MATH 213
ASSIGNMENT Nº 5

1. Use results obtained in class to find

$$\mathcal{L}^{-1}\left\{\frac{25s + 50}{2s^2 + 10\sqrt{2}s + 50}\right\}$$

2. Find the Laplace transforms of

$$t\cos\omega t\ u_{-1}(t)$$

and

$$t\sin\omega t\ u_{-1}(t)$$

3. Let

$$g(t) = \begin{cases} 1, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$$

Find $\mathcal{L}\{g(t)\}$.

4. Let

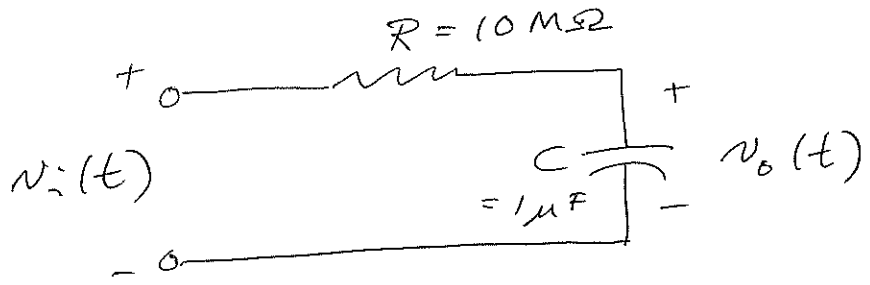
$$h(t) = \begin{cases} 1, & 2n \le t < 2n+1 \\ 0, & 2n+1 \le t < 2n+2 \end{cases},$$

for all $n = 0, 1, 2, \ldots$

a) Write down a recursive definition of $h(t)$ in terms of the function $g(t)$ in the previous question.

b) Use the results of part a) and the previous question to write down the Laplace transform of $h(t)$.

5. The following circuit:



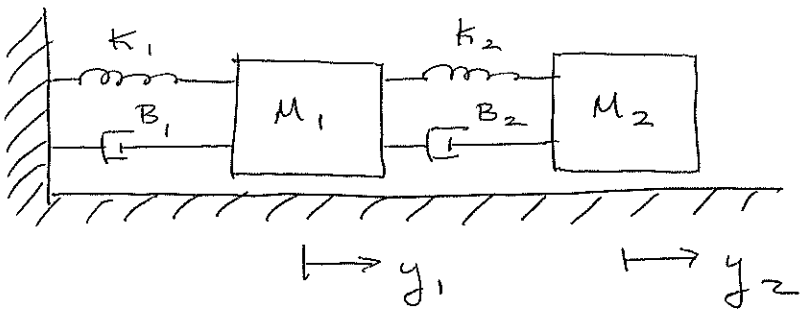
is modelled by the differential equation

$$10\,\dot{v}_o t + v_o(t) = v_i(t).$$

Suppose that $v_i(t) = 5 \sin 0.1t$ volts and that $v_o(0^-) = 10$ volts.

Use Laplace transforms to solve the initial-value problem.

6. The following mechanical system



is governed by the system of differential equations

$$M_1 \ddot{y}_1 + (B_1 + B_2)\dot{y}_1 + (K_1 + K_2)y_1 - B_2\dot{y}_2 - K_2 y_2 = 0$$

$$M_2 \ddot{y}_2 + B_2\dot{y}_2 + K_2 y_2 - B_2\dot{y}_1 - K_2 y_2 = 0$$

(Check this.)

Suppose that $M_1 = 2$ kg, $M_2 = 1$ kg,

$K_1 = 10$ N/m, $K_2 = 5$ N/M,

& $B_1 = 25$ N-s/m $B_2 = 15$ N-s/m.

Suppose also that

$y_1(0^-) = -1$ m and $y_2(0^-) = 1$ m.

& $\dot{y}_1(0^-) = \dot{y}_2(0^-) = 0$.

Find $y(t)$, for $t \geq 0$.