MATH 213

ASSIGNMENT No. 2

1. Find the complementary solutions — of the following equations:

a) $\dot{y}(t) + 10\,y(t) = f(t)$

b) $(D+3)(D+4)(D+5)\,y(t) = f(t)$

c) $(D+3)^3\,y(t) = f(t)$

d) $\ddot{y}(t) + 4\,y(t) = \dot{f}(t) + 2f(t)$

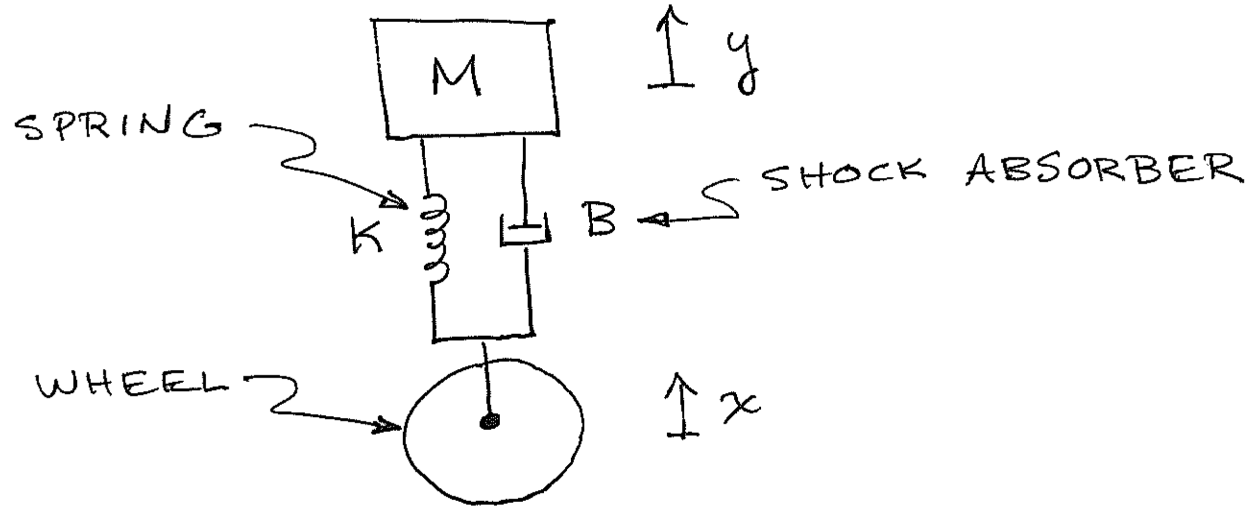
e) $(D+2)(D^2+9)\,y(t) = f(t)$

f) $(D+2)^2(D^2+9)\,y(t) = f(t)$

g) $(D^2+9)^2\,y(t) = f(t)$

2.

Here's a crude model of a vehicle suspension:



If $x(t)$ is the height of the axle, and $y(t)$ is the vertical displacement of the mass M from its equilibrium position when $x(t) \equiv 0$, then Newton's law gives the equation of motion

$$\ddot{y} + 2\zeta\omega_n \dot{y} + \omega_n^2 y = \frac{B}{M}\dot{x} + \frac{K}{M}x,$$

where $\omega_n = \sqrt{\frac{K}{M}}$ and $\zeta = \frac{B}{2\sqrt{KM}}$.

Find the complementary solution of the above equation when

a) $\omega_n = 100$ radians/sec

& $\zeta = 2$

b) $\omega_n = 100$ radians/sec

& $\zeta = \frac{1}{\sqrt{2}}$

c) $\omega_n = 100$ radians/sec

& $\zeta = 1$.

[An automotive suspension might have a "natural frequency" ω_n in the neighbourhood of the above value. The "damping ratio" ζ would likely lie in the range between those of b) and c).

A realistic model would at least take into account the springiness and shock-absorbing behaviour of the tire and the mass of the wheel, in addition to suspension parameters and vehicle mass.]

When $\zeta > 1$, as in a), a system of this form is said to be "overdamped"; when $\zeta < 1$ (b) it is "underdamped"; if $\zeta = 1$ (c) it is "critically damped."