



# CHAPTER 6.

## ANNUAL CASH FLOW ANALYSIS

We now examine annual worth (AW) analysis among the three main approaches of project valuation. All cash flows are converted into equivalent uniform annual worth (EUAW). Similar to present worth analysis, we apply the AW model in cases with equal, unequal and infinite project lives.

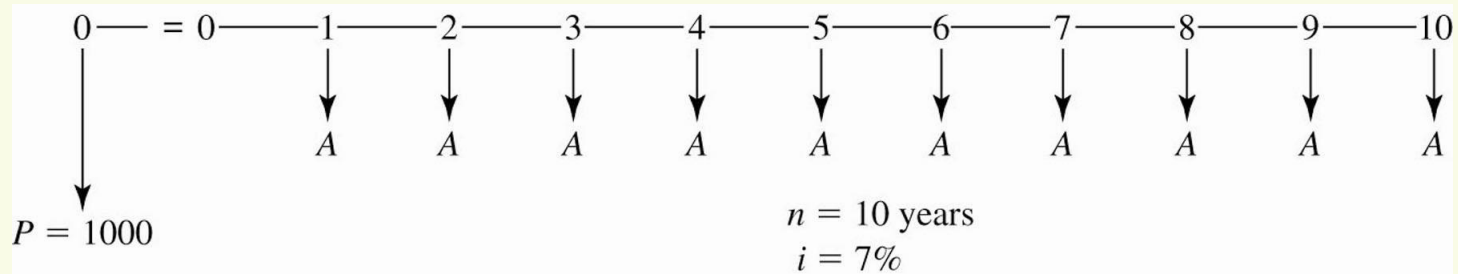


# 1. ANNUAL CASH FLOW ANALYSIS

- In this chapter, we focus on *annual cash flow* analysis.
  - Depending on the situation and type of information, it may be easier to express cash flows as *annual worth*.
  - Equivalent uniform annual worth (EUAW) = EUAB – EUAC
- Examples of conversions
  - What is the annual worth of an initial investment?  $A = P(A/P, i\%, n)$
  - What is the annual worth of salvage value which comes  $n$  years later?  $A = S(A/F, i\%, n)$
  - *Capital recovery cost*:  $EUAC = P(A/P, i\%, n) - S(A/F, i\%, n)$
- Decision rule
  - Among multiple *mutually exclusive* projects, choose the one with the largest EUAW.
  - This approach is equivalent to that of the PW method.

## EXAMPLE 1-1. EUAW CALCULATION

- A student bought \$1,000 worth of furniture. What is the equivalent annual cost if it is expected to last 10 years and the interest rate is 7%?

