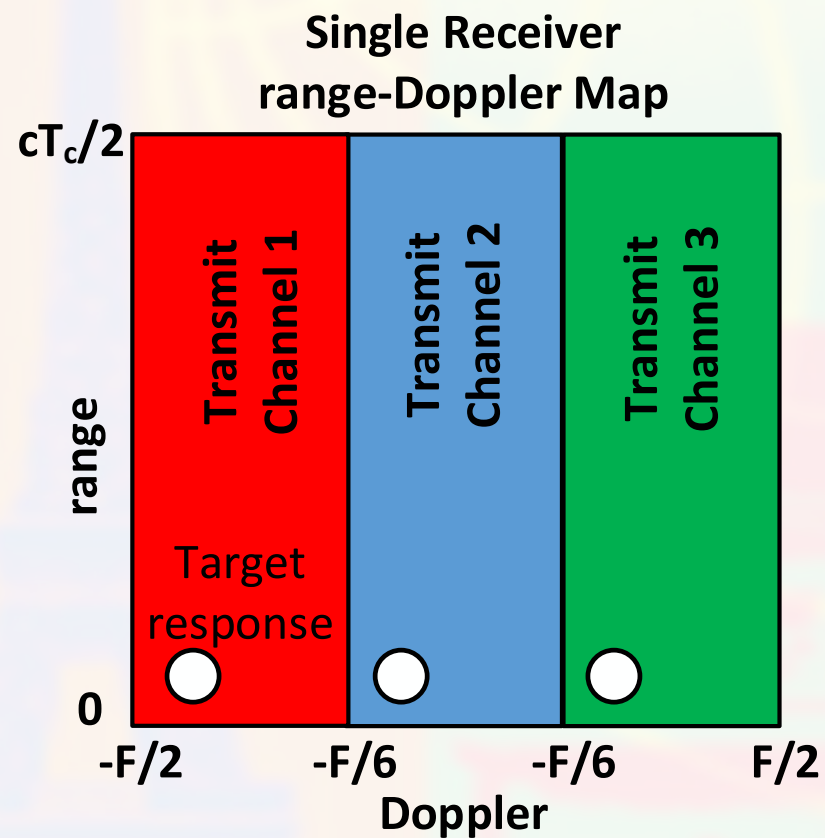


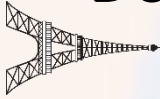
Single Receiver
range-Doppler Map





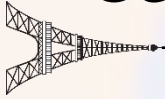
Doppler Division Multiplexing (DDM)





Doppler Division Multiplexing (DDM)

- DDM scheme requires advanced transmission hardware
- Consists of dedicated phase shifters with defined alphabet size at every transmit antenna
- Automotive radars have vastly applied this transmission scheme
- Requires identification of the peak corresponding to the real Doppler from the available aliased peaks
- Additional processing steps involved at the receiver



General Transmit Signal Model

$$\mathbf{u}_{m,q}(t) = w_{m,q} e^{j\phi(t)} \frac{1}{T_c} \text{rect}\left(\frac{2t}{T_c}\right)$$

where $\phi(t) = 2\pi(f_c t + \frac{1}{2} S t^2)$,

$m = 1, \dots, M$, $q = 1, \dots, Q$, and

$$w_{m,q} = \begin{cases} \delta[m - \text{mod}(q - 1, M) - 1], & \text{TDM}^\# \\ e^{-j2\pi m q / M}, & \text{DDM} \\ \text{Had}[m, \text{mod}(q - 1, M) + 1], & \text{BPM} \end{cases}$$

M : transmit antenna elements

T_c : transmit chirp duration

f_c : center frequency

S : chirp slope

Q : transmit pulses

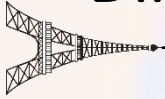
Had: Hadamard Matrix

δ : Dirac delta function

mod: modulo operation

[#] N. K. Sichani et al., "Waveform Selection for FMCW and PMCW 4D-Imaging Automotive Radar Sensors," 2023

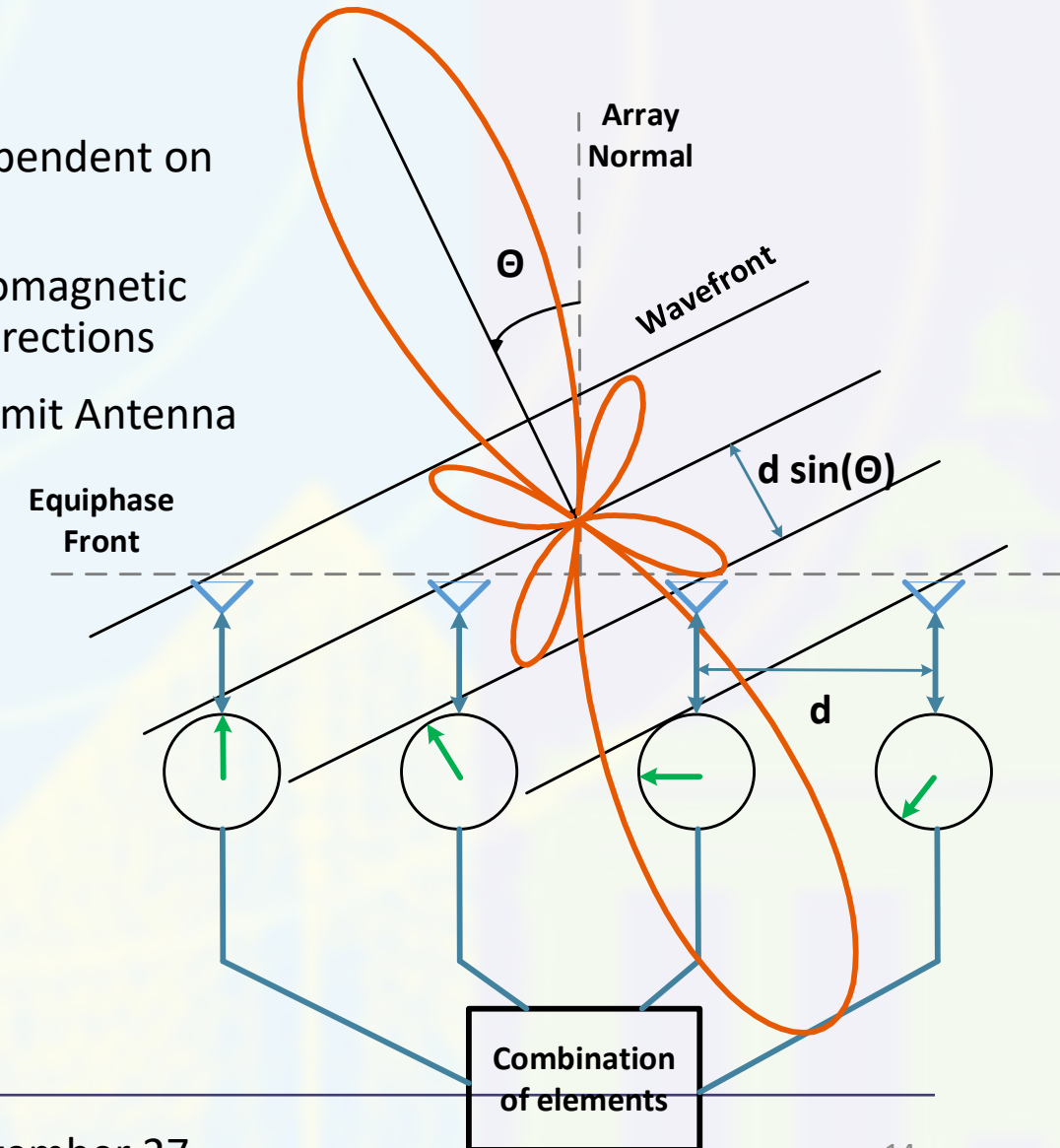
IEEE Radar Conference (RadarConf23), San Antonio, TX, USA, 2023, pp. 1-6, doi: 10.1109/RadarConf2351548.2023.10149733.



