MATH 239 Spring 2014: Assignment 9

Due: 3:00 PM, Monday July 21, 2014 in the dropboxes outside MC 4066

Last Name: First Name:

I.D. Number: Section:

Mark (For the marker only): /28

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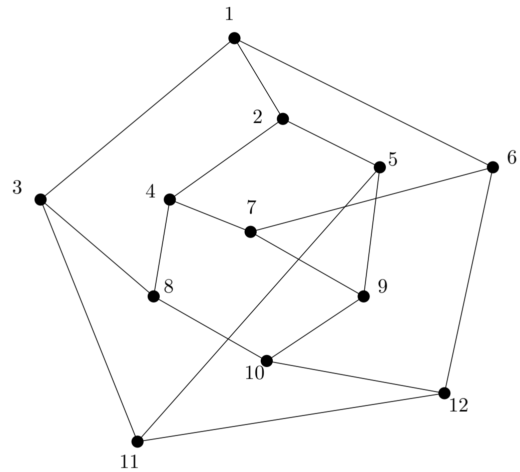
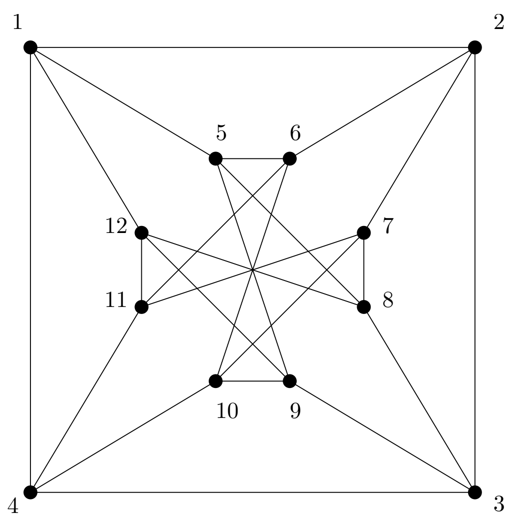
1. {5 marks} Let G be a 3-regular connected planar graph with a planar embedding where each face has degree either 4 or 6. In addition, each vertex is incident with exactly one face of degree 4. Determine the number of vertices, edges, faces of degree 4, and faces of degree 6 in this embedding. Draw a planar embedding of G.

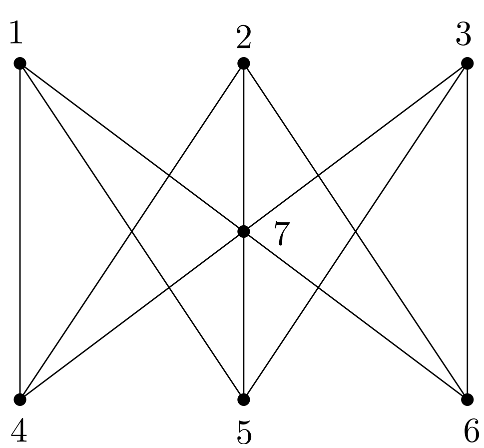
2. {4 marks} Prove that any planar embedding of a simple connected planar graph contains a vertex of degree at most 3 or a face of degree at most 3.

3. {4 marks} Let G be a connected planar graph with a planar embedding where every face boundary is a cycle of even length. Prove that G is bipartite. (Hint: Consider any cycle. Count the degrees of the faces inside this cycle.)

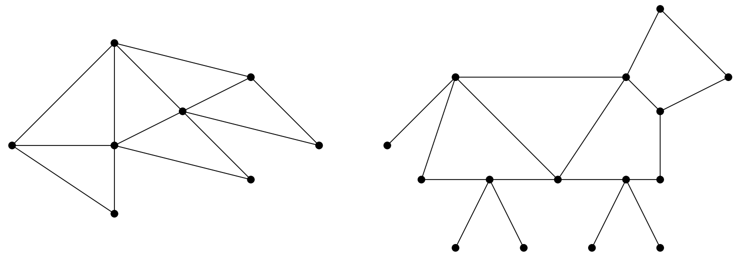
4. {9 marks} For each of the following graphs, determine whether or not it is planar. Prove your assertions.







5. A graph is *outerplanar* if it has a planar embedding where every vertex lies on the unbounded face. Two examples of outerplanar graphs are drawn below. An example of a planar graph that is not outerplanar is K_4.



(a) {3 marks} Prove that a connected outerplanar graph on n vertices has at most $2n-3$ edges. (Hint: Use Euler's formula. What is the minimum degree of the unbounded face?)

(b) {3 marks} Use Kuratowski's Theorem to prove that a graph is outerplanar if and only if it does not have an edge subdivision of K_4 or $K_{2,3}$ as a subgraph. (Hint: Modify the graph in a way so that you can apply Kuratowski's Theorem.)