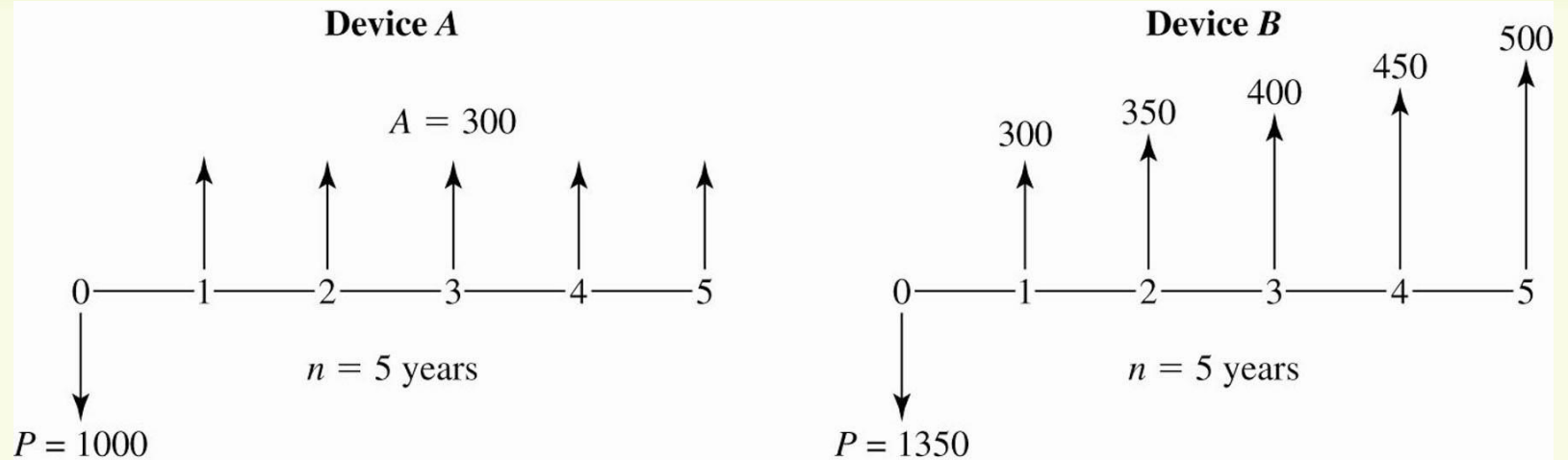


- From the above question, what is the equivalent annual cost if the furniture can be sold for \$200 after 10 years?

## EXAMPLE 1-2. ANNUAL WORTH ANALYSIS (1)

- A firm needs to select one of two devices to reduce costs. Both devices have a five year service life with no salvage value.
  - Device A costs \$1,000 and can be expected to result in \$300 savings annually.
  - Device B costs \$1,350 and provides cost savings of \$300 in the first year with an increase of \$50 annually from the second year.
- With an interest rate of 7%, which device should the firm purchase?
  - Do you remember example 2-1 in chapter 5? It asked the use of PW to choose the best alternative.
  - We now ask the same question using the annual worth approach.

## EXAMPLE 1-2. ANNUAL WORTH ANALYSIS (2)



## EXAMPLE 1-3. MONTHLY PAYMENT OF A CAR

- Helen purchases a new Ford Focus. She negotiates a price of \$19,000, trades in her 2007 VW for \$2,000, puts a down payment of an additional \$1,000, and borrows the remainder for 3 years at 6% interest. What is the amount of her *monthly payment*?

# 1.1. HOW TO PAY FOR A CAR

- When you buy a car, how do you pay for it?
- Cash
  - If you have enough money, you can purchase a car in cash. Rare!
- Financing (or loan)
  - You get a loan from a bank (or from a financing company), and pay monthly payments to the bank.
  - You own the vehicle at the end of your financing term.
- Leasing
  - You pay monthly payments to the lender (leasing company).
  - You are allowed a limited mileage allowance (24,000 km per year).
  - You return the vehicle at the end of the leasing term, or you can buy the vehicle at the residual value.

## EXAMPLE 1-4. ACQUIRING A CAR (1)

- Suppose you need a car for 4 years, and decide to get BMW 528i for \$54,600 (including all fees). Three options are considered.
  - Option A (Pay cash): The cash price is \$52,600 (with a \$2,000 cash rebate). The car can be sold for \$22,000 at the end of 4 years.
  - Option B (Leasing): The monthly payment is \$750 on a 48-month lease, payable at the end of each month. At the end of the 4-year period, the car is returned to the leasing company.
  - Option C (Leasing with an option to buy): You pay \$780 a month for 48 months. At the end of the 4 years, you buy the car for \$20,000 and sell it for \$22,000.
- Given an interest rate of 3%, which option should you choose? Use the *annual worth* method.