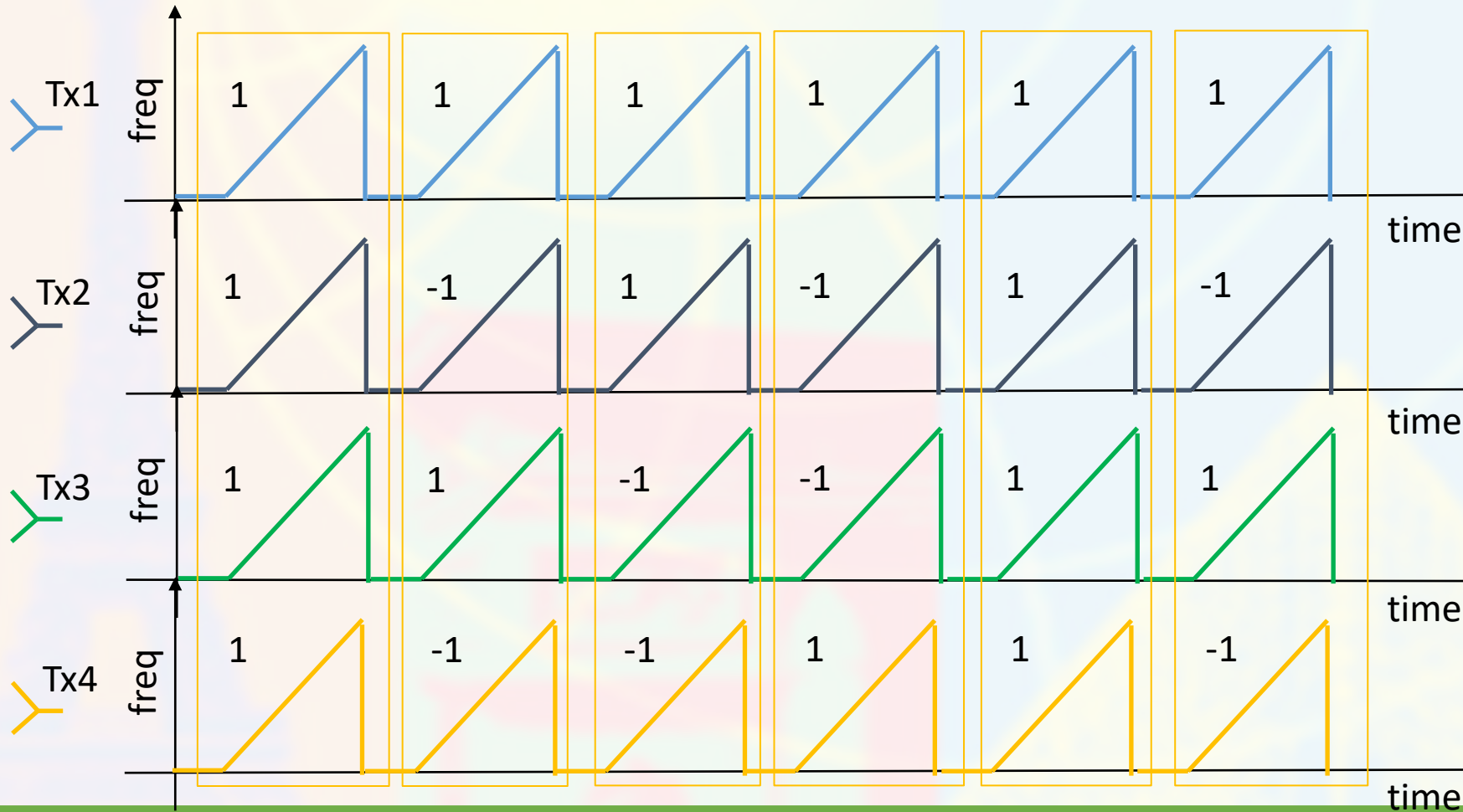
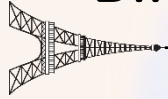
Directivity / Beampattern and ULA

- Radiated/received energy from/at radar antenna array is dependent on angle
- Beampattern is a visual representation of intensity of electromagnetic energy transmitted/received by the antenna in all angular directions
- An antenna with Uniform Linear Array layout with one Transmit Antenna and four Receive antennas (Figs) has a steering vector as

$$\mathbf{a}(\theta) \triangleq \begin{bmatrix} a_1(\theta) \\ \vdots \\ a_m(\theta) \\ \vdots \\ a_M(\theta) \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ e^{-j\frac{2\pi}{\lambda}md\sin(\theta)} \\ \vdots \\ e^{-j\frac{2\pi}{\lambda}Md\sin(\theta)} \end{bmatrix}.$$



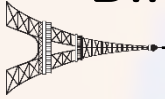
Binary Phase Modulation (BPM)



Hadamard Code

1	1	1	1
1	-1	1	-1
1	1	-1	-1
1	-1	-1	1

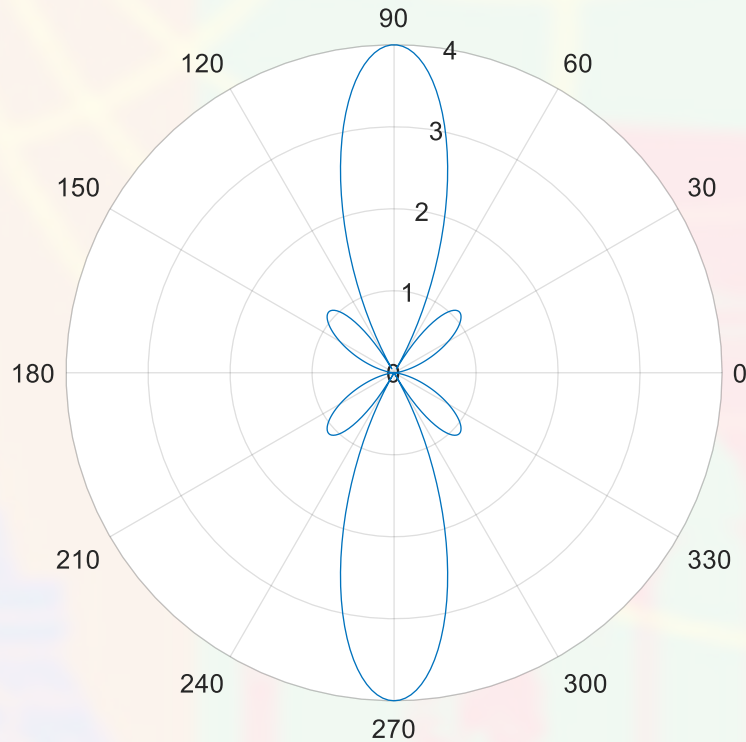
Binary Phase Modulation (BPM)



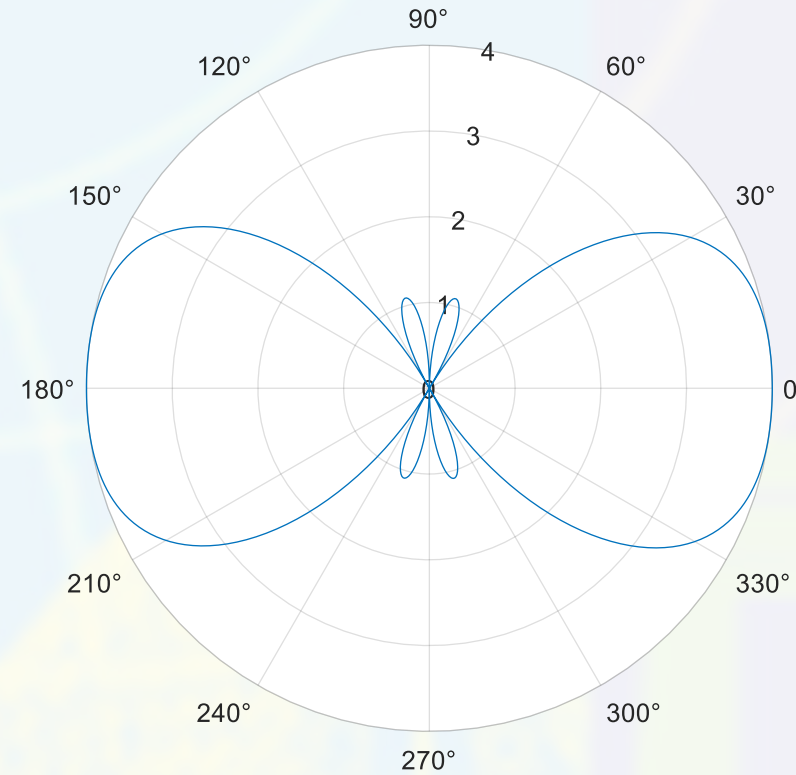
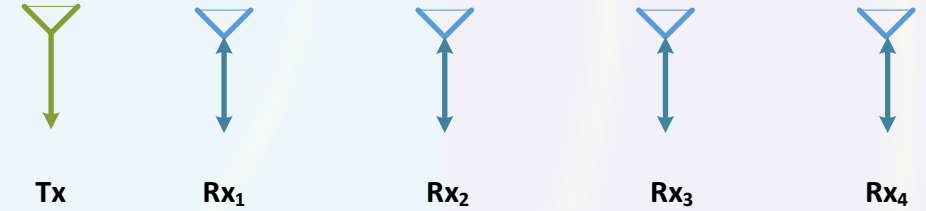
Filled Array

