ECE 457 B: COMPUTATIONAL INTELLIGENCE

ASSIGNMENT #2

<u>Due Date: Deposit in the Dropbox Assignment #2 Folder</u>
<u>by Midnight of March 7, 2017</u>

NOTE: To help with the adequate marking of your assignment, please follow these rules carefully. Copies that disregard one or more of these rules will be penalized or rejected:

1. All work should be carried out **<u>individually</u>**
2. The first page of the assignment must have the name and ID of the student
3. The items required must be numbered, labelled, and in numerical order.
4. The solutions can be handwritten (in very neat way) or typewritten
5. All pages must be numbered sequentially
6. Show your steps and state any additional assumptions that you make.
7. Presentation of your results and the organization of your copy count for **10% of the mark**

Problem 1

We are given a dynamic process guided by the fuzzy logic control system with the following two fuzzy control rules:

Rule 1	If x is A_1 and y is B_1 Then z is C_1
Rule 2	If x is A_2 and y is B_2 Then z is C_2

Where x_0 and y_0 are the sensor readings for the linguistic input variables x and y and z is the consequent linguistic variable. The fuzzy predicates for the linguistic variables are given by A_1, A_2, B_1, B_2, C_1 and C_2, which membership functions are as:

$$\mu_{A_1}(x) = \begin{cases} \frac{x-2}{3} & 2 \le x \le 5 \\ \frac{8-x}{3} & 5 < x \le 8 \end{cases} \qquad \mu_{A_2}(x) = \begin{cases} \frac{x-3}{3} & 3 \le x \le 6 \\ \frac{9-x}{3} & 6 < x \le 9 \end{cases}$$

$$\mu_{B_1}(y) = \begin{cases} \frac{y-5}{3} & 5 \le y \le 8 \\ \frac{11-y}{3} & 8 < y \le 11 \end{cases} \qquad \mu_{B_2}(y) = \begin{cases} \frac{y-4}{3} & 4 \le y \le 7 \\ \frac{10-y}{3} & 7 < y \le 10 \end{cases}$$

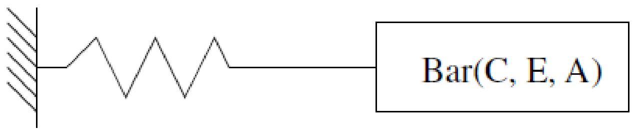
$$\mu_{C_1}(z) = \begin{cases} \frac{z-1}{3} & 1 \le z \le 4 \\ \frac{7-z}{3} & 4 < z \le 7 \end{cases} \qquad \mu_{C_2}(z) = \begin{cases} \frac{z-3}{3} & 3 \le z \le 6 \\ \frac{9-z}{3} & 6 < z \le 9 \end{cases}$$

1) Further assume that at time t_1 we are reading the sensor values at $x_0(t_1) = 4$ and $y_0(t_1) = 8$. Using the Mamdani inferencing system and the Mean of Maximum (MOM) defuzzifcation[1] strategy, find the final control output at time t_1.
2) Compare the results you obtain with the Center of Area defuzzification strategy. Discuss your results

[1] The MOM defuzzification strategy uses the formulae $z_{MOM} = \frac{\int_{Z'} z\,dz}{\int_{Z'} dz}$ where $Z' = \{z \mid \mu_C(z) = \mu^*\}$ with μ^* representing the maximum value of the aggregated membership taken over the universe of discourse Z.

Problem 2.

The calculation of the vibration of an elastic structure depends on knowing the material properties of the structure as well as its support conditions. Suppose we have an elastic structure, such as a bar of known material, with properties like wave speed (C), modulus of elasticity (E), and cross-sectional area (A). However, the support stiffness is not well known, hence the fundamental natural frequency of the system is not precise either. A relationship does exist between them, though, as illustrated in the following figure.



Define two fuzzy sets,

K = ”support stiffness," in pounds per square inch
and
f_1 = ”first natural frequency of the system," in hertz

With membership functions:

$$K = \left\{ \frac{0}{1e+3} + \frac{0.2}{1e+4} + \frac{0.5}{1e+5} + \frac{0.8}{5e+5} + \frac{1}{1e+6} + \frac{0.8}{5e+6} + \frac{0.2}{1e+7} \right\}$$

$$f_1 = \left\{ \frac{0}{100} + \frac{0}{200} + \frac{0.2}{500} + \frac{0.5}{800} + \frac{1}{1000} + \frac{0.8}{2000} + \frac{0.2}{5000} \right\}$$

a) Using the proposition, " if *x* is *K* then *y* is f_1"**,** find this relation *R* using the following forms of the implication:
   i) Classical: $\mu_R = \max[\min(\mu_K, \mu_{f_1}), (1-\mu_K)]$
   ii) Mamdani: $\mu_R = \min(\mu_K, \mu_{f_1})$
   iii) Product: $m_R = m_K . m_{f_1}$

b) Now define another antecedent, say *K'* ="damaged support,"
   $$K' = \left\{ \frac{0}{1e+3} + \frac{0.8}{1e+4} + \frac{0.1}{1e+5} \right\}$$

Find the system's fundamental (first) natural frequency due to the change in the support conditions; i.e., find f_1 ="first natural frequency due to damage support" using classical implication from part (a), section (i), and max-min composition operation

Problem 3.

