# Tutorial Problems 5

The TA will cover a subset of these problems during the tutorial. Solutions will not be provided outside of the tutorial.

1. In the last tutorial, you have found that the generating series for the set of strings
$$S = (1(0\{1\}^*0)^*1\{0\}^*)^*$$
is
$$\Phi_S(x) = \frac{1 - x - x^2}{1 - x - 2x^2}.$$
This set of strings describes all binary strings whose value is a multiple of 3. Use partial fraction expansion to determine an explicit formula for the coefficient of $[x^n]$ for $n \geq 1$.

2. Let $\{a_n\}$ be the sequence which satisfies
$$a_n - 7a_{n-1} + 16a_{n-2} - 12a_{n-3} = 2 \cdot 3^{n-1} + 6$$
for $n \geq 3$ with initial conditions $a_0 = 0, a_1 = 3, a_2 = 47$. Determine an explicit formula for $a_n$.

3. Let $\{b_n\}$ be the sequence
$$b_n = (5n - 3)3^n + 1$$
for each $n \geq 0$. Determine a rational expression for $B(x) = \sum_{n \geq 0} b_n x^n$.

4. Let $A_n$ be the tridiagonal matrix where the main diagonal consists of all 1's, the diagonal above the main diagonal are all $-2$'s, the diagonal below the main diagonal are all 3's, and the remaining entries are 0. For example,
$$A_1 = \begin{bmatrix} 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 1 & -2 \\ 0 & 3 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 1 & -2 & 0 & 0 \\ 3 & 1 & -2 & 0 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 3 & 1 \end{bmatrix}.$$

   (a) Let $a_n = \det A_n$. Prove that for $n \geq 3$,
   $$a_n = a_{n-1} + 6a_{n-2}.$$

   (b) Find an explicit formula for $a_n$ for $n \geq 1$.