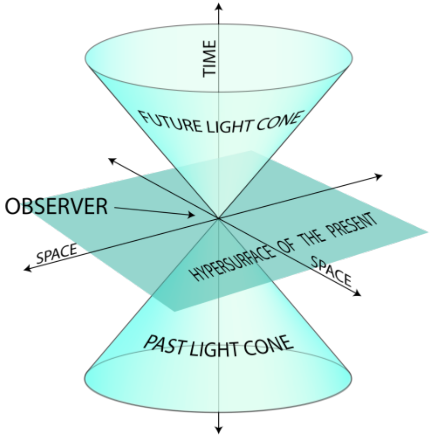
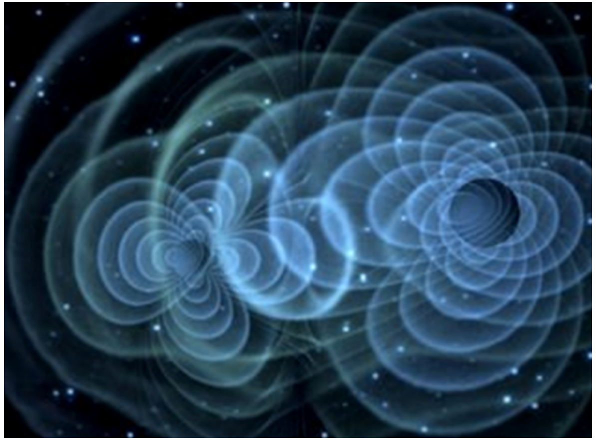
Introduction

In this third and final section of the course, "**Everything**," we will explore the idea of a "**theory of everything**," and begin to address the question: "**What is the ultimate nature of reality?**"

We will start with the major **revolution** in our understanding of reality that took place in the early 1900s, now called "**modern physics**," which includes **special relativity**, **general relativity**, and **quantum mechanics**. The central theme in all cases is UNIFICATION.



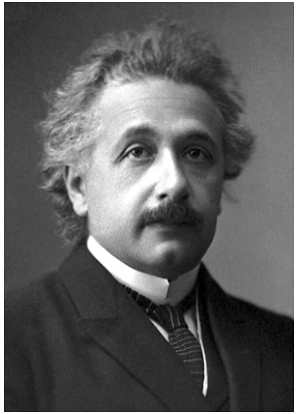


Einstein’s Two Relativity Theories

- A central figure in this revolution was **Albert Einstein**. He played a key role in birth of **quantum mechanics**, and developed two **theories of relativity**:

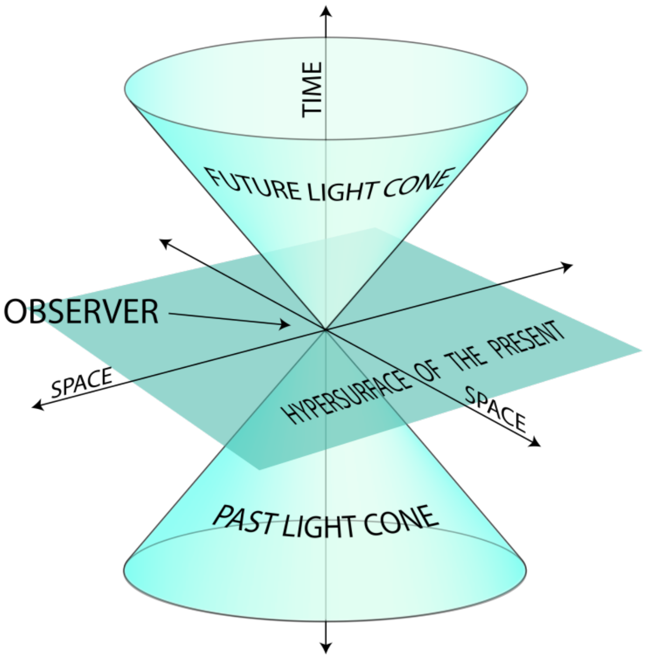
  - **Special Relativity (SR)**:
    - ✓ 1905
    - ✓ <u>**Unified**</u> **space & time** → 4D <u>**flat**</u> **spacetime**
    - ✓ **Includes:** Time dilation, length contraction, relativity of simultaneity, universal speed limit (speed of light), etc.

  - **General Relativity (GR)**:
    - ✓ 1915
    - ✓ <u>**Unified**</u> **spacetime & gravity** → 4D <u>**curved**</u> **spacetime** (which says gravity is not a force, but rather, curvature of spacetime)
    - ✓ **Includes:** “bending” of light (discussed earlier), gravitational waves (recently observed), black holes, cosmology, etc.



We will first focus on **special relativity** (SR), a theory about the **geometrical unification** of space & time into a 4-dimensional (geometrically “flat”) spacetime.

In particular, we will understand the **geometrical nature of time**.



- **Principle of relativity: There is no way to tell, from inside a closed room, if the room is moving**. [Assuming motion with **constant velocity** (speed & direction) and no rotation.]

    - **Example:** everything that happens inside a train (walking around, juggling balls, playing billiards) happens the **same way** whether the train is **at rest** on the tracks, or **moving** with constant speed along a straight track. Ignoring bumps and vibrations (non-constant velocity), the only way to tell the train is moving is to look out the window.

    - In other words: there is **no experiment** you can do inside a closed room to detect its motion. This principle of relativity was first suggested by Galileo in 1632, and was accepted as true for **mechanical** experiments (walking around, juggling balls, playing billiards, etc.). But is it true for ***all*** experiments, ***including experiments involving light***?

    - Early 1800s: **wave nature of light** suggested the answer is **NO**. Einstein showed how it could be **YES** if the nature of space & time was **different** than we had been ***assuming***.

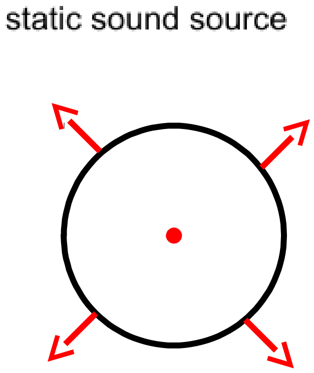
- **Nature of Light:**

  - 1690: **Huygens** published a theory that **light is a wave**. But waves require a medium—something that is "waving", e.g., air for sound waves, water for water waves. Huygens imagined that light consists of waves in some kind of "light medium," called the **ether**, that uniformly fills all of space. (Recall: "Ether" was thought to be the stuff that made up "the heavens"; what the "gods breathed".)

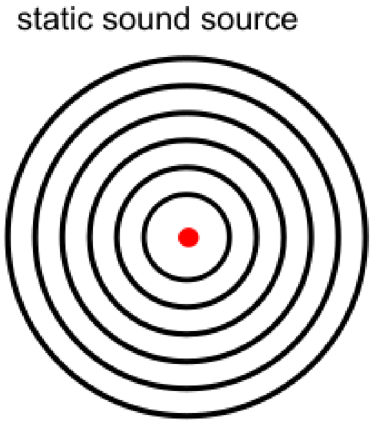
  - 1704: **Newton** published a theory (after Huygens' death) in which he imagined that light is a **shower of particles** that move through the **vacuum** of empty space—his so-called "absolute space".

- **Nature of Light:**
  - Huygens' **wave-in-ether** and Newton's **particle-in-vacuum** models of light were **debated** for a century.
  - In the **early 1800s**, **Thomas Young** did simple experiments (wave interference) that showed, definitively, the **wave nature of light**. Throughout the 1800s, virtually all physicists accepted the **wave-in-ether** model of light.

- **Key fact about waves in a medium (e.g., sound wave-in-air, but same for light wave-in-ether):**
  - **Clap hands once**: The disturbance of the air creates an expanding spherical wave front (sound wave). Once the wave front is created, it **takes on a life of its own**, propagating (by one air molecule bumping into its neighbor, etc., like dominoes) with a **fixed speed** (call it c) **relative to the still air**.

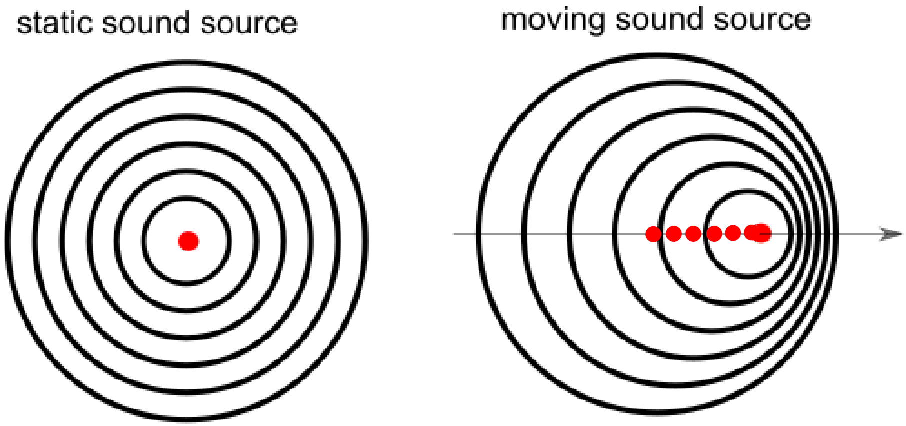


- **Key fact about waves in a medium:**
  - **Keep clapping**: You create a **sequence** of nested, expanding spherical wave fronts, each moving with speed c relative to the still air.