- 1 Tx1
- 1 Tx2
- 1 Tx3
- 1 Tx4

Tx 1 beampattern



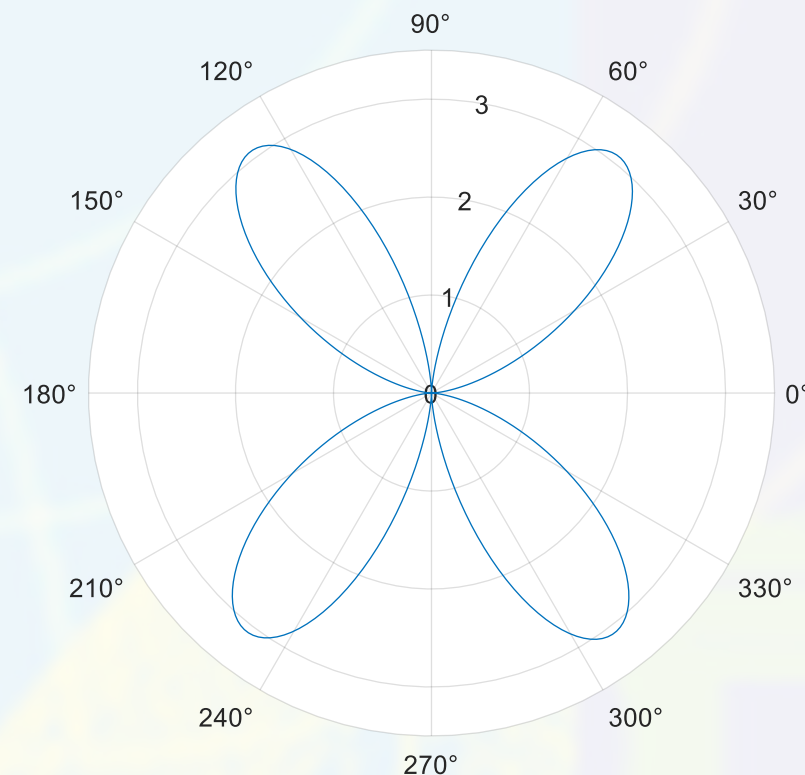
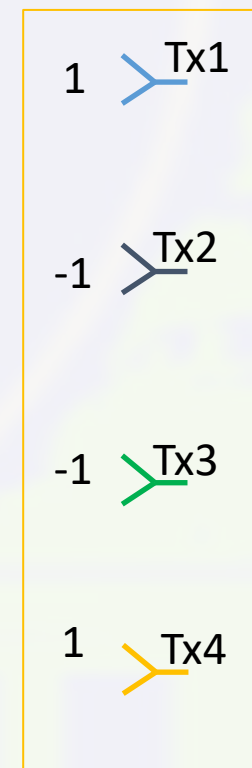
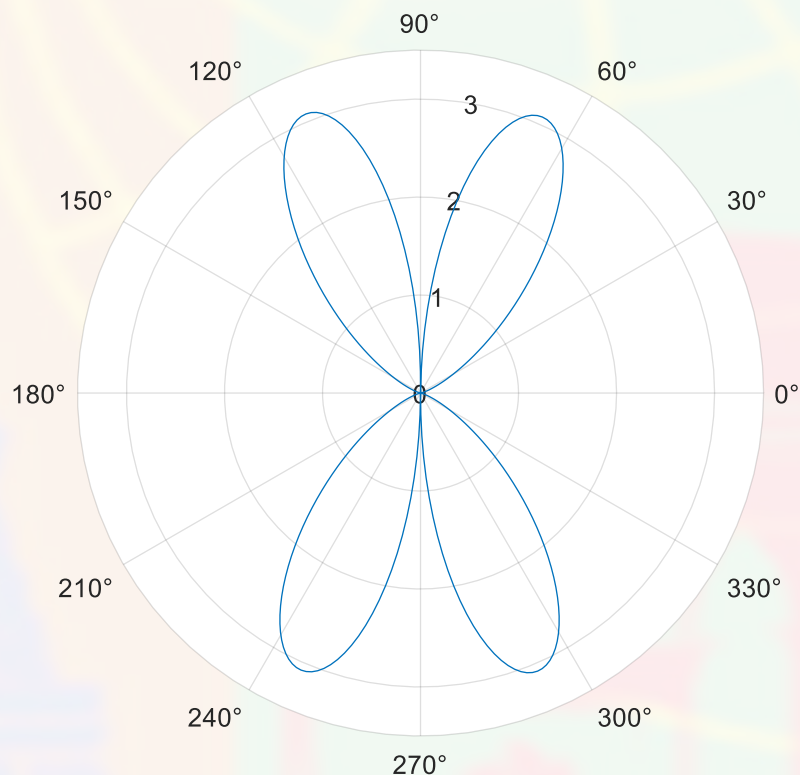
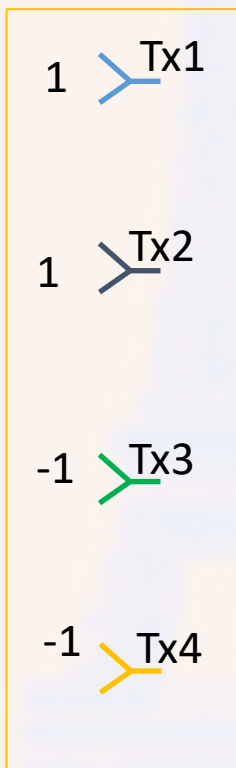
ULA (filled)

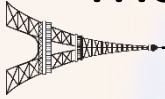


- 1 Tx1
- 1 Tx2
- 1 Tx3
- 1 Tx4

Binary Phase Modulation (BPM)