## EXAMPLE 1-4. ACQUIRING A CAR (2)

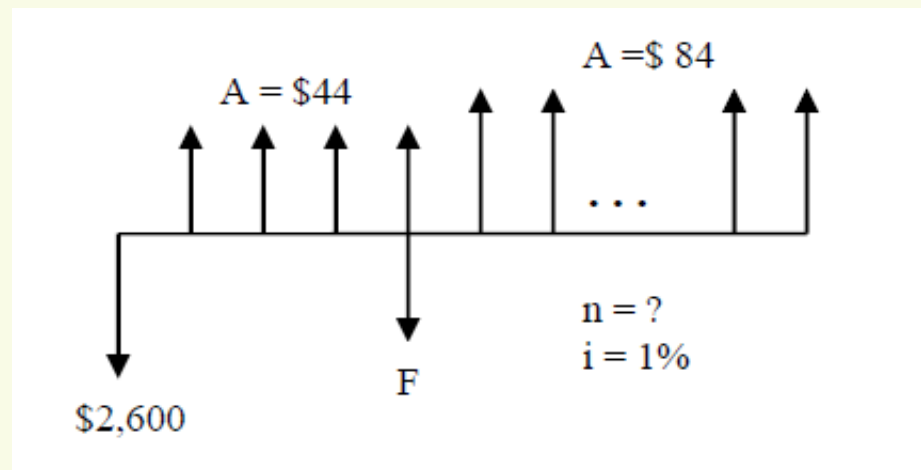
- We will compute equivalent uniform monthly cost for each alternative. Note that the monthly interest rate is 0.25%.

## EXAMPLE 1-5. PERPETUAL SCHOLARSHIP

- Jim has made a fortune and wants to start a *perpetual* scholarship for students at his alma mater.
  - The scholarship provides a student with an annual stipend of \$2,500 for each of four years, plus an additional \$5,000 at the end of the fourth year.
  - Students graduate in four years, a new award is given every 4 years. The money is paid at the end of each year, and the interest rate is 8%.
- What is the EUAC of providing the scholarship to a student?
- How much money must Jim donate to the university?

## EXAMPLE 1-6. MONTHLY PAYMENT (1)

- A motorcycle is for sale for \$2,600. The dealer is willing to sell it on the following terms;
  - No down payment.
  - Pay \$44 at the end of each of the first four months.
  - Pay \$84 at the end of each month after that until the loan has been paid in full.
- At a 12% interest rate, how many \$84 payments will be required?



## EXAMPLE 1-6. MONTHLY PAYMENT (2)

- Compute future worth at the end of four months with the \$44 monthly payments for four years.
- Now, solve for the unknown  $n$ .

## EXAMPLE 1-7. MULTIPLE ALTERNATIVES (1)

- Three alternatives are being considered for improving an operation. Equipment costs vary, as do the annual benefits of each in comparison to the present situation. Each alternative has a 10-year life. If interest rate is 8%, which plan, if any, should be adopted? Use the *annual worth* method.

	Plan A	Plan B	Plan C
Installed cost of equipment	15,000	25,000	33,000
Material and labor savings per year	14,000	9,000	14,000
Annual operating expenses	8,000	6,000	6,000
End-of-life salvage value	1,500	2,500	3,300

## EXAMPLE 1-7. MULTIPLE ALTERNATIVES (2)

- We need to convert the initial cost and salvage value into annual worth.

# EXAMPLE 1-8. COST OF A PRIVATE JET (1)

- What is the cost of owning a private jet?
  - Rich individuals like Tiger Woods or large firms own their private jets.
  - Many others instead purchase a *partial ownership* of a jet. They are entitled to use a certain hours (say 100 hours) per year.
- Should your company buy a private jet?
  - Currently, suppose your employees make a trip to London (from Toronto) three times a week and pay a first-class ticket for \$5,500.
  - Instead, you are considering buying a Gulfstream G500 with the following cost factors.



## EXAMPLE 1-8. COST OF A PRIVATE JET (2)

Item	Cost(\$)
Gulfstream G500	35,000,000
Salvage value (after 15 years)	7,000,000
Crew costs per year (2 pilots)	300,000
Airport landing fee (per landing)	500
Annual maintenance/insurance cost	700,000
Fuel cost per km	1
Catering cost per passenger (round trip)	100

- The distance between Toronto-London is 5,500 km (one-way).
- How many passengers should be carried on an average trip to justify the jet, given an interest rate of 10%?



## EXAMPLE 1-8. COST OF A PRIVATE JET (3)

- Let  $X$  denote the number of round-trip passengers per year.