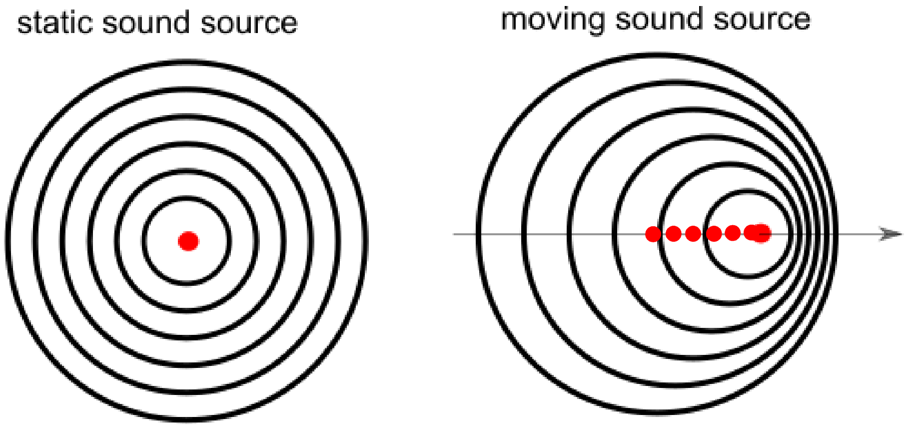


Wave Nature of Light

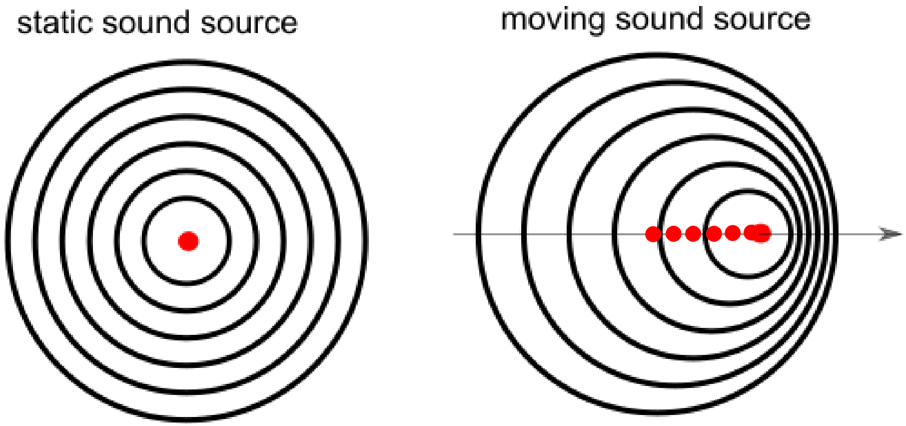
- **Key fact about waves in a medium:**
  - **Move while clapping**: Each time you clap, the spherical wave front you create expands outwards (with speed c relative to the still air) **from the point at which you created it** (it takes on a life of its own), ***regardless of the motion of your hands***. Simulation.



- **Key fact about waves in a medium:**
  - For someone at rest in the still air, **the *speed* of the sound waves is independent of the motion of the *source* of the waves** (your hands).
  - This is true for **any** wave-in-medium phenomenon (e.g., sound waves in air, water waves in water), **including light waves in ether**.



- **Key fact about waves in a medium:**
    - Doppler effect, YES. Speed change, NO.

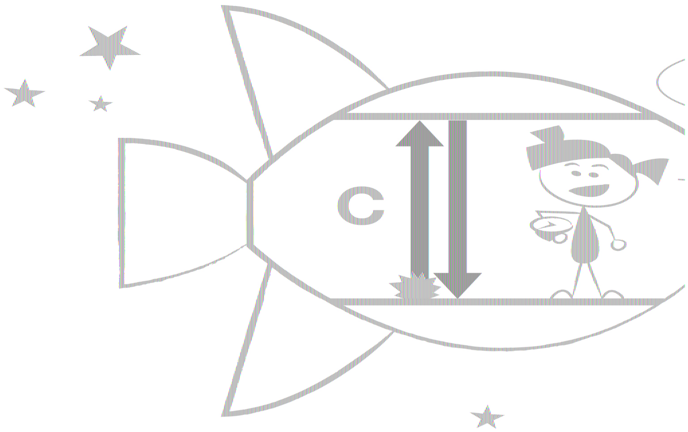


So what?

Key Thought Experiment

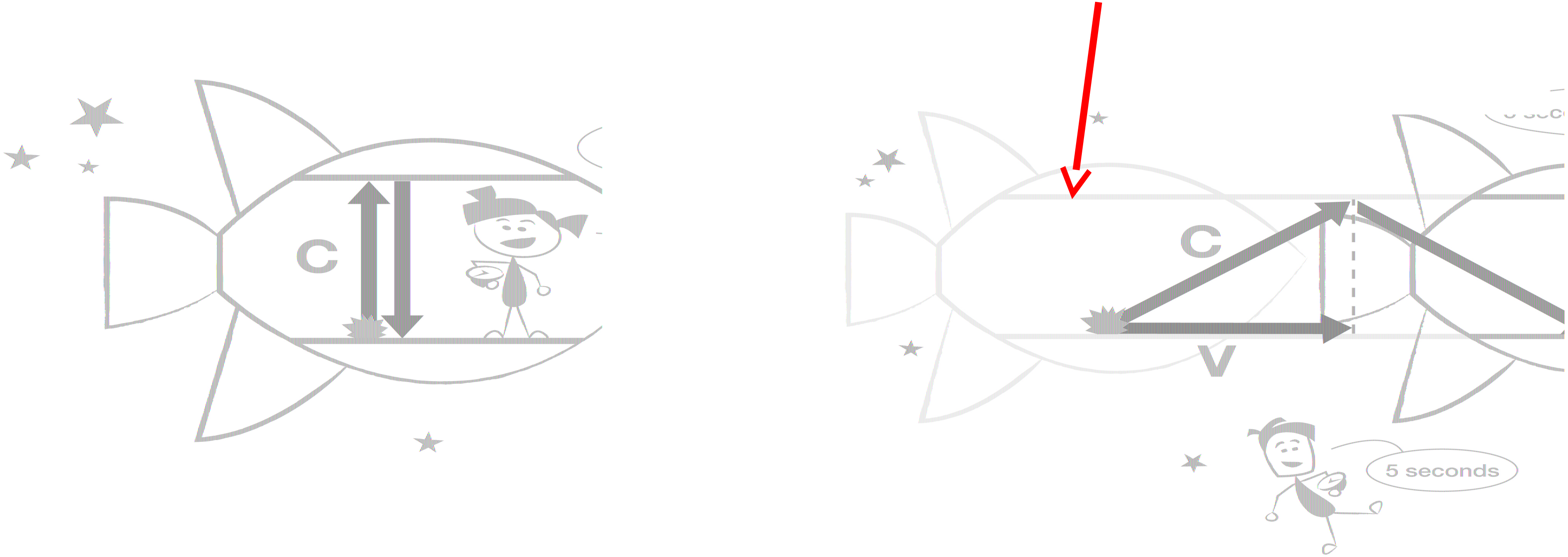
Assume light is a **wave-in-ether**, as experiments suggested, with Alice **at rest** in the ether.

Suppose her experiment with light takes **3 seconds**.



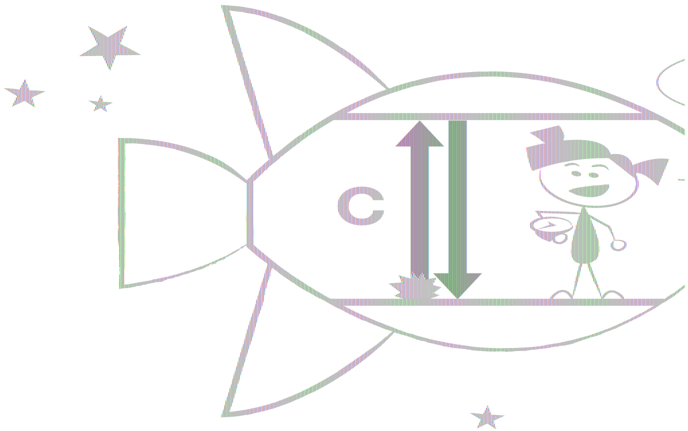
She goes to sleep and, unbeknownst to her, Bob gives her spaceship a push so it is drifting with constant speed v through the ether ("ether drift…"). She wakes up and repeats the experiment.

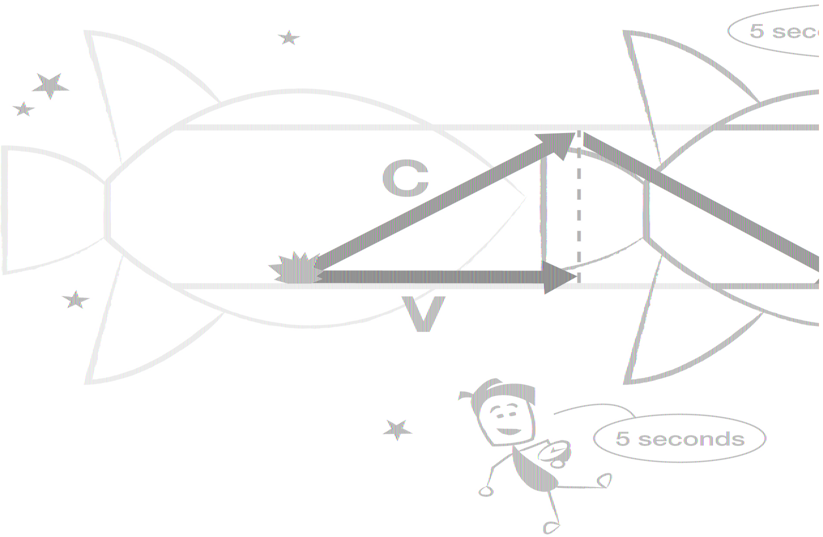
Bob says it takes longer, say, **5 seconds**. Why? For Bob, at rest in the ether, the speed of light is **independent of the motion of the source** (Alice); the speed is still c, but the **distance is longer**.