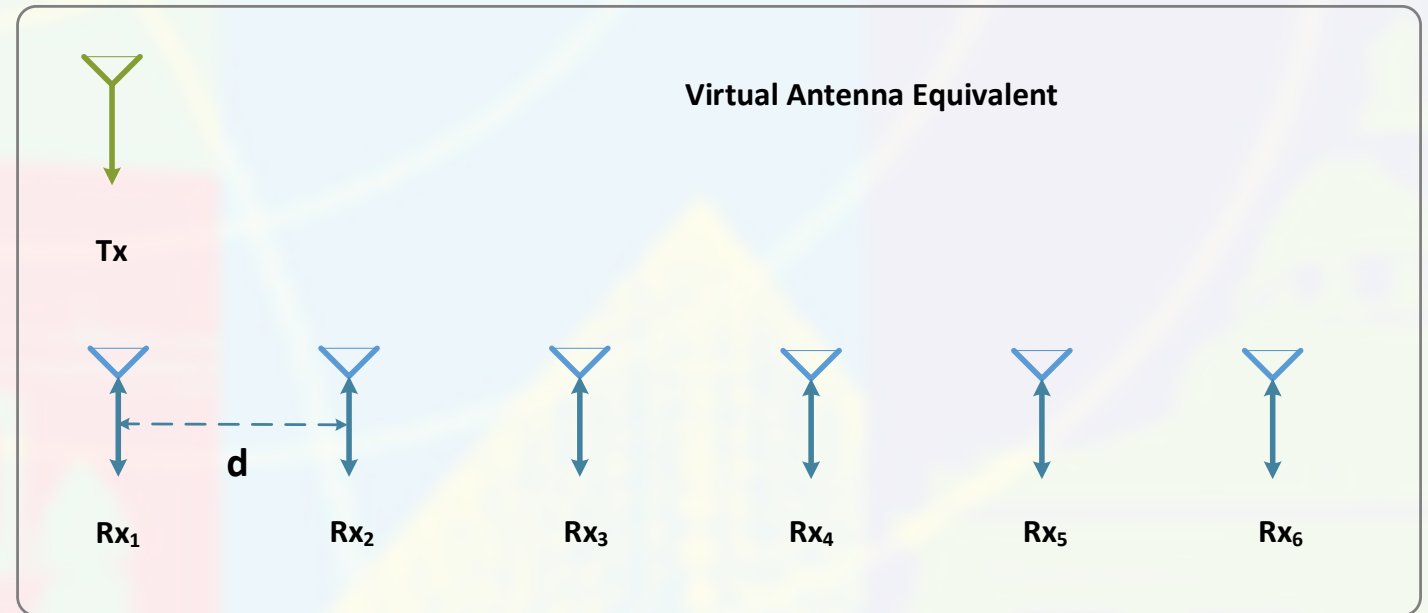
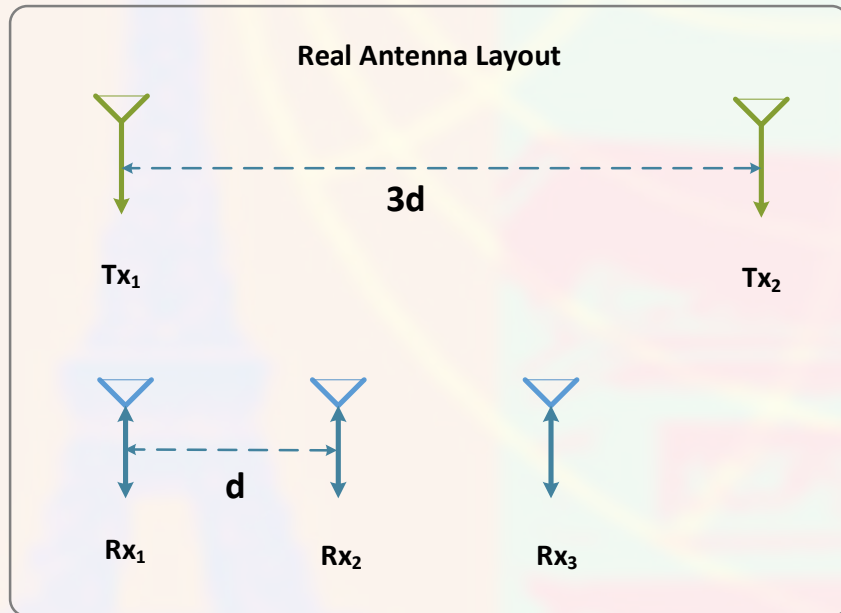
Filled Array





Magic of MIMO in Radars

- MIMO radar: it probes a channel by transmitting multiple signals (separated temporally, spectrally or spatially) and received with some similar multiplicity*.

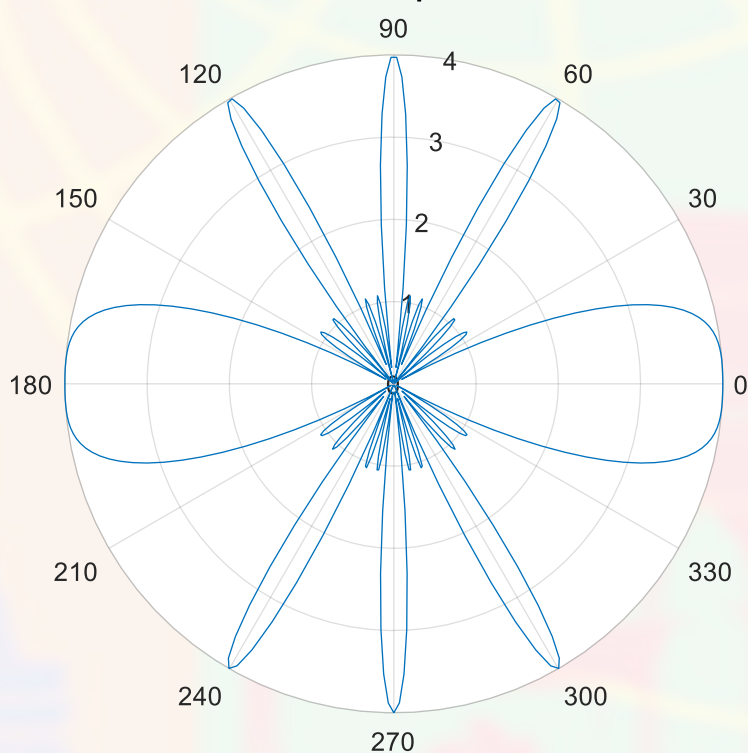


* D. W. Bliss and K. W. Forsythe, "Multiple-input multiple-output (MIMO) radar and imaging: degrees of freedom and resolution," *The Thirty-Seventh Asilomar Conference on Signals, Systems & Computers*, 2003, Pacific Grove, CA, USA, 2003, pp. 54-59 Vol.1, doi: 10.1109/ACSSC.2003.1291865.

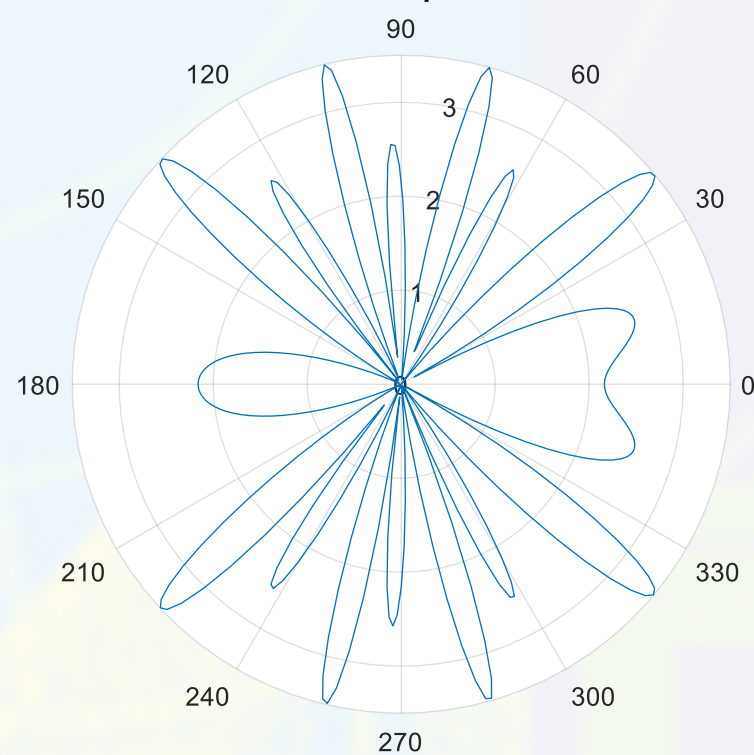
Binary Phase Modulation (BPM)