## 2. ANALYSIS PERIOD AND EUAW

- Unlike the present worth approach, we don't need to consider the *least common multiple* in the annual worth approach.
  - We simply compare the EUAW for alternatives based on their own service lives, assuming identical replacements in the future.
  - The same logic applies for an infinite analysis period.
- Choice between the one with 7 years and another with 13 years
  - Under PW, we need to find the least common multiple, 91 years.
  - Or, we need to limit the *analysis period*, say 10 years, with the estimation of the remaining salvage value.
  - Under EUAW, however, we simply calculate the annual worth for the respective service lives.
- To evaluate alternatives with unequal service lives, we better use the EUAW approach rather than the PW approach.

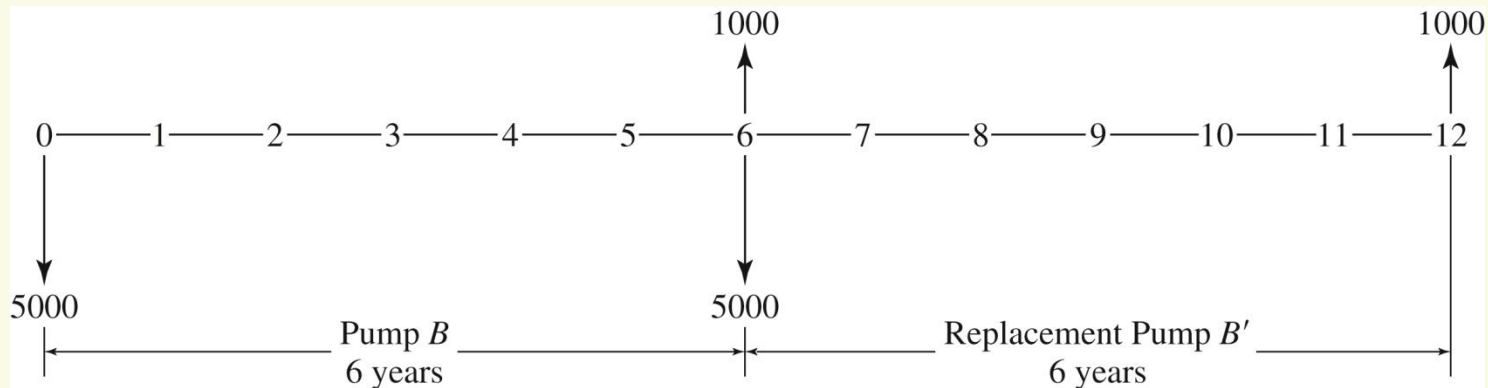
## EXAMPLE 2-1. UNEQUAL SERVICE LIVES (1)

- Two pumps are being considered for purchase. If interest rate is 7%, which pump should be bought?

	Pump A	Pump B
Initial cost	7,000	5,000
End-of-life salvage value	1,500	1,000
Useful life (years)	12	6

## EXAMPLE 2-1. UNEQUAL SERVICE LIVES (2)

- The previous solution is complete and correct.
- However, what if we consider the case of *least common multiple*, with another purchase of pump B at the end of 6 years?



- $$\begin{aligned} \text{EUAC}(B) &= [5,000 - 1,000(P/F, 7\%, 6) + 5,000(P/F, 7\%, 6) \\ &\quad - 1,000(P/F, 7\%, 12)](A/P, 7\%, 12) \\ &= [5,000 + 4,000(0.6663) - 1,000(0.4440)](0.1259) = \$909 \end{aligned}$$
- We don't need to consider the multiple purchases under EUAW!

## EXAMPLE 2-2. UNEQUAL SERVICE LIVES

- Two alternatives of an aqueduct are being considered. Given the following information, which one should be chosen given an interest rate of 6%?

	<b>Tunnel</b>	<b>Pipeline</b>
Initial cost	5.5 million	5 million
End-of-life salvage value	0	0
Useful life (years)	permanent	50

## EXAMPLE 2-3. UNEQUAL SERVICE LIVES

- Consider the following three mutually exclusive alternatives.

	A	B	C
Initial cost	10,000	150,000	20,000
Annual benefit	1,000	1,762	5,548
Useful life (years)	$\infty$	20	5

- Assuming 8% interest, which alternative should be selected?