Key Thought Experiment

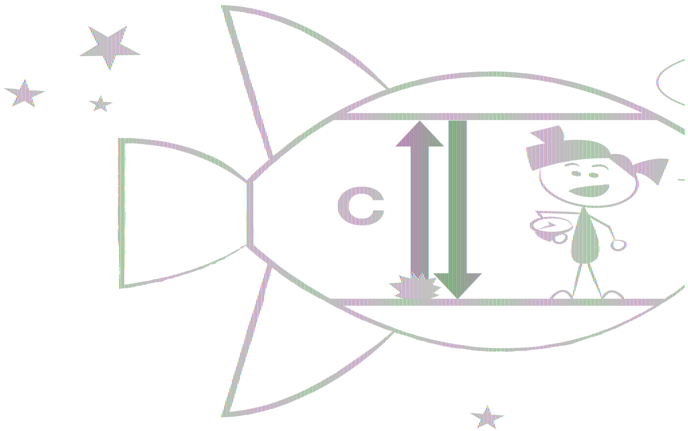
Question: What will Alice say: **5 seconds or 3 seconds**?

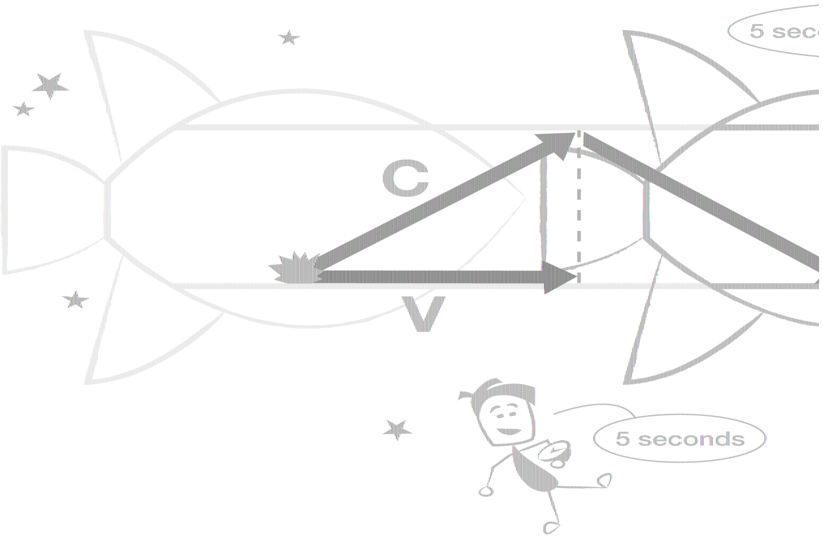




Assumed **answer**: **5 seconds**. “Universal Time”: when 5 seconds elapses for Bob (on some “great grandfather clock in the sky”), 5 seconds elapses for everyone else, including Alice.

Problem: This **violates the Principle of Relativity**. Alice could use her experiment with light, inside her closed spaceship, to **detect its motion**.

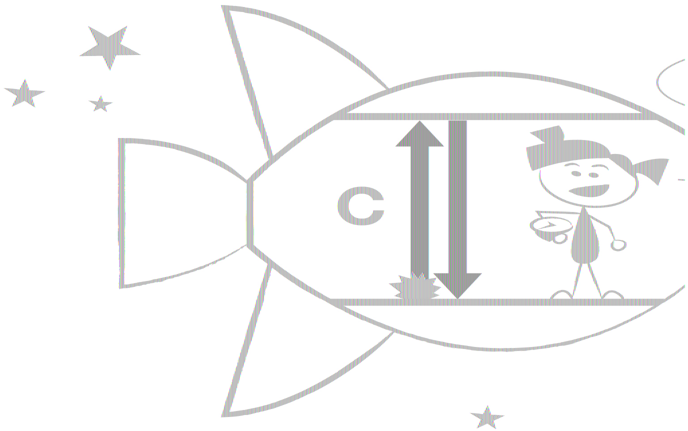


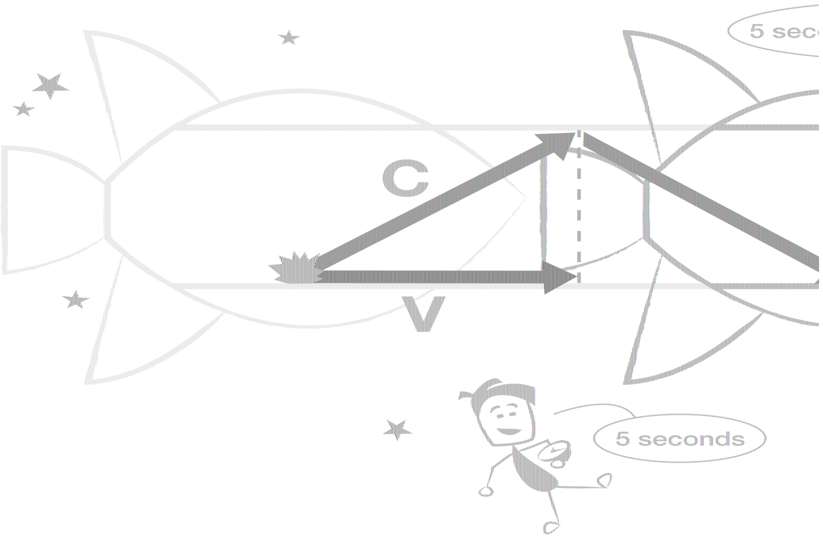


Key Thought Experiment

Einstein's **answer**: **3 seconds**. "Universal Relativity" (relativity extended to include light): There is ***no*** experiment Alice can use inside her closed spaceship, including light, to detect its motion.

Problem: This **violates Universal Time**. The "great grandfather clock in the sky" *assumption* must be abandoned.





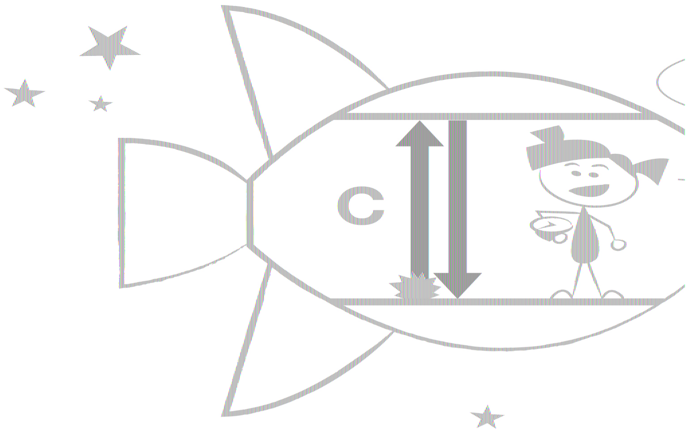
Summary: if we assume a wave-in-ether model of light (as experiments suggested), then **Universal Time** and **Universal Relativity** are **incompatible**. At least one is WRONG. Which one?

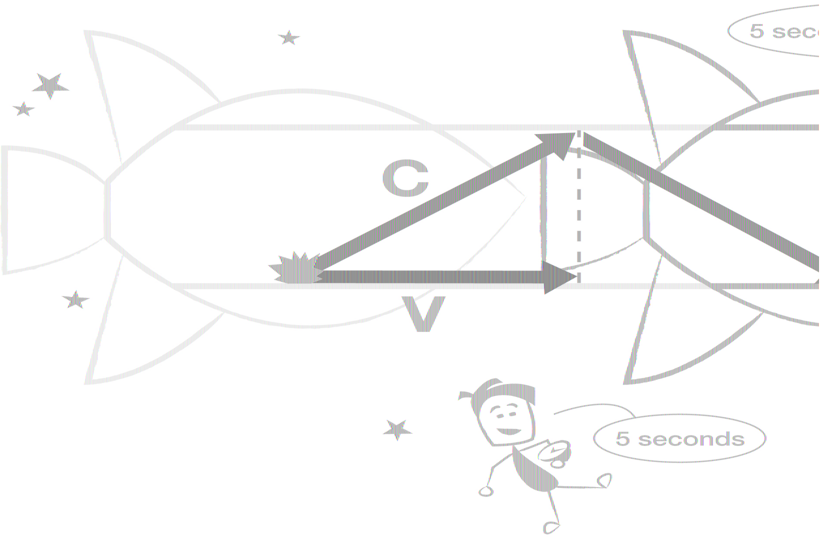
Einstein: "My solution was really for the very concept of time, that is, that time is not absolutely defined."

Key Thought Experiment

Summary: if we assume a wave-in-ether model of light (as experiments suggested), then **Universal Time** and **Universal Relativity** are <u>**incompatible**</u>. At least one is <u>WRONG</u>. Which one?

Only way to **test** Einstein's intuition about Universal Relativity □ <u>time dilation</u> is by **experiment**.





