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By filling out the names above, the group members acknowledge that a) they have jointly authored this submission, b) this work represents their original work, c) that they have not been provided with nor examined another person's assignment, either electronically or in hard copy, and d) that this work has not been previously submitted for academic credit.			

LAB 5. LEAD AND LAG COMPENSATOR DESIGN

Always include this page at the beginning of your prelab and postlab submission.

Select your lab session:		☐ morning lab; ☒ afternoon lab; ☒ Tue; ☐ Wed; ☐ Thu
Assigned plant number:		GroupNumber - rounddown(GroupNumber/52.5, 0) * 52 45
Plant parameters found on LEARN in the Lab5 folder:	a	1.5
	b	8
	T	100
	K_i	40
Design specification 1: compensated phase-margin (±3°)		44
Design specification 2: closed-loop step-tracking e_{ss}		20%

1 Question 1

1.1 a: transfer function

$$\text{P(S)} = \frac{K_i bT}{s(s + aT) + K_i bT}$$
$$\text{P(S)} = \frac{40 * 8 * 100}{s(s + 1.5 * 100) + 40 * 8 * 100}$$
$$\text{P(S)} = \frac{32000}{s^2 + 150s + 32000}$$

1.2 b: natural frequency and dampening ratio

$$\begin{aligned}
\omega_n &= \sqrt{K_i bT} \\
&= \sqrt{32000} \\
&= 178.89 \\
\zeta &= \frac{aT}{2\omega_n} \\
&= \frac{150}{2 * 178.89} \\
&= 0.42
\end{aligned}$$

1.3 c: time to first peak

$$T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$
$$T_p = \frac{\pi}{178.89\sqrt{1 - 0.42^2}}$$
$$T_p = 0.019s$$

1.4 d: overshoot

$$\begin{aligned}
OS &= 100e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \\
&= 100e^{\frac{-0.42\pi}{\sqrt{1-0.42^2}}} \\
&= 23\%
\end{aligned}$$

1.5 e: low frequency gain

$$\begin{aligned}
\text{low frequency gain} &= 20\log P(j\omega) \\
&= 20\log(\frac{32000}{(0.01)^2 + 150j(0.01) + 32000}) \\
&= 20\log(\frac{32000}{32000}) \\
&= 20\log 1 \\
&= 0
\end{aligned}$$

2 Question 2

Since the plant is a system of type zero:

$$\begin{aligned}
K_p &= lim_{s\to 0} KP(S) \\
&= lim_{s\to 0} K \times \frac{32000}{s^2 + 150s + 32000} \\
&= K \\
e_{ss} &= \frac{1}{1+K_p} \\
&< 0.2 \\
&\implies K > 4
\end{aligned}$$

3 Question 3

We need to a gain cross over frequency of 275 rad/s. The gain at this frequency is 6.62

The gain at the desired cross over frequency must be changed to be 0.

$$\begin{aligned}
20\log\alpha &= -6.62 \\
\alpha &= 0.467
\end{aligned}$$

The phase needs to stay the same at the new cross over frequency.

$$\begin{aligned}
\frac{10}{\alpha\tau} &= 275 \\
\frac{10}{0.467\tau} &= 275 \\
\tau &= 0.078
\end{aligned}$$

Plug these values into C(S)

$$\begin{aligned}
\text{C(S)} &= K\frac{\alpha\tau s + 1}{\tau s + 1} \\
&= 4\frac{0.467 * 0.078s + 1}{0.078s + 1} \\
&= 4\frac{0.036s + 1}{0.078s + 1}
\end{aligned}$$