## EXAMPLE 2-4. BRIDGE TOLL

- A proposed steel bridge has an infinite life. The initial cost of the bridge is \$5 million, and annual maintenance costs are \$50,000. The bridge deck will be resurfaced every 10 years for \$100,000 starting 10 years from now, and anticorrosion paint will be applied every 5 years for \$400,000 starting 5 years from now. If the interest rate is 6%, what is EUAC?
  
  
  
  
  
  
  
  
  
  
- If 500,000 cars cross the bridge each year, what is the toll per car?

# 3. MORTGAGE

- What is a mortgage?
  - Technically, it is a legal document on real estate.
  - Practically, it is a long-term amortized *loan* that is used for buying real estates such as a house or land.
- Mortgage in Canada
  - When you buy a house, you often pay a “down-payment,” 5 – 20% of the house value.
  - The rest of the house value is loaned from a mortgage company.
  - The duration that the loan is paid is called “amortization period,” typically 20 – 25 years.
  - The interest rate used in this loan is called “mortgage rate.”
  - The mortgage rate (and monthly payment) is agreed for a limited “term,” say 5 years, after which the rate is re-negotiated.



## EXAMPLE 3-1. MORTGAGE PAYMENT

- Cherry Garcia bought a house for \$500,000 with a down payment of \$100,000. He obtained a 30-year mortgage loan for the remaining amount. Mortgage payments are made monthly and the mortgage rate is 6%. After 10 years (120 payments), he decided to pay the remaining balance on the loan.
- What was his monthly mortgage payment?
- What must he have paid (in addition to his regular 120<sup>th</sup> monthly payment) to pay the remaining balance of his mortgage?

## EXAMPLE 3-2. MORTGAGE LOAN

- A lender requires that monthly mortgage payments be no more than 33% of gross monthly income with a maximum term of 25 years. If you can make only a 10% down payment, what is the *minimum monthly income* needed to purchase a \$300,000 house when the mortgage rate is 6%?

## EXAMPLE 3-3. REFINANCING MORTGAGE (1)

- For her house, Silvia borrowed \$80,000 at 10% mortgage rate to be repaid in 25 equal end-of-year payments. After making 10 payments, she found she could refinance the balance due on his loan at 9% interest for the remaining 15 years.
  - To refinance the loan, she must pay the original lender the balance due on the loan, plus a penalty charge of 2% of the balance due. To the new lender she also must pay a \$1,000 service charge to obtain the loan.
- Should Silvia refinance the loan for the remaining 15 years? Use the *annual worth* approach.
  - You need to compare the annual mortgage payment under the original loan with the new loan.

## EXAMPLE 3-3. REFINANCING MORTGAGE (2)

- Original loan
- New loan

## EXAMPLE 3-4. RENTING AN APARTMENT (1)

- A 300-unit apartment building lasts 30 years. When the owner sells it 30 years later, he can recover 10% of its construction cost and the full value of land. The following is the information
  - Land: \$3 million
  - Building: \$5 million
  - Annual operating and maintenance cost: \$800,000
  - Annual property taxes and insurance: 5% of initial investment
  - Vacancy rate: 10%
- If the owner wants a 12% rate of return, what does the *monthly rent* for each unit have to be?

## EXAMPLE 3-4. RENTING AN APARTMENT (1)

- If the owner converts 2 units into an exercise facility, the vacancy rate drops to 7%. Is this a good decision if the monthly rent is the same as above?

## EXAMPLE 3-5. AMORTIZATION SCHEDULE

- An *amortization schedule* lists of each payment period: the loan payment, interest paid, principal paid and remaining balance.
- We can create an amortization schedule using the methods we learned, but it is much easier with Excel.
- Suppose you purchased a car of \$20,000 with 24 month, 6% nominal loan.
- How much do you pay per month?
- After paying 10 months, what is the remaining balance?
- With this information, let's create an amortization schedule (See Excel file).

# SUMMARY OF CHAPTER 6

- Equivalent uniform annual worth (EUAW)
- EUAW with unequal service life
  - No need for least common multiple
  - Assume infinite replacement
- Mortgage in Canada
  
- Selective end-of-chapter problems
  - 9, 16, 21, 27, 30, 33, 41, 53, 55, 60