Sparse Array

Tx 1 beampattern



Tx 2 beampattern



1 Tx1

1 Tx2

1 Tx3

1 Tx4

1 Tx1

-1 Tx2

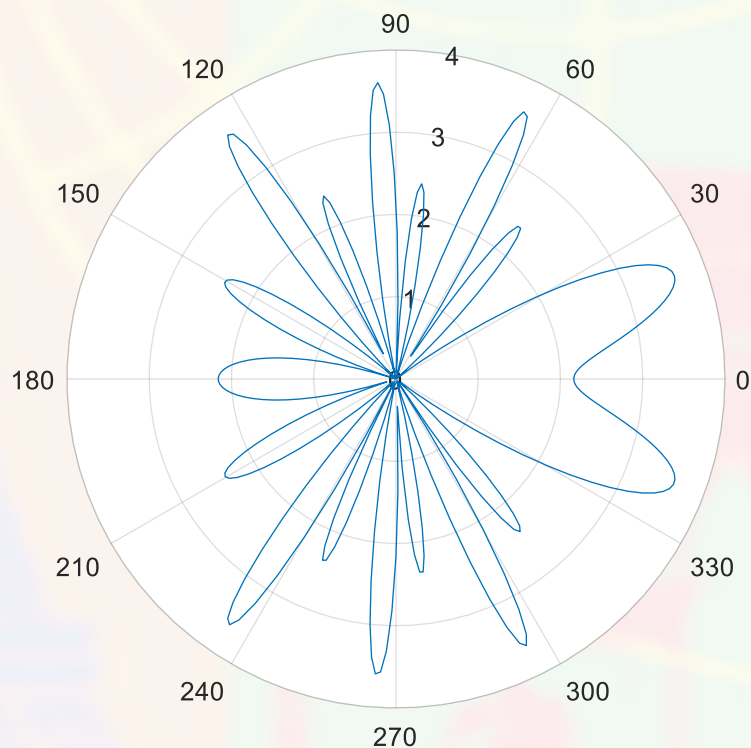
1 Tx3

-1 Tx4

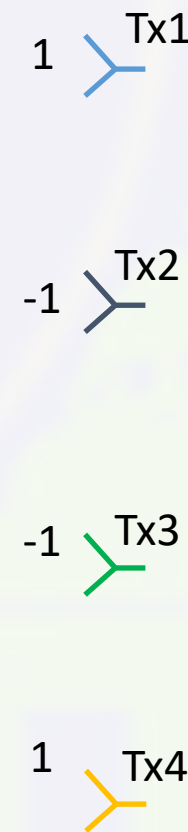
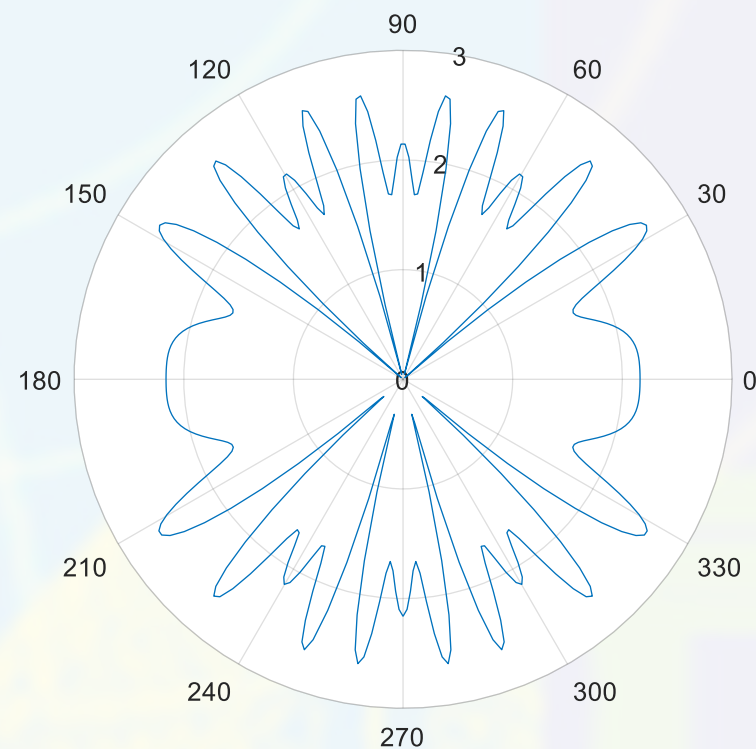
Binary Phase Modulation (BPM)