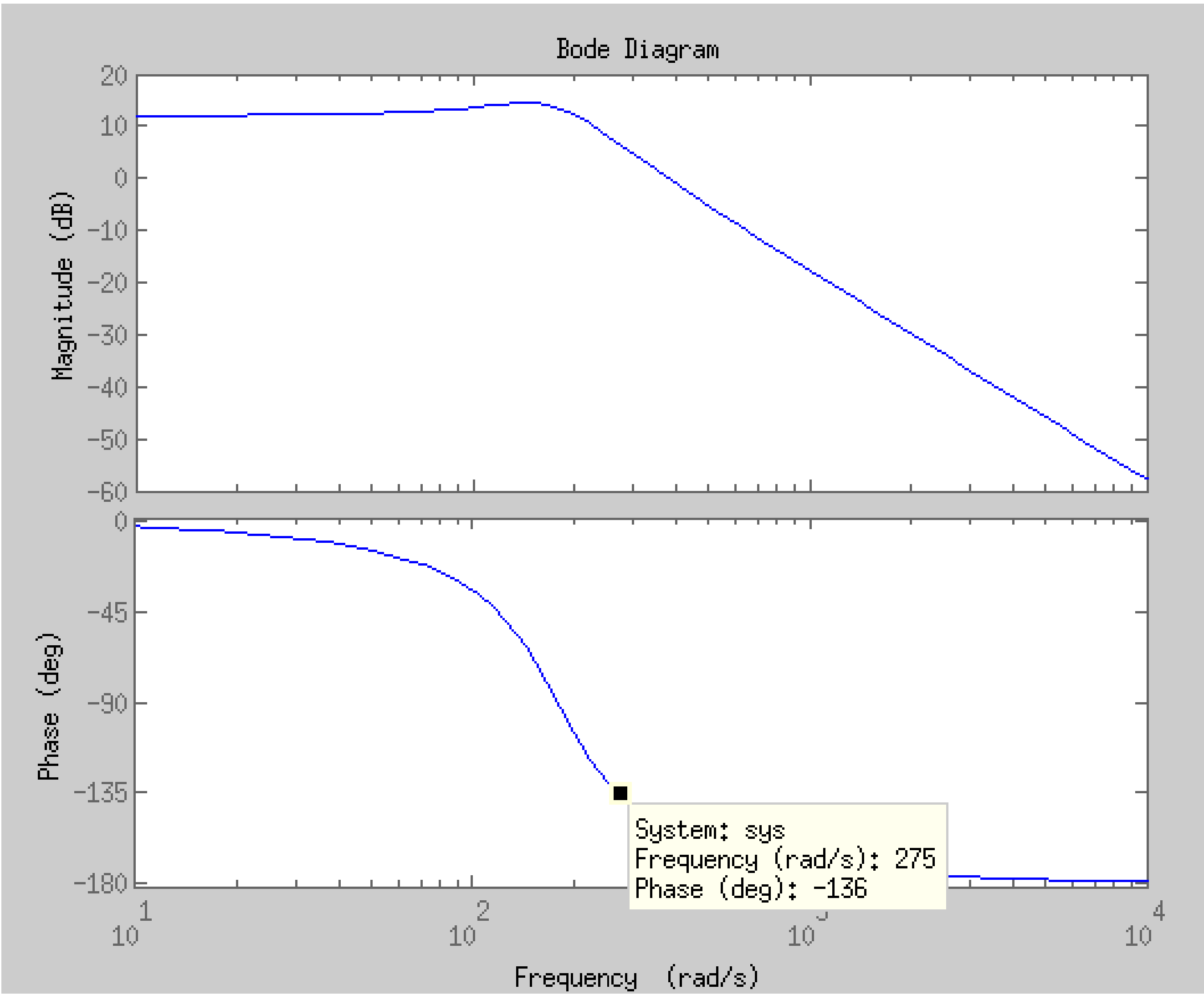

Figure 1: Bode Plot of $KP(S)$

From the transfer function of C(S) we can calculate the location of our zero and pole

$$
\begin{aligned}
0.036z_{lead} + 1 &= 0 \\
z_{lead} &= -27.48 \\
0.078p_{lead} + 1 &= 0 \\
p_{lead} &= -12.82
\end{aligned}
$$

