Group Members			
Name	Userid	Name	Userid
Lara Janencka	20460089 lajaneck	Tyler Babaran	20457511 tbabaran
By filling out the names above, the group members acknowledge that a) they have jointly authored this submission, b) this work represents their original work, c) that they have not been provided with nor examined another person's assignment, either electronically or in hard copy, and d) that this work has not been previously submitted for academic credit.			

LAB 3. DIRECT CURRENT MOTOR SPEED AND POSITION CONTROL

On your pre-lab and post-lab submissions, always include this page at the beginning of the document.

Select your lab session:	☐ morning lab; ☒ afternoon lab; ☒ Tue; ☐ Wed; ☐ Thu
Bench number:	15

Transfer function P(S)

$$\begin{aligned}\mathrm{P(S)} &= \frac{\frac{K_pK_aK}{\tau_m s+1}}{1+\frac{K_pK_aK}{\tau_m s+1}}\\ &= \frac{K_pK_aK}{\tau_m s+1+K_pK_aK}\end{aligned}$$

Time constant for P(S)

$$\begin{aligned}\frac{1}{s}\mathrm{P(S)} &= \frac{1}{s}\times\frac{K_pK_aK}{\tau_m s+1+K_pK_aK}\\ &= \frac{1}{s}\times\frac{\frac{K_pK_aK}{1+K_pK_aK}}{\frac{\tau_m}{1+K_pK_aK}s+1}\\ \tau &= \frac{\tau_m}{1+K_pK_aK}\end{aligned}$$

Steady-state error for P(S)

$$\begin{aligned}\frac{\mathrm{E(S)}}{\mathrm{V(S)}} &= \frac{1}{1+\frac{K_pK_aK}{\tau_m s+1}}\\ &= \frac{\tau_m s+1}{\tau_m s+1+K_pK_aK}\end{aligned}$$

Since this is stable the final value theorem applies

$$\begin{aligned}e_{ss} &= \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(S)\\ &= \lim_{s\to 0} s\times\frac{\tau_m s+1}{\tau_m s+1+K_pK_aK}\times\frac{1}{s}\\ &= \lim_{s\to 0}\frac{\tau_m s+1}{\tau_m s+1+K_pK_aK}\\ &= \frac{1}{1+K_pK_aK}\end{aligned}$$

Transfer function Q(S)

$$\begin{aligned}\mathrm{Q(S)} &= \frac{\frac{K_pK_a\bar{K}}{s(\tau_m s+1)}}{1+\frac{K_pK_a\bar{K}}{s(\tau_m s+1)}}\\ &= \frac{K_pK_a\bar{K}}{s(\tau_m s+1)+K_pK_a\bar{K}}\\ &= \frac{K_pK_a\bar{K}}{\tau_m s^2+s+K_pK_a\bar{K}}\end{aligned}$$

Steady-state error for Q(S)

$$\begin{aligned}
\frac{\text{E(S)}}{\text{V(S)}} &= \frac{1}{1+\frac{K_pK_a\bar{K}}{s(\tau_m s+1)}} \\
&= \frac{s(\tau_m s+1)}{s(\tau_m s+1)+K_pK_a\bar{K}} \\
&= \frac{\tau_m s^2+s}{\tau_m s^2+s+K_pK_a\bar{K}}
\end{aligned}$$

Since this is stable the final value theorem applies

$$\begin{aligned}
e_{ss} &= \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(S) \\
&= \lim_{s\to 0} \frac{1}{s} \times \frac{\tau_m s^2+s}{\tau_m s^2+s+K_pK_a\bar{K}} \times s \\
&= \lim_{s\to 0} \frac{\tau_m s^2+s}{\tau_m s^2+s+K_pK_a\bar{K}} \\
&= 0
\end{aligned}$$

Natural frequency for Q(S)

$$\begin{aligned}
Q(S) &= \frac{K_pK_a\bar{K}}{\tau_m s^2+s+K_pK_a\bar{K}} \\
\omega_n &= \sqrt{K_pK_a\bar{K}}
\end{aligned}$$

Dampening ration for Q(S)

$$\begin{aligned}
Q(S) &= \frac{K_pK_a\bar{K}}{\tau_m s^2+s+K_pK_a\bar{K}} \\
\zeta &= \frac{1}{2\omega_n} \\
&= \frac{1}{2\sqrt{K_pK_a\bar{K}}}
\end{aligned}$$