University of Waterloo
Department of Electrical and Computer Engineering

MATH 213, Advanced Mathematics for Software Engineering
Midterm Examination
June 20, 2013

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Instructions:

- Time allowed: 90 minutes.
- No aids allowed.
- The exam comprises 4 questions with a total value of 100 points; answer all of them.

Question No. 1 (32 points)

Find the complementary solutions of the following differential equations:

(a) $\ddot{y} + 3\dot{y} + y = e^{\frac{3}{2}t}$

(b) $\ddot{y} + 2\dot{y} + y = e^{-t}$

(c) $\ddot{y} + 2\dot{y} + 1 = e^{-t} + t$

(d) $\ddot{y} + \dot{y} + y = \sin(\sqrt{3}/2)t$

(e) $\ddot{y} + 4\dot{y} = e^{j2t}$

(f) $(D+3)^3 y = t^2 e^{-3t}$

(g) $(D^2+4)^2 y = e^{-2t}$

(h) $(4D+5)y = (D+2)t^2$

Question No. 2 (32 points)

of parts (a) - (e)

Find particular solutions of each of the differential equations of the previous question.

Question No. 3 (16 points)

An electric circuit is modelled by the following differential equation

$$v_i(t) = RCv_o(t) + v_o(t)$$

and the initial condition $v_o(t) = 5$ volts. Suppose that $R = 5$ MΩ and $C = 1\mu$F and that

$$v_i(t) = \begin{cases} 0 & , \ t < 0 \\ 10 \text{ volts} & , \ 0 \leq t < 5 \text{ seconds} \\ 0 & , \ t > 5 \text{ seconds} . \end{cases}$$

(a) Does the initial-value problem have a unique solution?

(b) Find the most general solution.

Question No. 4 (20 points)

A mass-spring-damper system is modelled by the following differential equation

$$(D^2 + D + 4)y = (D + 5)x$$

and the initial condition $y(0) = 1$. Suppose that

$$x(t) = \begin{cases} 0 & , \ t < 1 \\ 1 & , \ t \geq 1 \end{cases}$$

(a) Does the initial-value problem have a unique solution?

(b) Find the most general solution. There's no need to simplify.