4 Question 4

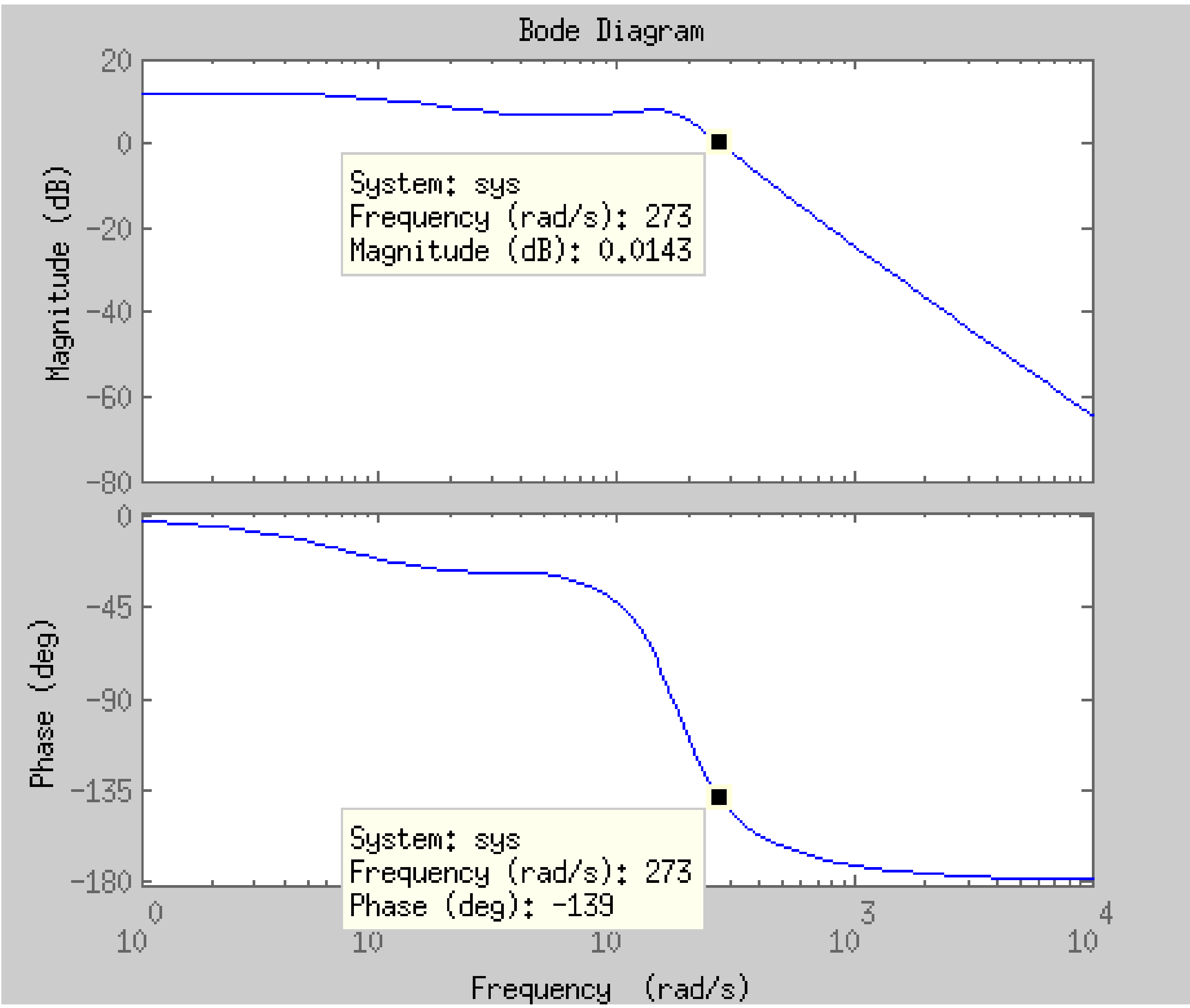

Figure 2: Frequency response of lag compensated system

This shows the desired phase margin(or close to it) at the cross over frequency indicating our compensator works.