A metallurgical process consists of heat treatment of a bulk of material for a specified duration of time at a suitable temperature. The heater is controlled by its fuel supply rate. A schematic diagram of the system is shown in Figure 3 (a).

The following fuzzy quantities are defined, with the corresponding states:

T : Temperature of the material (LW = low; HG = high)
M: Mass of the material (SM = small; LG = large)
P : Process termination time (FR = far; NR = near)
F : Fuel supply rate (RD = reduce; MN = maintain; IN = increase)

The membership functions of these quantities are given in Figure 3 (b). A simple rule-base that is used in a fuzzy controller for the fuel supply unit is given below:

If T is LW and P is FR then F is IN
or if T is HG then F is RD
or if M is SM and P is NR then F is MN
or if M is LG and P is FR then F is IN
or if P is NR then F is RD
End if.

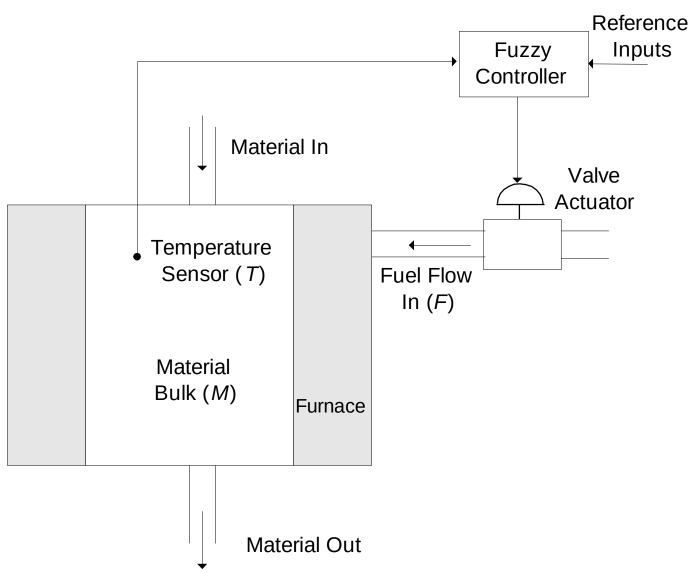


Figure 3 (a). A metallurgical heat treatment process.

At a given instant, the following set of process data is available:

Temperature = 300 °C

Material mass = 800 kg
Process operation time = 1.3 hr

Determine the corresponding inference membership function for the fuel supply, and a crisp value for the control action. Comment on the suitability of this inference.