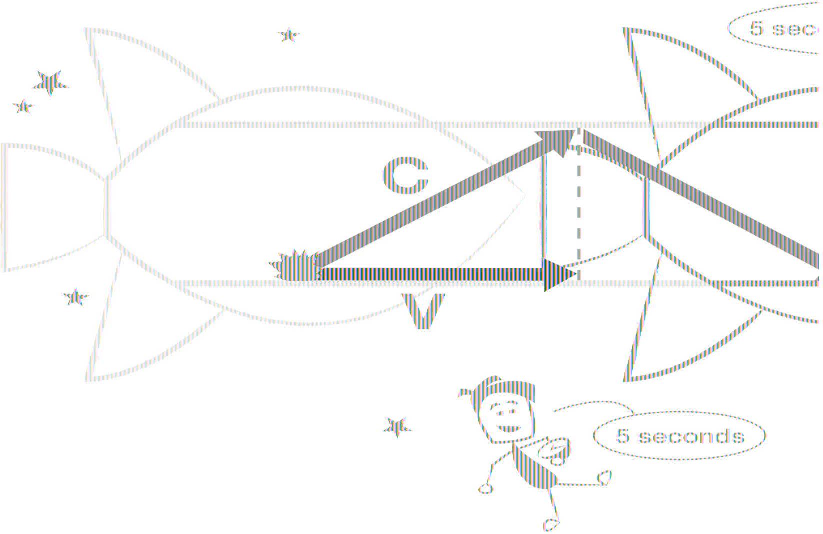
But first, a bit of mathematics:

Look at the triangle: For Bob,
$c_{\text{vertical}} = \sqrt{c^2 - v^2} = c\sqrt{1 - v^2/c^2} < c.$

For Bob, the up/down speed of the light pulse is **slower** than c by a factor of $\sqrt{1 - v^2/c^2}$.

For Alice to see this speed as c (universal relativity: she can't tell she is moving), her clock must be running **slower**, **relative** to Bob's, by the **same factor**: $\sqrt{1 - v^2/c^2}$

$$\sqrt{1 - v^2/c^2} = \frac{3}{5} \quad \Rightarrow \quad v = \frac{4}{5}c$$



Testing Time Dilation:

- **Problem:** For everyday speeds, $v \ll c \Rightarrow \sqrt{1 - v^2/c^2}$ is almost equal to 1 (almost **no time dilation**). This is why we never noticed it! Time dilation becomes noticeable only when v is an appreciable fraction of c. Thus, we need to *make a clock go at close to the speed of light*.

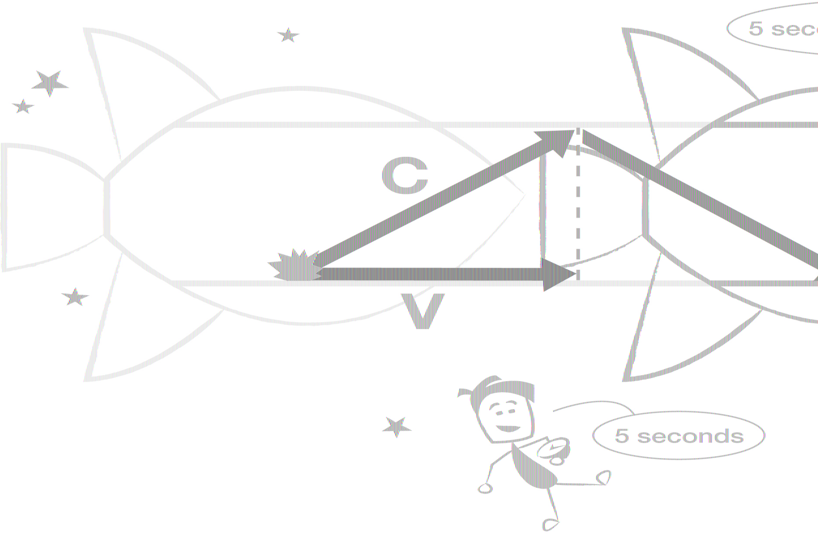
- **Solution:** Use an **unstable** elementary particle that is travelling at **nearly the speed of light**.
  - "**Unstable**" means it will decay into another particle(s) in a fixed average time. Example: a certain type of particle (a "kaon"), at rest in the lab, will spontaneously decay into two other particles ("pions") in an average time of about 9×10^{-11} seconds.
  - In particle accelerators, this type of kaon is easily created, travelling at nearly the speed of light. Using our ***assumed*** ideas about space & time, it would be able to travel, on average, only about $(3 \times 10^8 \, \frac{\mathrm{m}}{\mathrm{s}}) \times (9 \times 10^{-11}\ \mathrm{s}) = 2.7$ cm before decaying.

Testing Time Dilation:

- However, it is **observed** to travel **much farther**. How? In **Einstein's spacetime**, time is moving **much slower** for the kaon ("Alice") ***relative*** to time in the lab ("Bob"), so when 9×10^{-11} seconds elapses for the kaon, **much more** time elapse in the lab, allowing it to travel farther.

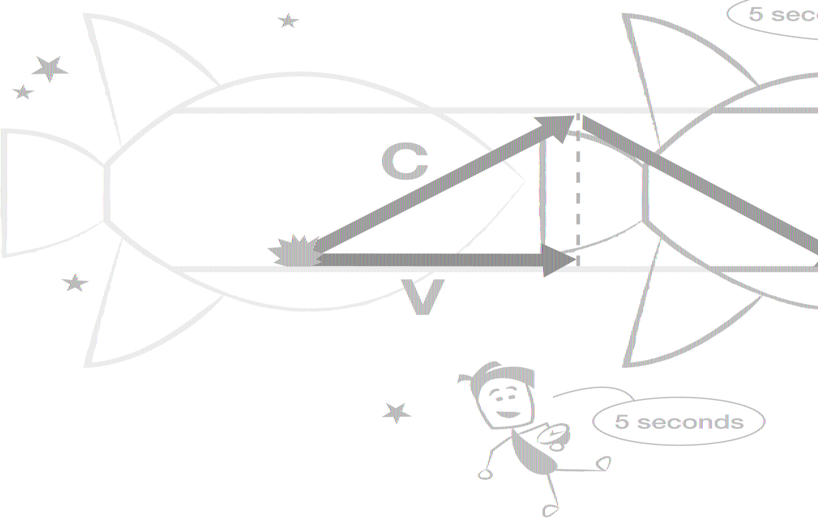
- **Example:** If the kaon travels at 0.995 c, then $\sqrt{1 - v^2/c^2} = 0.1$, so the amount of time elapsed in the lab is $1/0.1 = 10$ times the lifetime of the kaon. So instead of travelling only 2.7 cm, it travels $10 \times 2.7 = 27$ cm before decaying.

- Such **time dilation** is one of the most **accurately tested** aspects of the **nature of reality**.



Problem:

- If we are at rest in the lab ("Bob"), and see the kaon ("Alice") moving at $0.995\ c$ relative to us, then the kaon will see *us* moving at $0.995\ c$ relative to *it* (in the opposite direction).
- From the kaon's perspective, it is "at rest" and "lives" for 9×10^{-11} seconds. It "sees" the lab rushing by at $0.995\ c$. Moving at this speed, for only 9×10^{-11} seconds, only 2.7 cm of the lab will rush by. **So how does the kaon move 27 cm in the lab** before decaying?
- **Logic** implies that, along with time dilation, there must also be **length contraction**: in its direction of motion, the kaon must see the length of the lab **contracted** by the **same factor** as time dilation: $\sqrt{1 - v^2/c^2} = 0.1$.

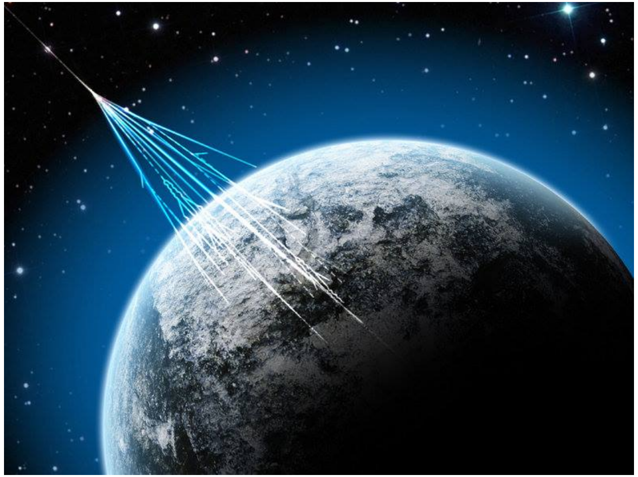


- Extreme example: **Ultra-high-energy cosmic rays (UHECR)**

- Space is filled with **cosmic rays**: extremely **high-energy** (fast moving) particles (e.g., **protons**), with energies **millions** of times that achievable at the LHC.

- When they slam into atomic nuclei in the Earth's upper atmosphere they create a cascading "**air shower**" of **secondary** particles that can be detected and analysed.

- The **origin** of UHECR is still **mysterious** (jets in Active Galactic Nuclei? Supernova explosions?)



Length Contraction & Time Dilation

- **Example:** For a proton to have an energy equal to the GZK limit (maximum energy for long distance travel through CMB), it would have to be moving at $0.99999999999999999999998\ c$. At this speed, the TD & LC factor is $\sqrt{1-v^2/c^2} = 2 \times 10^{-11}$.

- Suppose it came from the **Andromeda galaxy**, which is 2.5 million light years (ly) away.
  - From **our perspective**, it would take the proton 2.5 million years to cover the distance of 2.5 million ly.
  - From the **proton's perspective**, it would take only $2 \times 10^{-11} \times 2.5$ million years = **30 minutes**, to cover a length-contracted distance of $2 \times 10^{-11} \times 2.5$ million ly = 3 AU, **less than the size of the Solar System**!
  - Some cosmic rays are truly **inter*galactic* travellers**!