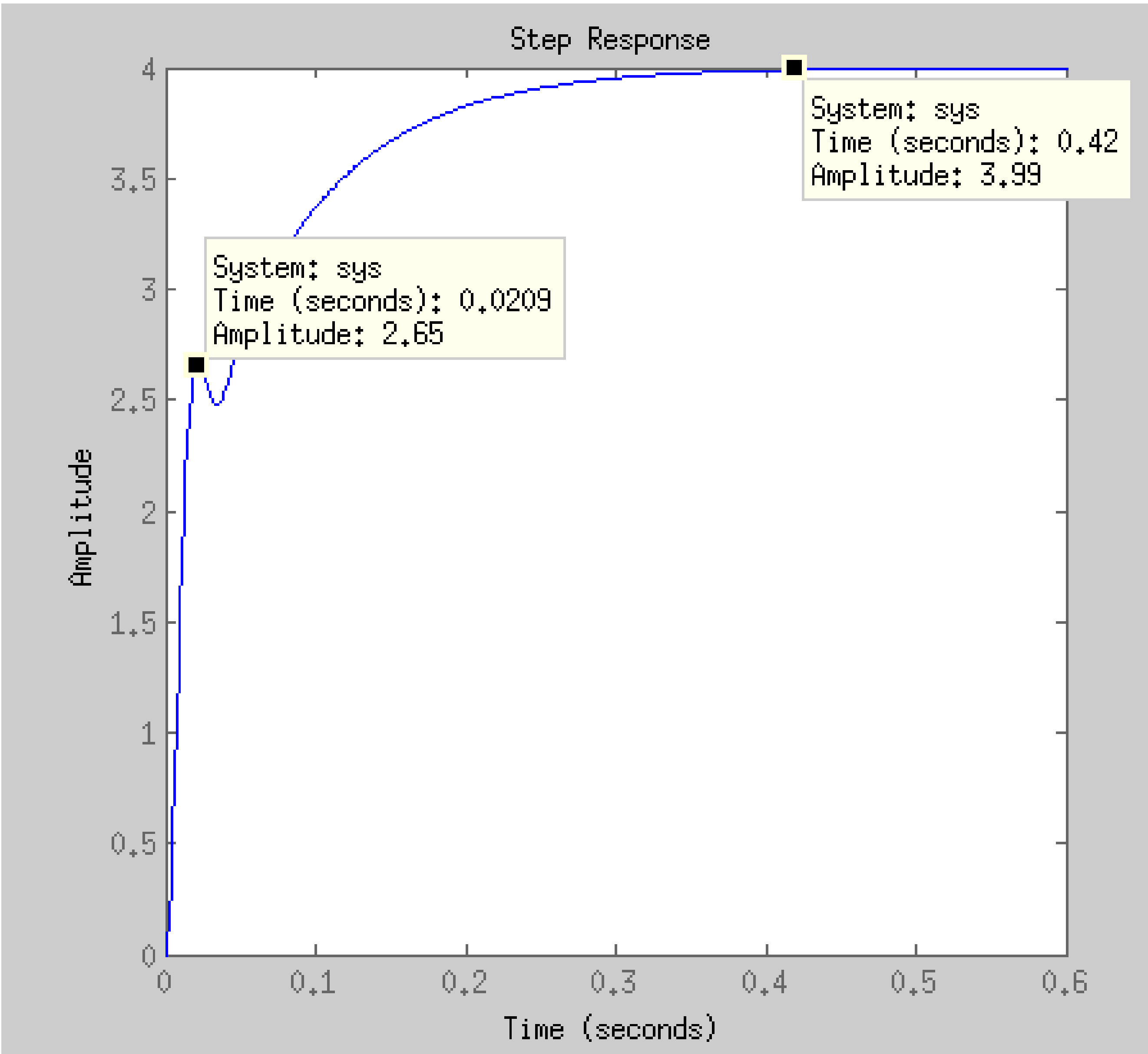

Figure 3: Step response of lag compensated system

5 Question 5

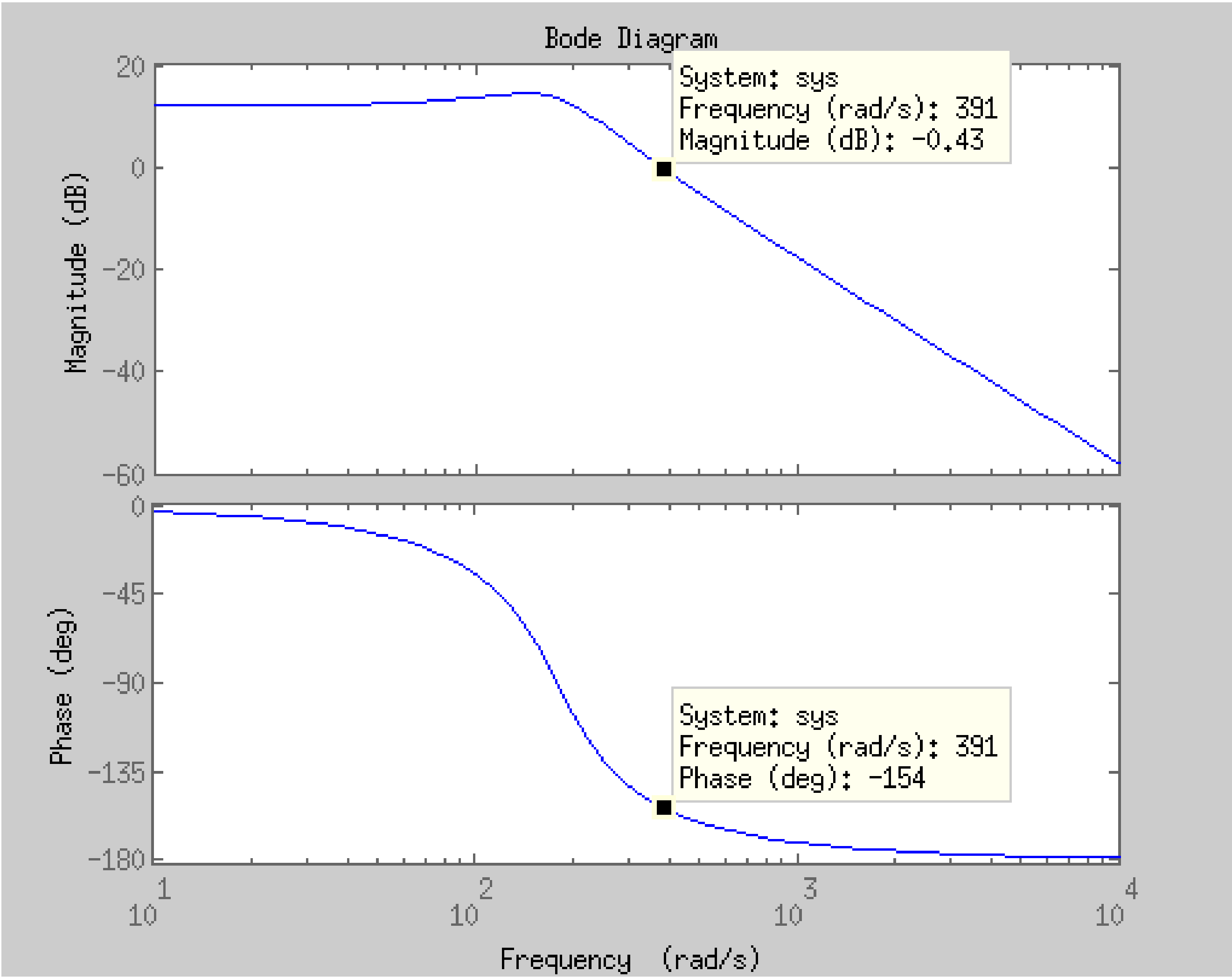

Figure 4: Bode Plot of $KP(S)$

The cross over gain frequency: $\omega_{gc} = 390$ rad/s

phase margin: $PM = 25$

We need to add 19 degrees of phase margin, we'll add 24 degrees instead.

Here we calculate for α

$$24 = \sin^{-1}\left(\frac{\alpha - 1}{\alpha + 1}\right)$$
$$0.41 = \frac{\alpha - 1}{\alpha + 1}$$
$$0.41\alpha + 0.41 = \alpha - 1$$
$$\alpha = 2.39$$

By setting the $\omega_{max} = \omega_{gc}$ we can calculate τ.

$$\omega_{max} = \frac{1}{\sqrt{\alpha}\tau}$$
$$390 = \frac{1}{\sqrt{2.39}\tau}$$
$$\tau = 0.00166$$

We then plug in the known values of α and τ to get the transfer function of C(S).

$$\begin{aligned} \mathrm{C(S)} &= K\frac{\alpha\tau s+1}{\tau s+1} \\ &= 4\frac{2.39 * 0.00166s+1}{0.00166s+1} \\ &= 4\frac{0.00396s+1}{0.00166s+1} \end{aligned}$$