Sparse Array

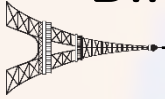
Tx 3 beampattern



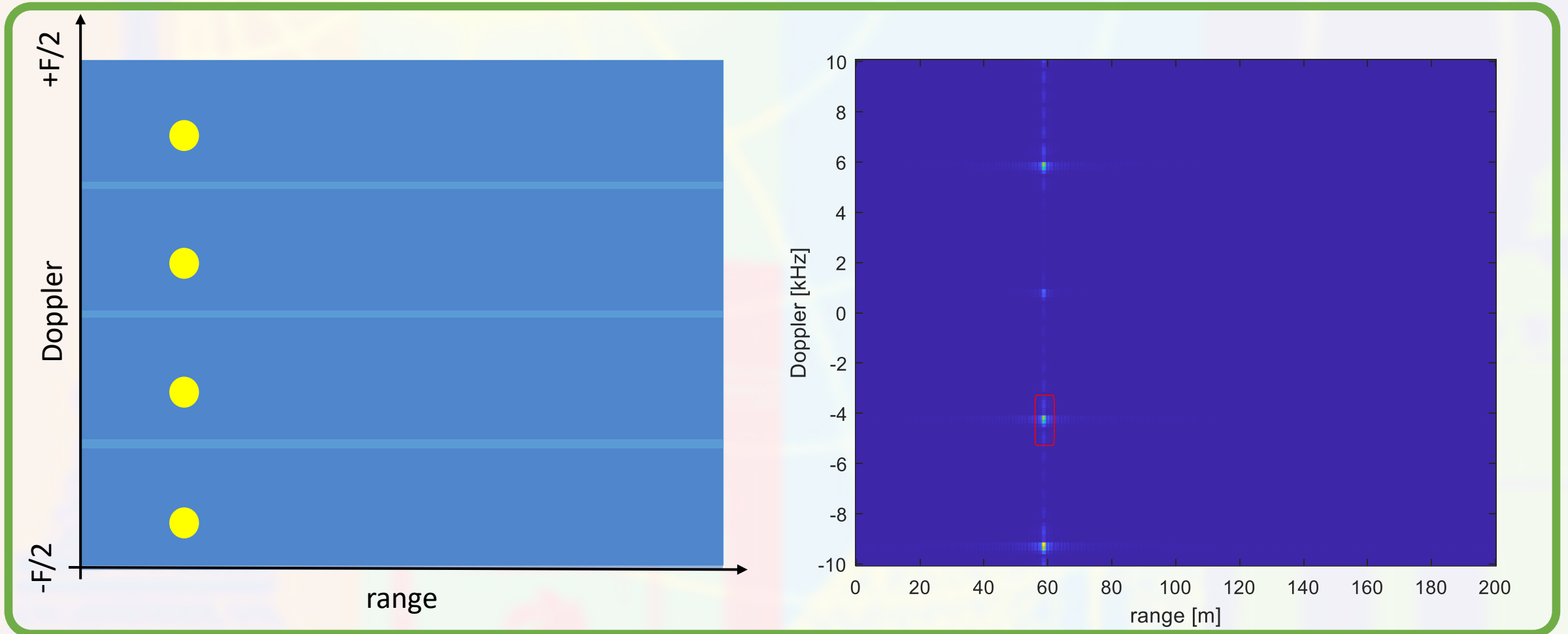
Tx 4 beampattern



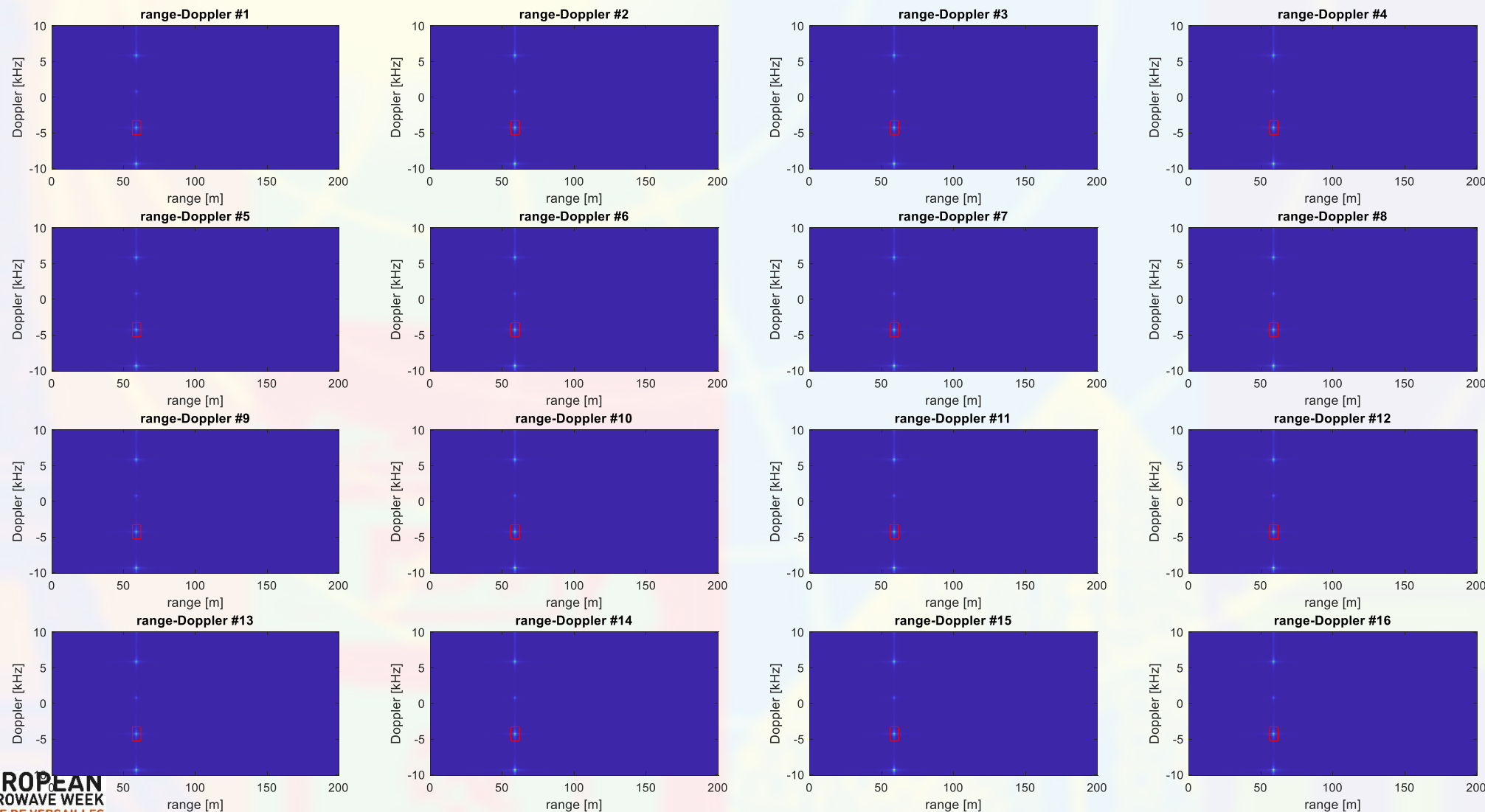
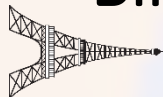
Binary Phase Modulation (BPM)

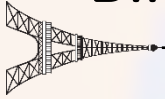


Doppler aliased clutter and targets appear in other channels



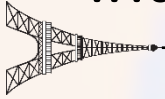
Binary Phase Modulation (BPM)





Binary Phase Modulation (BPM)

- BPM-MIMO makes use of the device's **full transmission capabilities** (because all the transmitters are active at any time)
- BPM-MIMO may not achieve perfect orthogonality between channels. As a result, when BPM-MIMO is used, **more sidelobes** are expected.
- BPM-MIMO reduces the **unambiguous Doppler range**



Inter-Pulse Code Division Multiplexing (CDM)

$$\mathbf{u}_{m,q}(t) = w_{m,q} e^{j\phi(t)} \frac{1}{T} \text{rect}\left(\frac{2t}{T}\right)$$

where $\phi(t) = 2\pi f_c t$,

$m = 1, \dots, M$, $q = 1, \dots, Q$, and

$w_{m,q} = Z(r, n)$, Zadoff-Chu Sequence Set

Further, $Z(r, n) = e^{-j \frac{\pi r n(n+1)}{N}}$,

where $n = 0, 1, \dots, N-1$, and

r is the root of the sequence coprime with N .

N : chips of duration T_{cp}

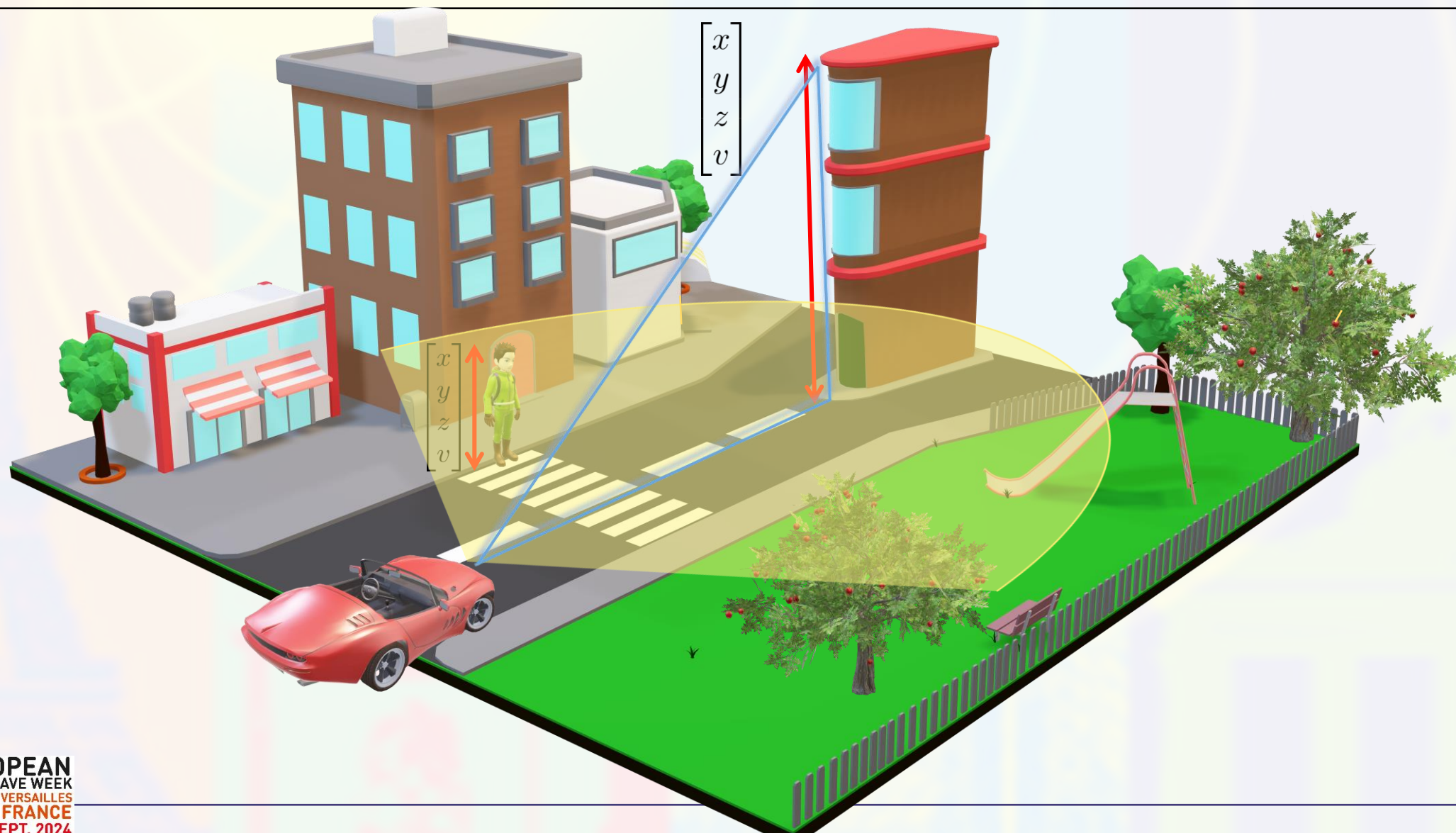
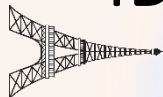
M : transmit sequences