- All objects along the proton's path, including the Milky Way galaxy, would be length-contracted in the direction of motion.

- Notation: $\gamma = \frac{1}{\sqrt{1-v^2/c^2}} \geq 1$

- **Objects in motion relative to you occupy less of "your space" than their "own space." Space is "relative."**

Length Contraction & Time Dilation

- **Summary (so far):**

  - Experiments showed that **light behaves like a wave** ⇒ **wave-in-ether** model of light.

  - In ***assumed*** space & time, a wave-in-ether model of light would **violate** the principle of relativity (in a closed room we could detect its motion using an experiment with light).

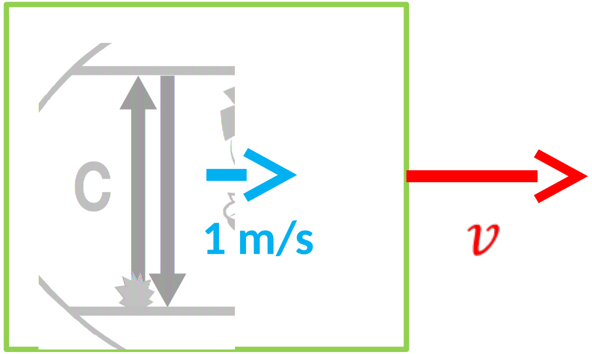
  - **Einstein *corrected* our understanding of time** by demanding the principle of relativity be true even for experiments involving a wave-in-ether model of light ⇒ **time dilation**.

  - But time dilation logically requires also **length contraction**. LC & TD are two sides of the same coin. You can't have one without the other. They go hand-in-hand. Together, they imply that space and time are unified into a 4D spacetime with a certain geometry…

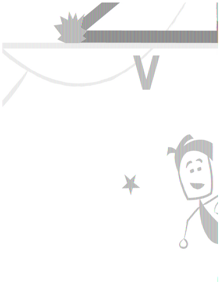
  - But first: LC & TD also work together to **enforce a universal speed limit** (the speed of light), and allow it to **make sense**…

Universal Speed Limit

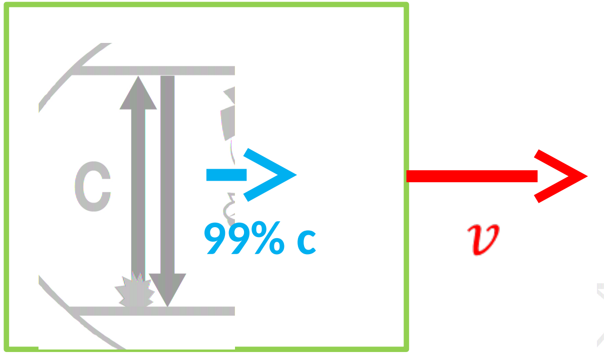
- Suppose Alice is **at rest** in a spaceship moving with **speed v** relative to Bob. $\gamma = \frac{1}{\sqrt{1-v^2/c^2}} > 1$
- She then starts **running forward** at 1 m/s inside her spaceship: she covers 1 m every 1 s according to **her** ruler and **her** clock.



  - But Bob sees Alice's 1 m **length-contracted** to $1/\gamma$ m
  - And for Bob, Alice's 1 s is **time-dilated** to γ s
  - Thus, from **Bob's perspective**, it takes Alice **longer** than 1 s to cover a distance of **less** than 1 m. Bob sees Alice's **additional** speed to be only $1/\gamma^2$ m/s (**less** than 1 m/s)
  - As v approaches c, γ approaches infinity, and Alice's **additional** speed **approaches zero**.
  - LC and TD **enforce a universal speed limit → makes sense!**

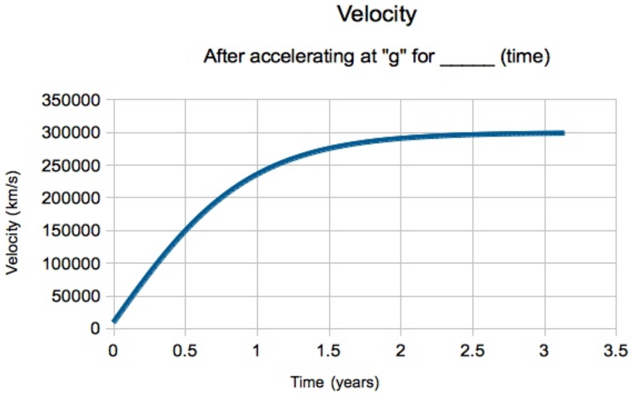


- In **Einstein's** space-time, Alice can throw a ball at 99% c relative to herself, then get in a rocket, **accelerate and catch up with the ball**, then throw the ball again at 99% c relative to herself. She can **repeat this process indefinitely**.



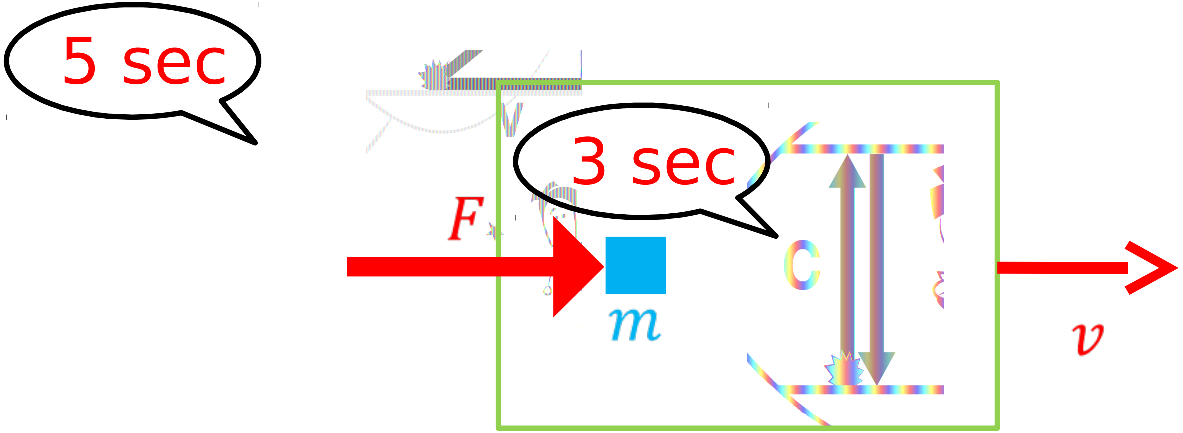
- Yet the speed of the ball will always be **less than c** from Bob's perspective, who stays at rest in Alice's original reference frame.

- There is a **speed limit** (enforced by LC & TD), but **no acceleration limit**. Alice can accelerate in a rocket with, say, constant acceleration $= g$, **<u>forever</u>**. From Bob's perspective she will just get closer and closer to the speed of light, but never reach it.



“Increasing mass”? Depends on who you ask!

- Due to **time dilation**, Bob’s force acts for a **shorter time** in Alice’s frame, and so does not cause as much acceleration (change in velocity) as he expected. Although the mass m does ***not change***, it has a greater ***effective inertia*** from Bob’s perspective due to TD (and LC…).



- In particle accelerators like the LHC, each time a particle comes around the loop it is given a **"kick" (acceleration)**, which **boosts its energy** by a certain amount, say ΔE.

- Although its **speed hardly increases** (just a little closer to c each time around), its **energy (and momentum) can be increased indefinitely.** Its energy is $\boldsymbol{E = \gamma mc^2}$ (Einstein's "$E = mc^2$"), which increases without limit as $v \to c$ ($\gamma \to \infty$).

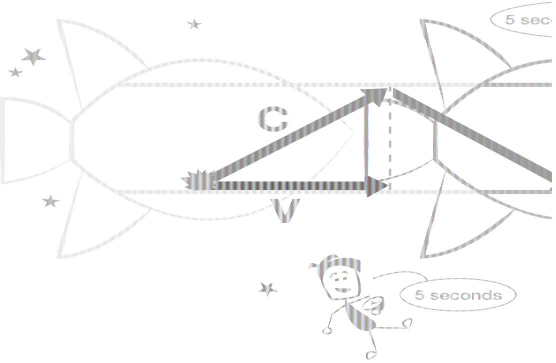
The Geometrical Nature of Einstein's Spacetime

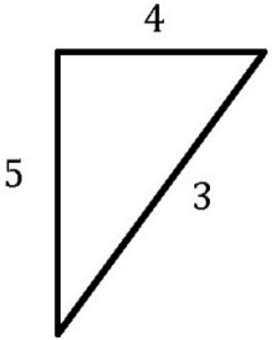
The most important consequence of TD and LC is that they logically imply that space and time are *unified* into a **4D spacetime** with a certain **geometry**. This discovery was an extremely deep insight into the ultimate nature of reality, which influenced all of physics afterwards…

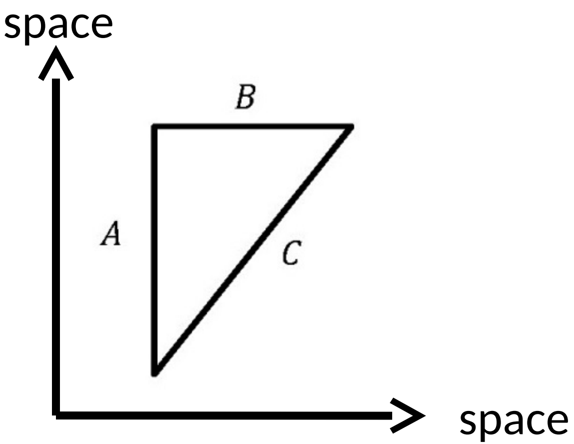
The mathematics is not difficult (grade 10 level). For those who are interested, see the article posted on LEARN. (Material in the posted article won't be on the test!)