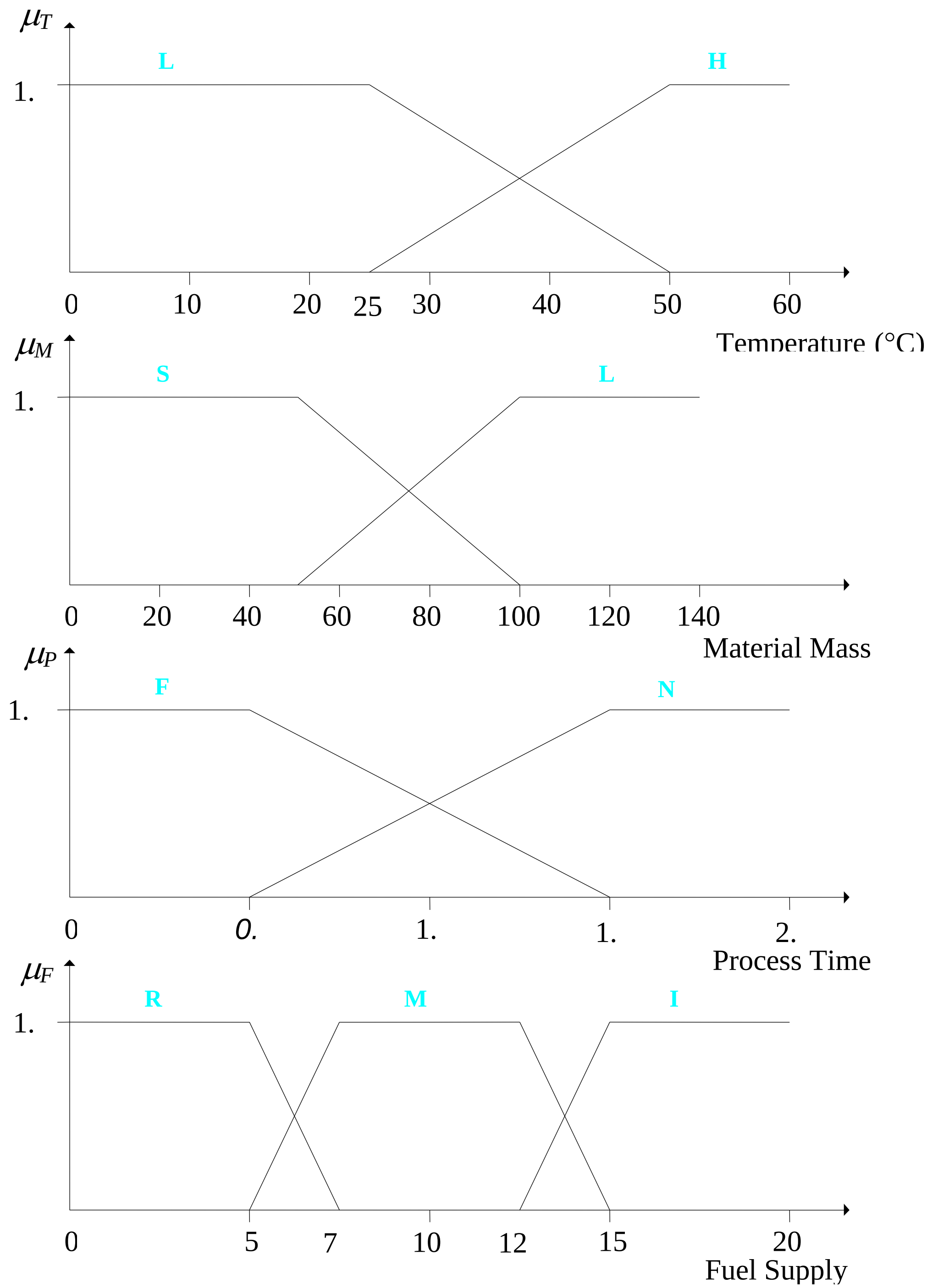


Figure 3 b. Membership functions for problem 3.

Problem 4.

We are required to use the Widrow-Hoff learning rule (LMS) to train the Adaline with signum function as its activation function for the following set of input and desired output training vectors belonging to two classes (C1=1) and (C2=-1):

$$(\mathbf{x}^{(1)} = [1, -2, 0, -1]^T \; ; t^{(1)} = -1),$$
$$(\mathbf{x}^{(2)} = [0, 1.5, -0.5, -1]^T \; ; t^{(2)} = -1),$$
$$(\mathbf{x}^{(3)} = [-1, 1, 0.5, -1]^T \; ; t^{(3)} = +1),$$

with initial weight vector $\mathbf{w}^{(1)} = [1, -1, 0, 0.5]^T$, and the learning rate $\eta = 0.1$.

Signum is given by the activation function:

$$\text{signum}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

- Run the training through one epoch cycle, and obtain the values of the weights.
- Write a small program and check whether the classes are linearly separable and in how many epochs the system would converge (if it does). Please attach the code of your program and your results. (you can use Matlab or other software program)
- To which class (C1 or C2), the following input vector belongs to: $\mathbf{u} = [-1, -1, 0, 0.5]^T$

Problem 5.

In the Adaline training (see training steps on slides 42) we consider the cumulative error to be sum squared between the target samples and the output of the combiner as follows:

$$E(\mathbf{w}) = \sum_{l} ((t^{(k)} - (\sum_{i} w_i x_i^{(k)} + \theta)))^2$$

We use here notation as in the slides. In the case, where we consider the error to be between the target samples and a filtered value of the combiner output going through the sigmoid function S, where S is given by .

$$S(x) = \text{sigmoid}(x) = \frac{1}{1 + \exp(-x)}$$

and the new error is given by:

$$E(\mathbf{w}) = \sum_{l} ((t^{(k)} - S(\sum_{i} w_i x_i^{(k)} + \theta)))^2$$

Using the LMS learning rule for the Adaline

$$\Delta \mathbf{w} = -\eta \nabla_{\mathbf{w}} E(\mathbf{w})$$

Derive the weight update formulae Δw_i for this special Adaline structure and compare it with the standard Adaline weight updating formulae $\Delta w_i = \eta(t^{(k)} - \sum_i w_i x_i^{(k)}) x_i^{(k)}$ seen in class (notes).