From the transfer function of C(S) we can calculate the location of our zero and pole

$$\begin{aligned} 0.00396z_{lead}+1 &= 0 \\ z_{lead} &= -252.3 \\ 0.00166p_{lead}+1 &= 0 \\ p_{lead} &= -602.41 \end{aligned}$$

6 Question 6

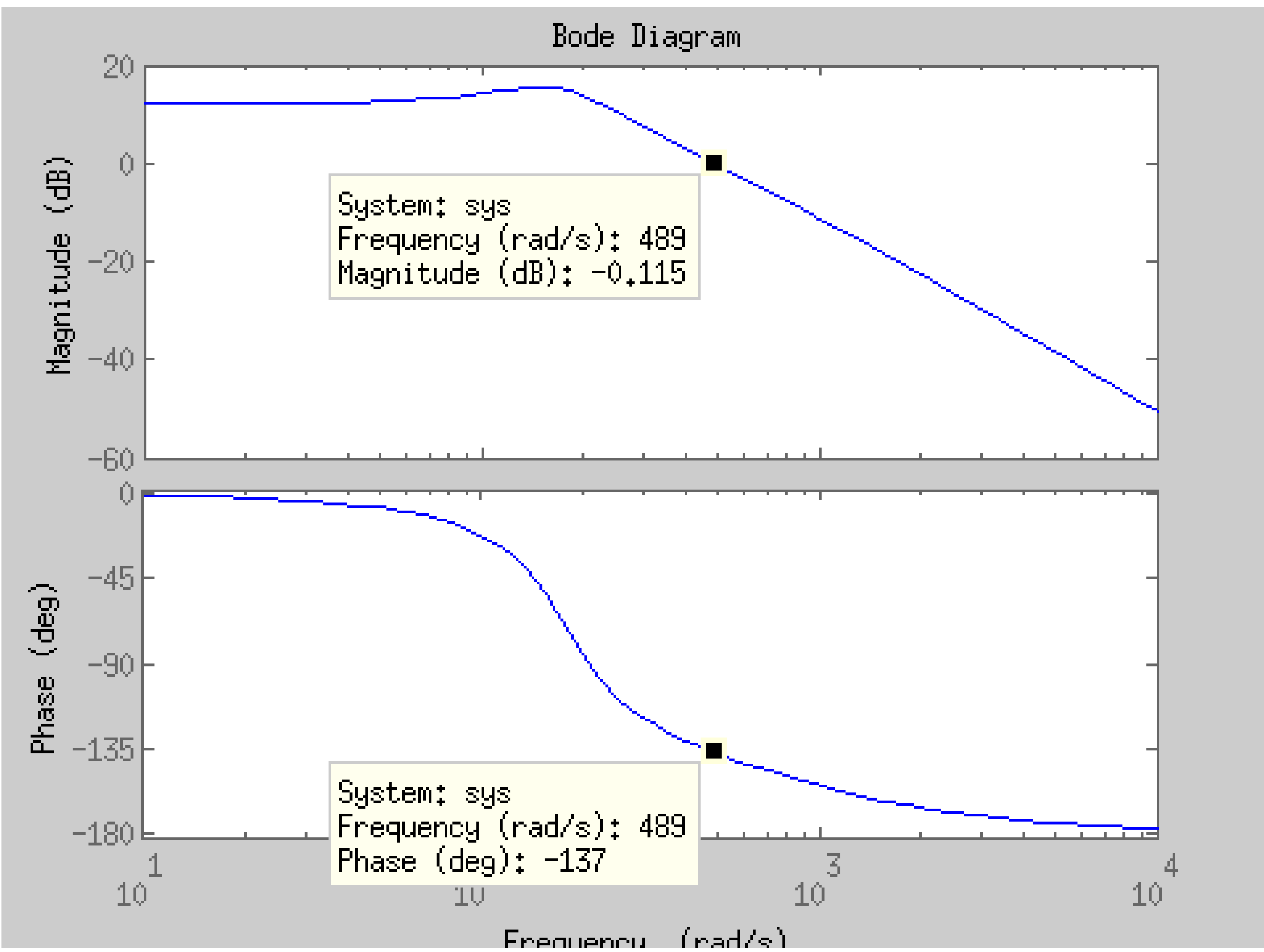

Figure 5: Frequency response of lead compensated system

This shows the phase margin of 43 which is very close to the desired phase margin of 44.