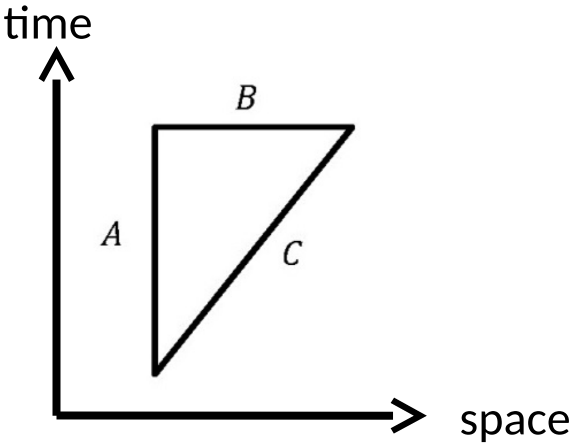
The upshot is…

- Know: space has **Euclidean geometry**: $C^2 = A^2 + B^2$
- Space*time* has **Minkowski geometry**: $C^2 = A^2 - B^2$
- **Example:** the hypotenuse (Alice’s time, 3 s) is **shorter** than the longest side (Bob’s time, 5 s). **Time dilation is geometry!**





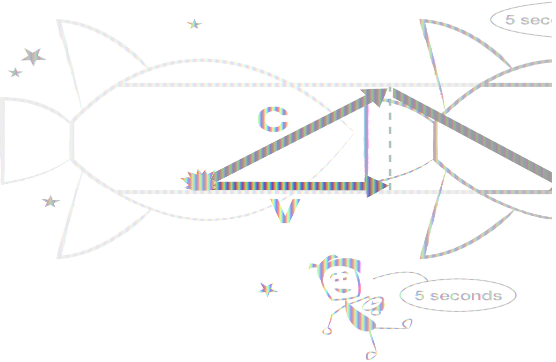


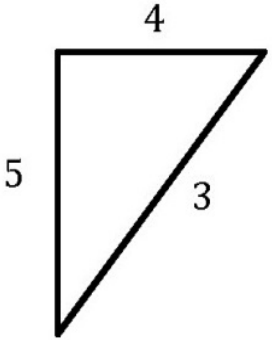


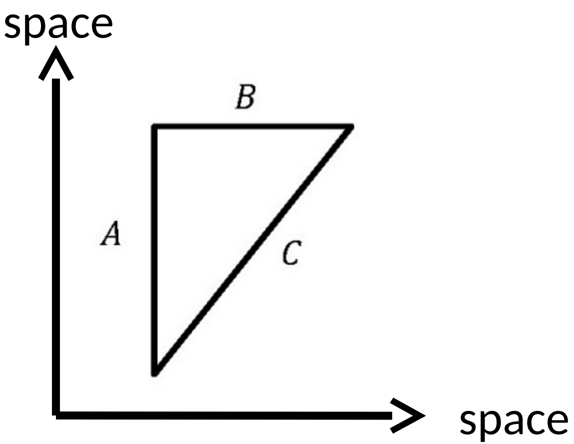
The Geometrical Nature of Einstein’s Spacetime

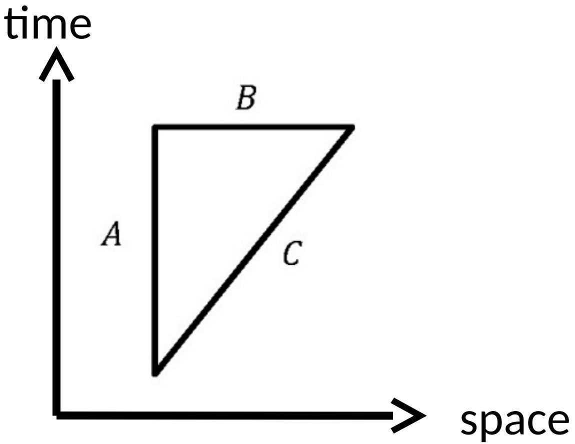
The upshot is…

- Lesson: “**Time**” **is actually a** **<u>geometrical distance</u>** in spacetime, *which depends on the path followed*! Alice’s elapsed time is the **geometrical length** of her path through Bob’s spacetime, computed using Minkowski’s version of Pythagoras’ Theorem: $C^2 = A^2 - B^2$.
- **A chronometer is a spacetime odometer. This is what time *is*.**



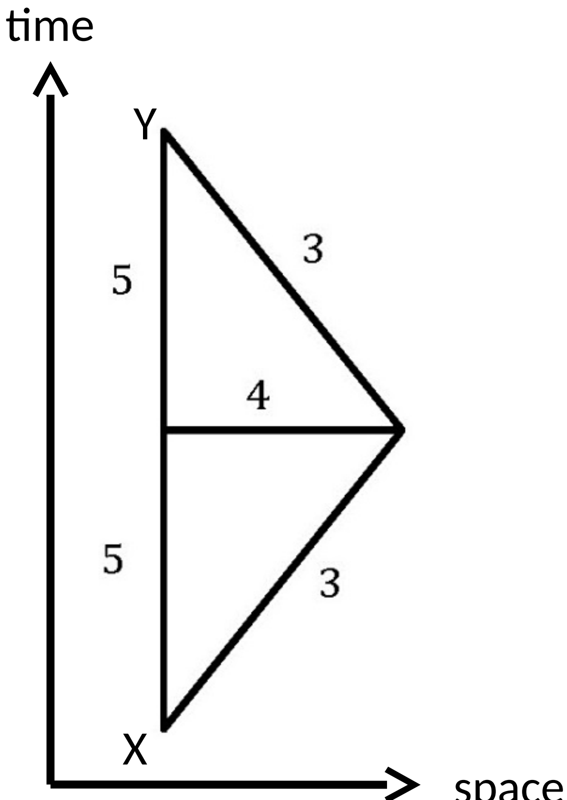




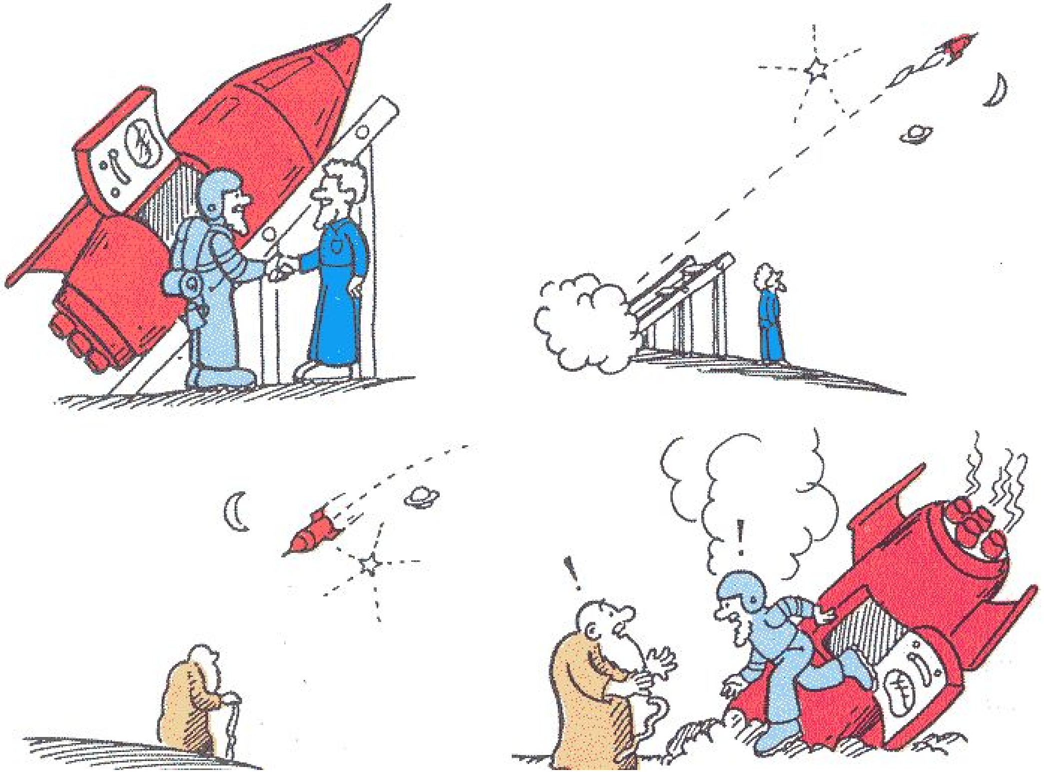


Example: Twins "Paradox" (there are no paradoxes in SR!)

- Bob **stays on Earth** and **Alice travels to a planet**, 4 ly away, at 80% the speed of light, then returns to Earth.
- **According to Bob**, it takes her 5 years to get there and 5 years to get back. **He has aged 10 years**.
- **According to Alice**, it takes her only 3 years to get there and 3 years to get back. **She has aged only 6 years**.
- **Why the difference?** It's not surprising that if Alice and Bob drove from X to Y along two different paths, **their odometer readings would be different**. Same here: Alice and Bob are just taking two different paths between here/now (X) and here/then (Y). Their **chronometer (= spacetime odometer) readings would obviously be different**. It's simple geometry!



Example: Twins "Paradox" (there are no paradoxes in SR!)

