Tutorial Problems 7

There is no tutorial for assignment 7. However, here are some tutorial problems with solutions to be posted on a different file.

1. Prove that a graph with n vertices where every vertex has degree at least $n/2$ is connected.

2. Prove that when $k \geq 2$, any k-regular graph contains a cycle of length at least $k + 1$.

3. Prove or disprove: If there exists a u, v-walk in G of odd length, then there exists a u, v-path in G of odd length.

4. Prove that the edges of a graph G can be partitioned into edge-disjoint cycles if and only if every vertex of G has even degree.

5. Let G be a graph and $X \subseteq V(G)$. Let C be a cycle in G. Prove that the number of edges in C that is in the cut induced by X is even.

Practice Problems for Assignment 7

1. How many Hamilton cycles are there in K_n where the vertices are labelled with $1, 2, \ldots, n$? We consider two Hamilton cycles to be the same if they use the same set of edges.

2. Suppose that P and Q are two paths of maximum length in a connected graph G. Prove that there exists at least one vertex that is in both P and Q.

3. Determine (with proof) a bipartite graph with the fewest number of edges such that it is NOT the subgraph of any n-cube.

4. Prove that for any integer $n \geq 2$, the n-cube contains a Hamilton cycle.

5. How many cycles of length 4 are there in an n-cube? For example, the 2-cube has 1 cycle of length 4, the 3-cube has 6 cycles of length 4.