This shows a time to first peak of 0.0164 which is very close to our calculated time to first peak of 0.019. The overshoot on this is roughly 30% which is also close to our calculated value of 23%.

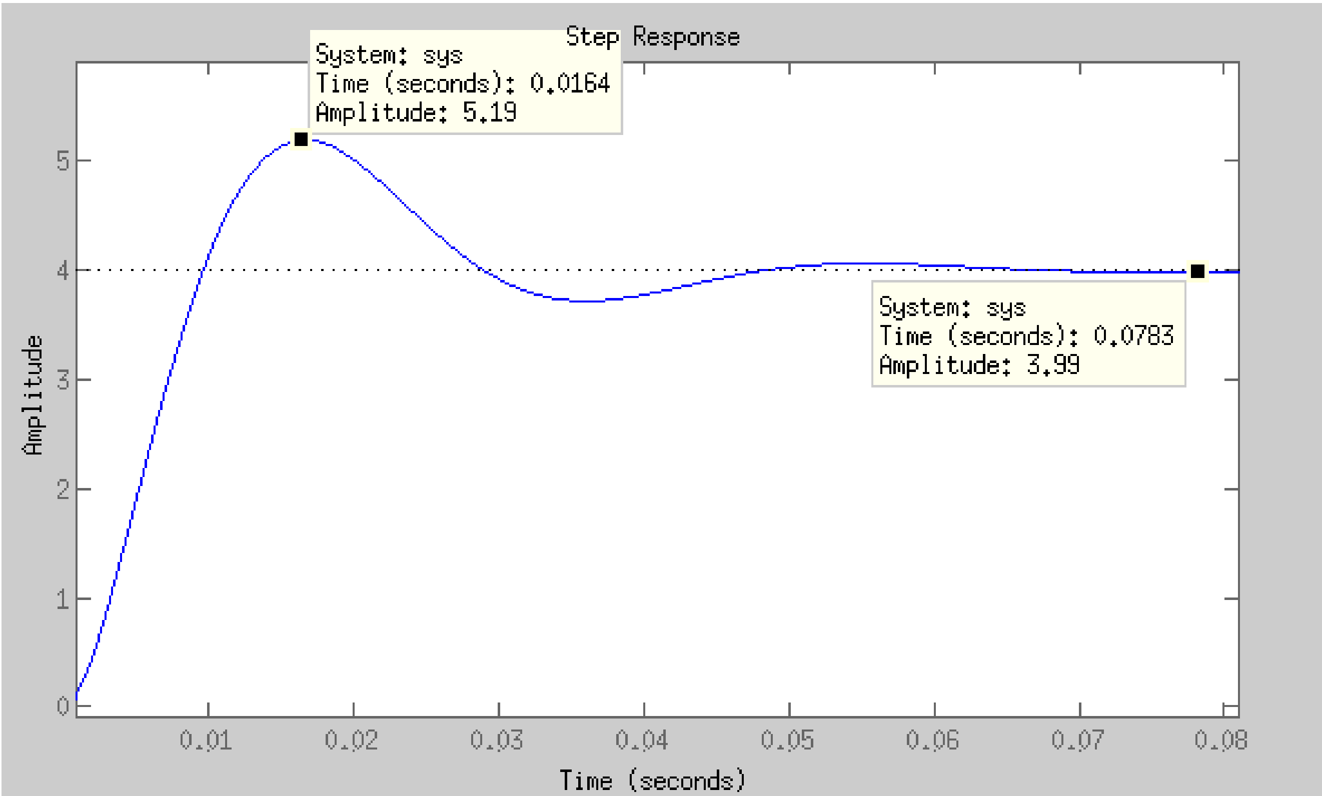

Figure 6: Step response of lead compensated system

z_{lead}	-252.3
p_{lead}	-602.41
$Gain_1$	4
z_{lag}	-27.48
p_{lag}	-12.82
$Gain_2$	4

Table 1: Prelab results for the lead and lag compensator