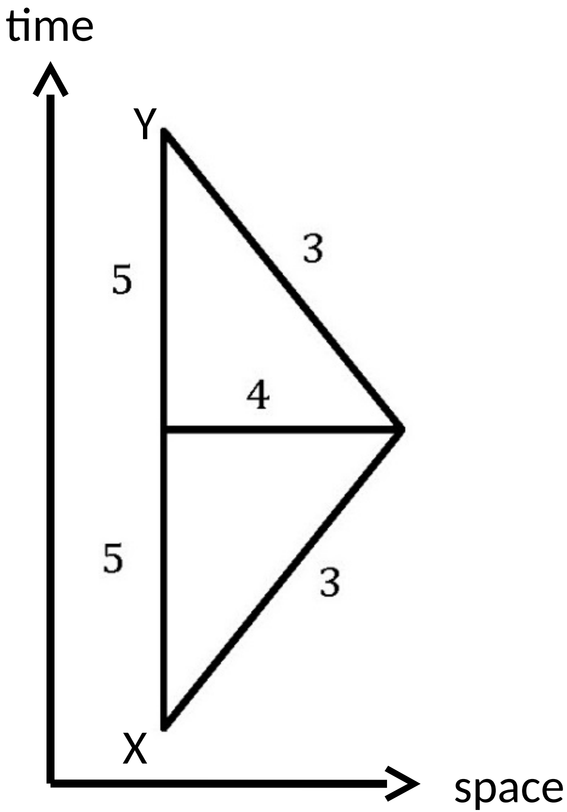


Summary (so far):

- We used to think that only 3D space had a geometry (Euclidean), and time was something **unrelated** to space.
- Based on Einstein’s ideas, his mathematics professor, Minkowski, **unified** space and time into **spacetime**; the **three dimensions** of space plus the **one dimension** of time became a **four-dimensional** spacetime, unified through **geometry**:

Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality—Minkowski 1908

- Since everything happens in space & time, this was a **big step** towards understanding the **ultimate nature of reality**.



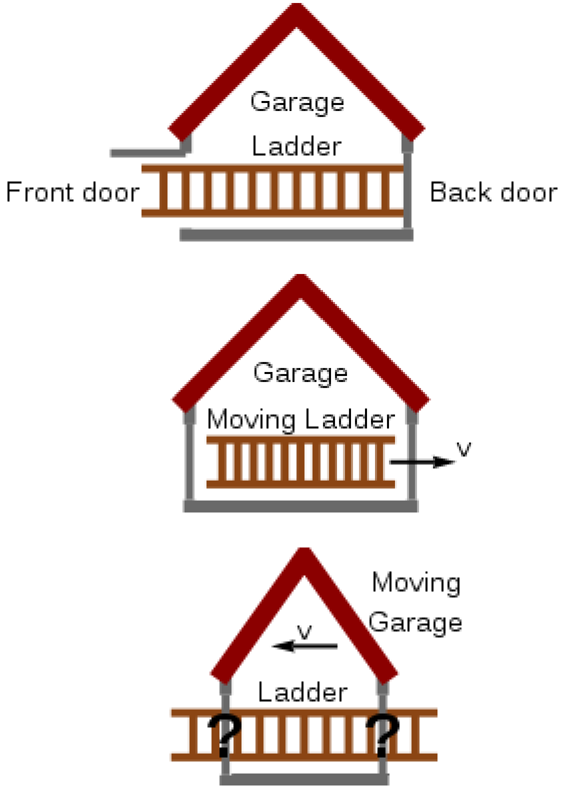
But it gets much more interesting…

Example: Ladder "Paradox" (there are no paradoxes in SR!)

- Ladder **at rest** is longer than garage. **Won't fit**.

- Ladder **moving**: From the point of view of the **garage**, the **moving** ladder is **length-contracted**. Although the ladder maintains the **same length** in its "own space", it is **shorter** in the "garage's space". If it moves fast enough, it **will fit** (***momentarily, both doors can be closed—simultaneously!***).

- <u>But</u> from the point of view of the **ladder**, it is the **garage** that is **length-contracted**. (By the principle of relativity, the garage and the ladder are equally allowed to consider themselves to be "at rest," so things like LC and TC must be **reciprocal**.) The "**won't fit**" situation appears to be **even worse**!



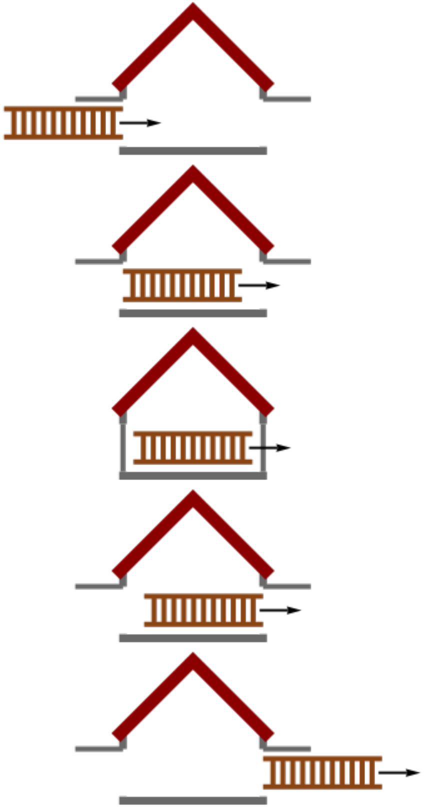
Example: Ladder "Paradox" (there are no paradoxes in SR!)

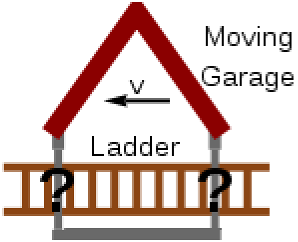
- **Recap:** From the point of view of the garage, the ladder can fit in the garage with **both doors momentarily shut**, **simultaneously**.

- But from the point of view of the ladder, the ladder cannot fit in the garage with **both doors momentarily shut**, **simultaneously**.

- This appears to be *logically impossible*!

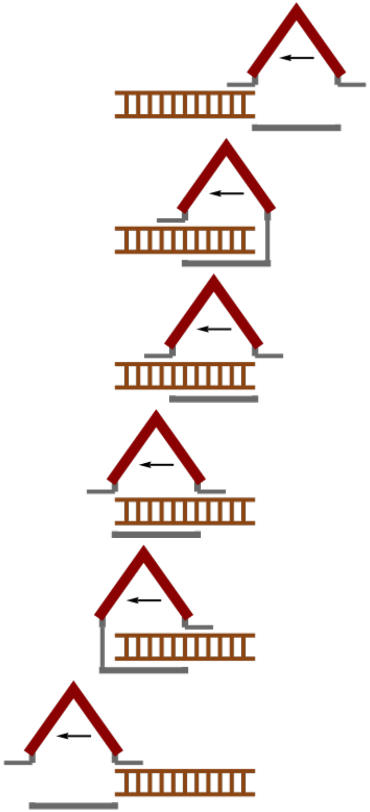
- But wait: What if "**simultaneous**" for the garage is NOT "**simultaneous**" for the ladder?





Example: Ladder "Paradox" (there are no paradoxes in SR!)

- This is exactly what the Minkowski geometry of spacetime says! From the point of view of the <u>ladder</u> (ladder at rest, garage moving):
  - …the back door momentarily closes and opens **first** (while a length of the ladder is protruding from the **front** of the garage).
  - …**later**, the front door momentarily closes and opens (while a length of the ladder is protruding from the **back** of the garage).

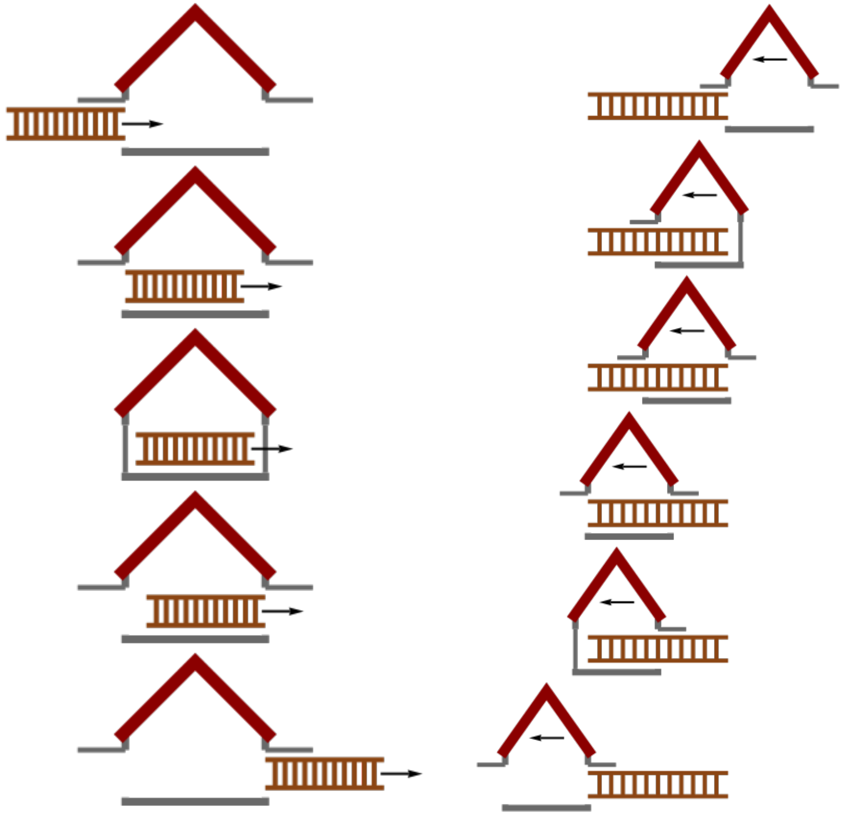


Example: Ladder "Paradox" (there are no paradoxes in SR!)

