MATH 213
ASSIGNMENT No. 2
SOLUTIONS

1a) characteristic equation:

$$m + 10 = 0$$

$$\Leftrightarrow \quad m = -10$$

The complementary solution is therefore

$$y_c(t) = c\, e^{-10t}$$

b) Roots of char. eq$^{\underline{n}}$:

$$m = -3, -4, -5$$

The complementary solution is

$$y_c(t) = c_1 e^{-3t} + c_2 e^{-4t} + c_3 e^{-5t}$$

c) The characteristic equation has 3 roots at $m = -3$. The complementary solution is therefore

$$y_c(t) = (c_1 + c_2 t + c_3 t^2)\, e^{-3t}$$

d)

$$m^2 + 4 = 0$$

$$\Leftrightarrow \quad m = \pm j2$$

So

$$y_c(t) = c_1 e^{j2t} + c_2 e^{-2jt}$$

e) Roots of characteristic equation:

$$m = -2, \quad m = \pm j3$$

So

$$y_c(t) = c_1 e^{-2t} + c_2 e^{j3t} + c_3 e^{-j3t}$$

f) Char. eq. has 2 roots at $m = -2$, and 2 more at $m = \pm j3$.

So

$$y_c(t) = (c_1 + c_2 t) e^{-2t} + c_3 e^{j3t} + c_4 e^{-j3t}$$

g) Char. eq$^{\underline{n}}$:

$$(m^2 + 9)^2 = 0$$

$$\Leftrightarrow \quad ((m+j3)(m-j3))^2 = 0$$

$$\Leftrightarrow \quad (m+j3)^2 (m-j3)^2 = 0$$

So, 2 roots at $+j3$, 2 at $-3j$

Hence,

$$y_c(t) = (c_1 + c_2 t) e^{j3t} + (c_3 + c_4 t) e^{-j3t}$$

2. Auxiliary equation:

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = 0$$

a) $$\ddot{y} + 400\dot{y} + 10\,000\,y = 0$$

characteristic equation:

$$m^2 + 400m + 10000 = 0$$

$$\Leftrightarrow \quad m = -200 \pm \tfrac{1}{2}\sqrt{160000 - 40000}$$

$$\Leftrightarrow \quad m = -200 \pm 50\sqrt{12}$$

$$= -200 \pm 100\sqrt{3}$$

$$\cong -27, \; -373$$

So
$$y_c = c_1 e^{-27t} + c_2 e^{-373t}$$

b) $$\ddot{y} + \sqrt{2}\cdot 100\,\dot{y} + 10\,000\,y = 0$$

$$m^2 + 100\sqrt{2}\,m + 10\,000 = 0$$

$$m = -50\sqrt{2} \pm 50\sqrt{2-4}$$

$$= -50\sqrt{2} \pm j\,50\sqrt{2}$$

So
$$y_c(t) = c_1 e^{(-50\sqrt{2} + j\,50\sqrt{2})t} + c_2 e^{(-50\sqrt{2} - j\,50\sqrt{2})t}$$

$$= e^{-50\sqrt{2}t}\left(c_1 e^{j50\sqrt{2}t} + c_2 e^{-j50\sqrt{2}t}\right)$$

c) $\ddot{y} + 200\dot{y} + 10000y = 0$

$$m^2 + 200m + 10000 = 0$$

$$m = -100 \pm 50\sqrt{4-4}$$

So 2 roots at -100.

therefore,

$$y_c(t) = (c_1 + c_2 t)e^{-100t}$$

For a fixed ω_n, the critically-damped case $(\zeta = 1)$ gives the fastest-decaying real exponentials.