T : transmit pulse duration

Q : transmit pulses

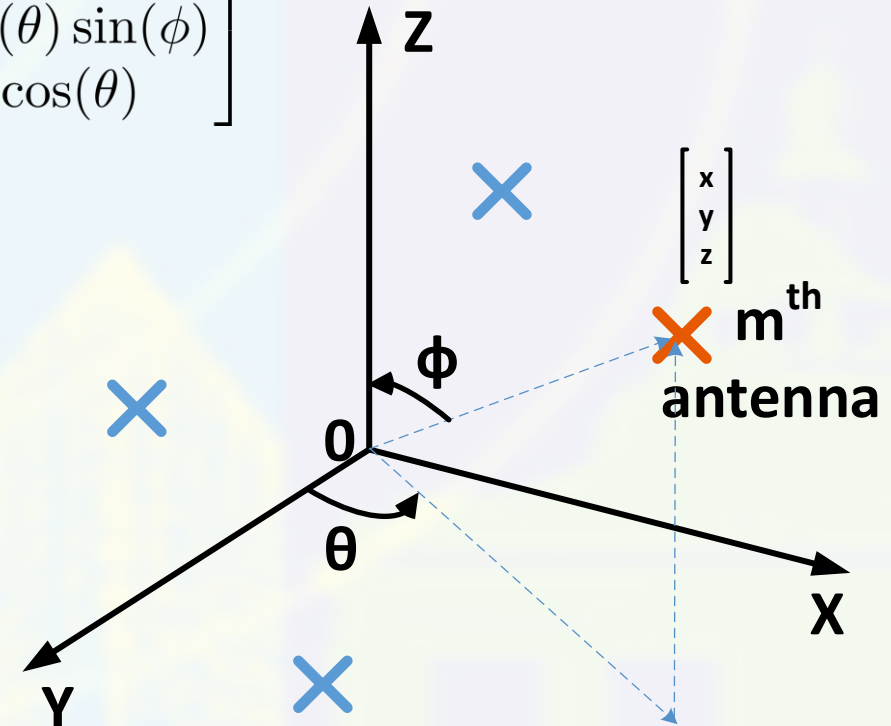
Z : Zadoff Chu Matrix

4D MIMO Radars

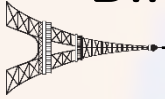


- Arbitrary position vector for the m^{th} antenna, $\mathbf{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{r} \triangleq \begin{bmatrix} \sin(\theta) \cos(\phi) \\ \sin(\theta) \sin(\phi) \\ \cos(\theta) \end{bmatrix}$
- Steering vector for the m^{th} antenna is

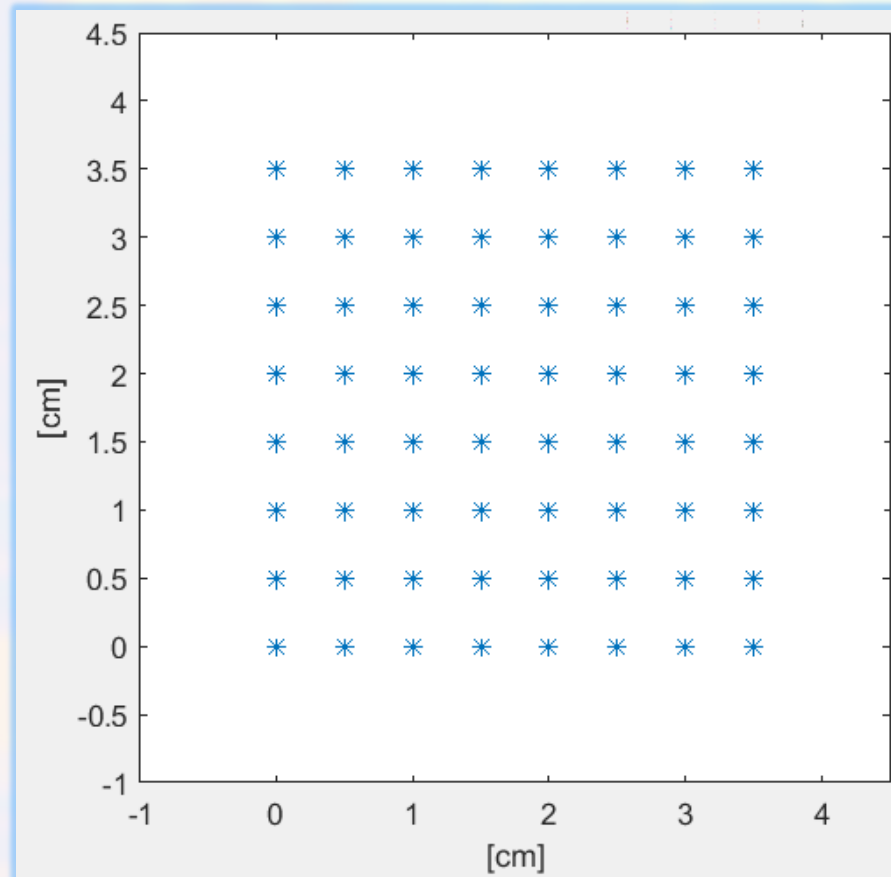
$$\mathbf{a}(\theta, \phi) = \begin{bmatrix} a_1(\theta, \phi) \\ \vdots \\ a_m(\theta, \phi) \\ \vdots \\ a_M(\theta, \phi) \end{bmatrix}, \text{ where } a_m(\theta, \phi) \triangleq e^{-j \frac{2\pi}{\lambda} \mathbf{p}_m^T \mathbf{r}(\theta, \phi)}.$$



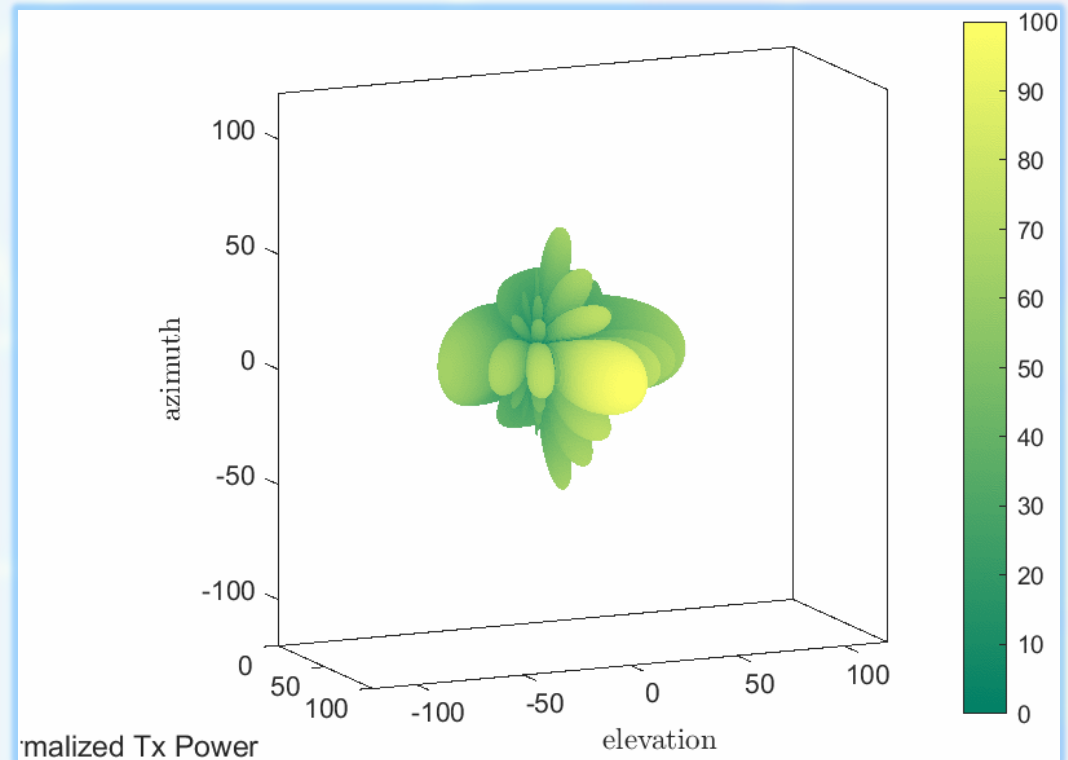
Binary Phase Modulation (BPM)