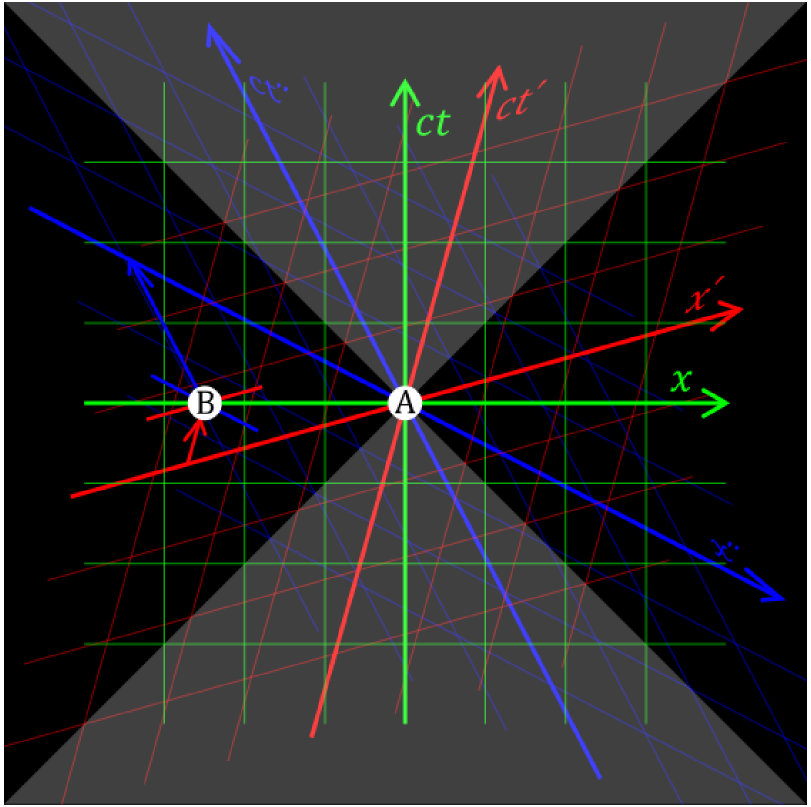
- **So what?**

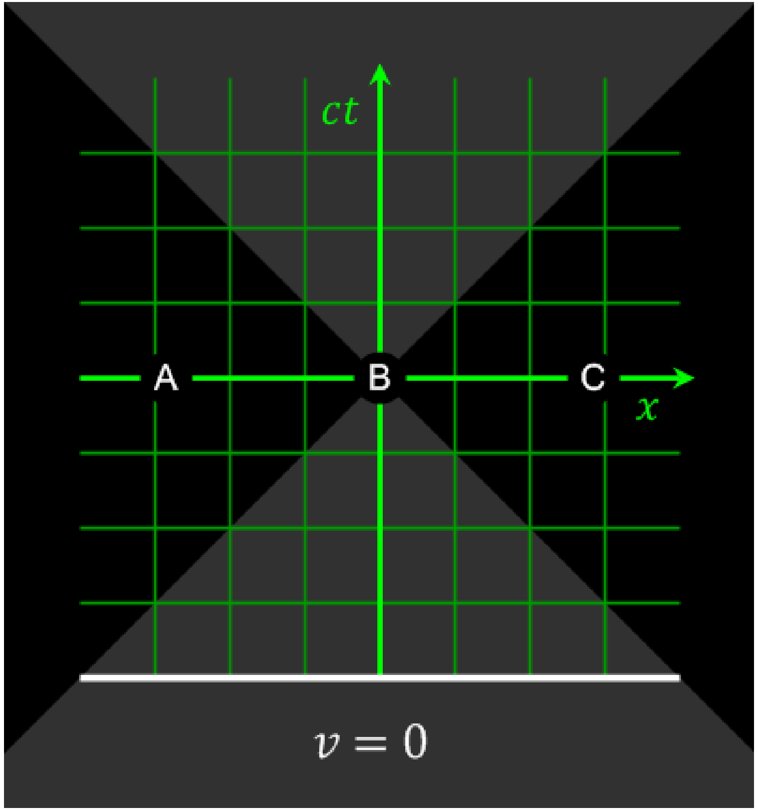
- What is **simultaneous** for one person is **not simultaneous** for another, moving relative to the first. **Simultaneity is not absolute, it is relative**. Called "**Relativity of Simultaneity**".

- Time dilation, length contraction, and relativity of simultaneity all **work together** to make Einstein's geometrically unified model of spacetime **make sense** (both intuitively, and be mathematically self-consistent).

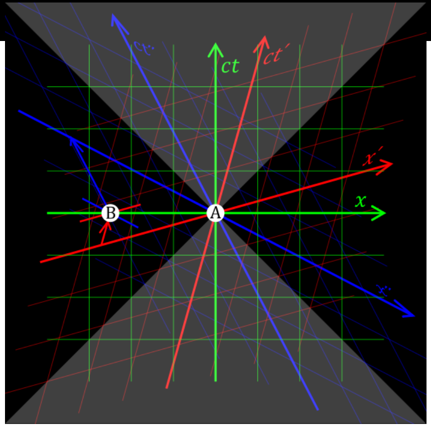


The Geometry of Relativity of Simultaneity





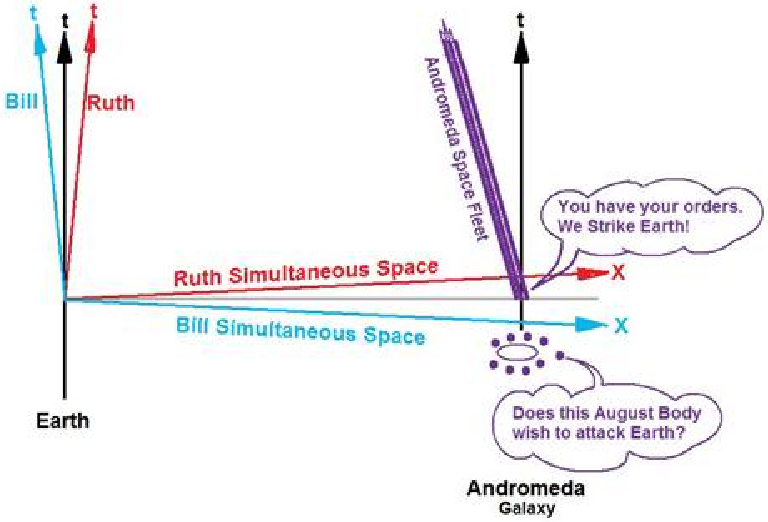
Relativity of Simultaneity



Example: Andromeda "Paradox" (there are no paradoxes in SR!)

- Bill and Ruth are on the Earth, with Ruth walking **towards** the Andromeda galaxy & Bill walking **away**.

- At the moment they cross paths:
  - Bill's "now" is simultaneous with the aliens **still debating** whether or not to attack Earth.
  - Ruth's "now" is simultaneous with the **aliens already sending** their attack fleet.



○ **So what?** *There is no such thing as an absolute "now"*. "Now" is a **relative** concept. What's happening "now" depends on who's asking!

Fate of the Ether

Einstein’s spacetime is compatible with **both** historical models of light:

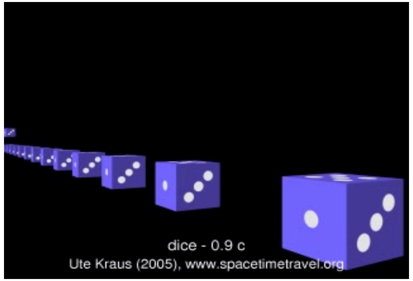
1. Huygens’ **wave-in-ether** model of light
2. Newton’s **particle-in-vacuum** model of light

Important because we now understand light to be neither a wave nor a particle, but a third thing (“quantum particle”) that exhibits **wave or particle** behaviour, depending on the situation!

Aside: Often it is said that the ether does not exist. We actually don’t know this:

> “More careful reflection teaches us however, that the special theory of relativity does **not compel us to deny ether**…”—**Einstein (1920)**

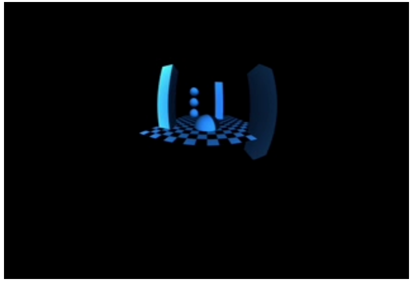
Ether fell out of favour not because experiments did not detect it, but because, in Einstein’s spacetime, it is impossible to detect it. **Ether might exist, or it might not. We can’t tell**! For most physicists, this meant the ether then falls to “Occam’s razor,” or “Popper’s falsifiability.”



A fast-moving cube will appear **rotated**. You can see the **back face** of it: the 4 face, opposite the 3. Why? Called aberration.

Guide to SR Flight Simulators

Relativity Visualized



Flight through set of objects: **aberration** and **Doppler shift**

MIT Game Lab: A Slower Speed of Light

- Modern physics is about **unifications**:

  - Special relativity: space & time ▭ spacetime

  - General relativity: spacetime & gravity ▭ curved spacetime

  - Quantum mechanics: wave & particle ▭ quantum particle