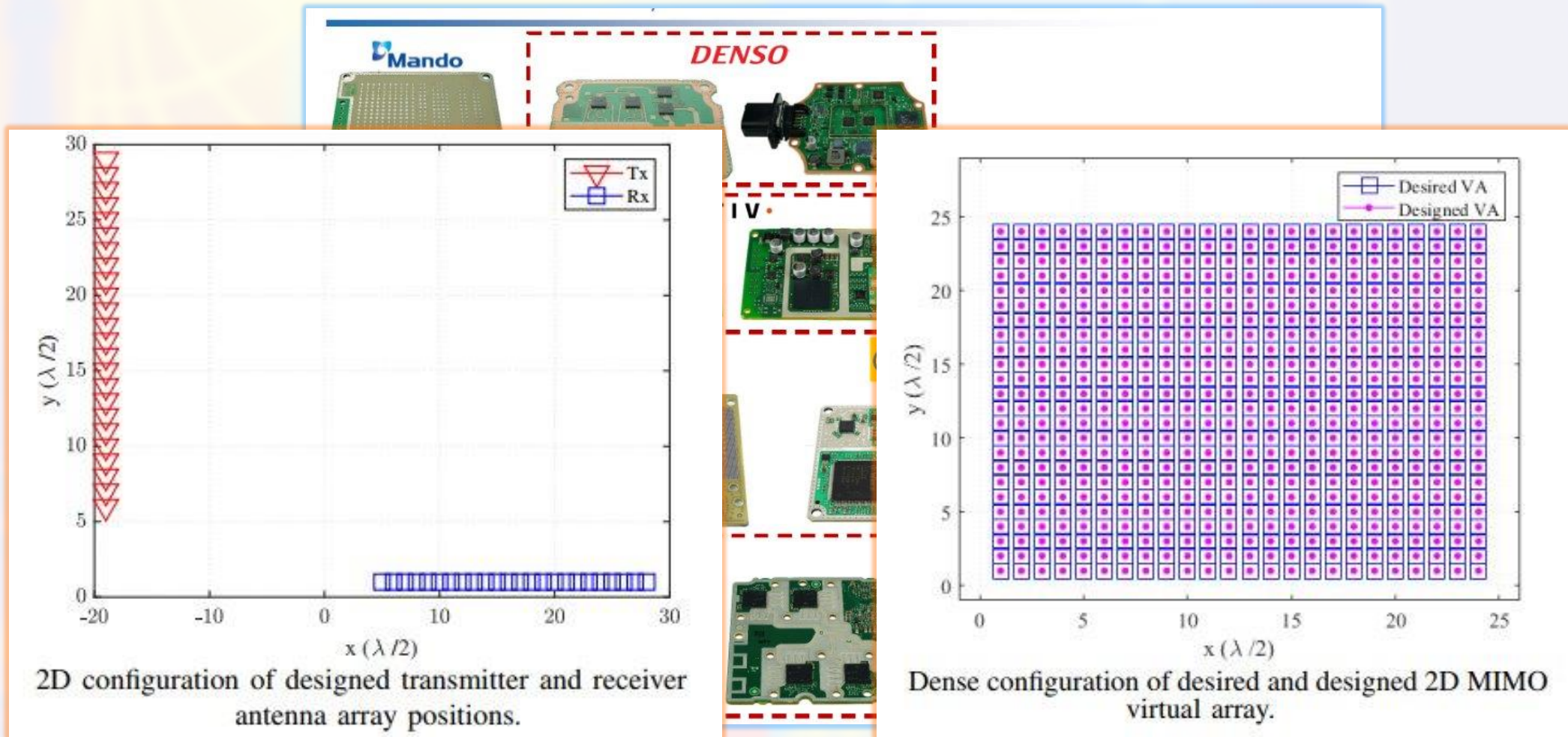
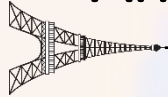
Antenna Layout



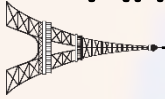
Beam Pattern GIF for Azimuth-Elevation



Antenna Layouts (Real and Virtual)

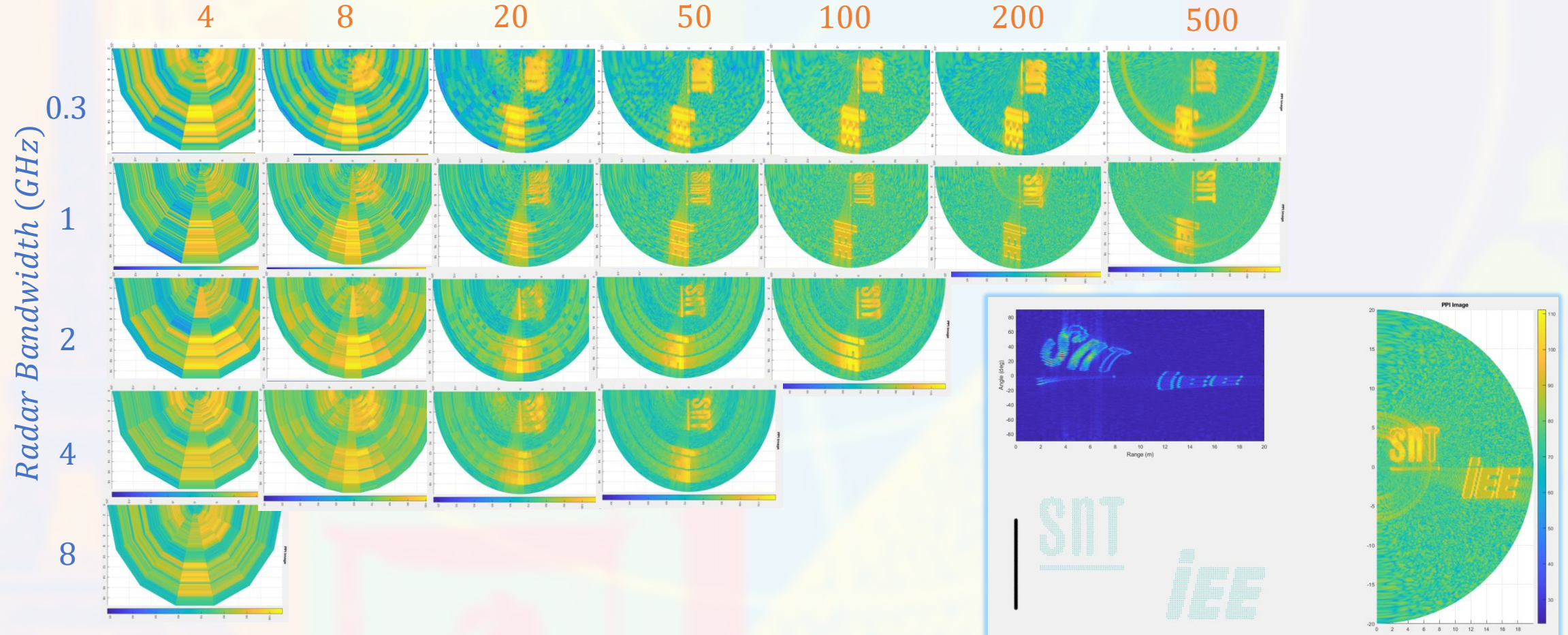


Real Antenna Position for Virtual array layout



Angular Resolution Enhancement with Virtual Aperture

MIMO Radar Virtual Array Length (Half wavelength)



Enhancement in angular resolution by increasing the Virtual aperture



Thanks!

