Fuzzy Reasoning

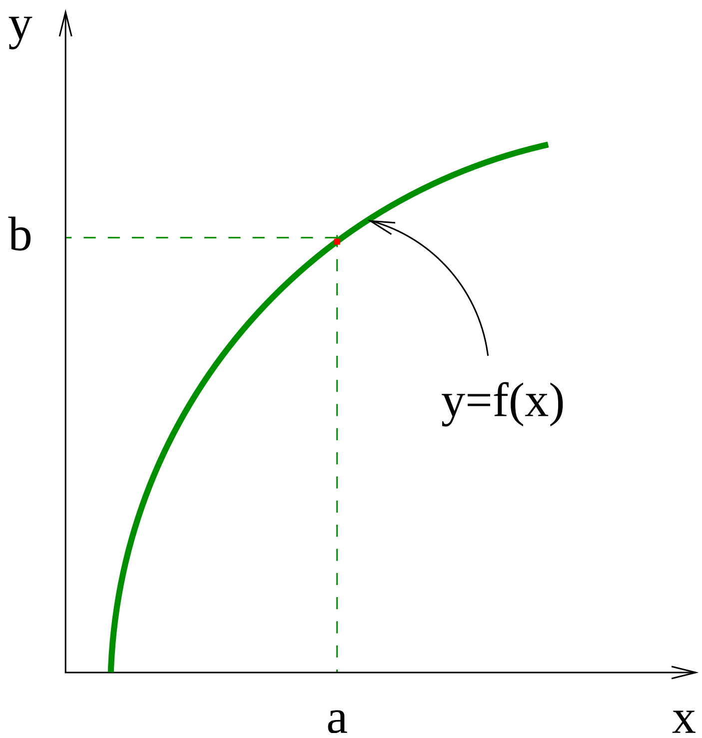
Definition: Fuzzy Reasoning

Fuzzy reasoning, also known as approximate reasoning (AR), is an inference procedure that derives conclusions from a set of if-then rules.

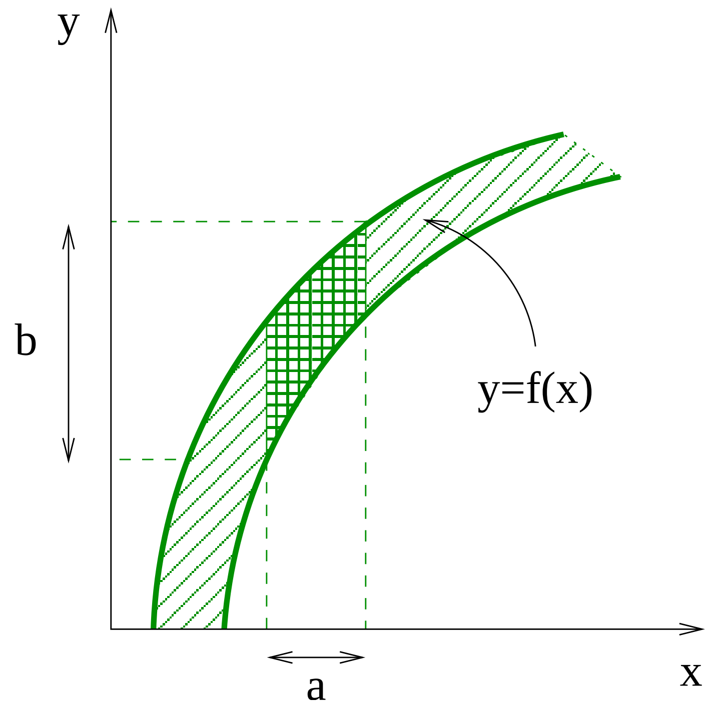
Definition: Composition Rule of Inference

- Let us assume that F is a fuzzy relation on $X \times Y$.
- Let A be a fuzzy set in X.
- To obtain the resulting fuzzy set B, we first construct a cylindrical extension of A, $C(A)$.
- The inference of $C(A)$ and F leads to the antecedent of the projection of $C(A) \cap F$.

Derivation of $y = b$ from $x = a$ and $y = f(x)$ in crisp logic setting:



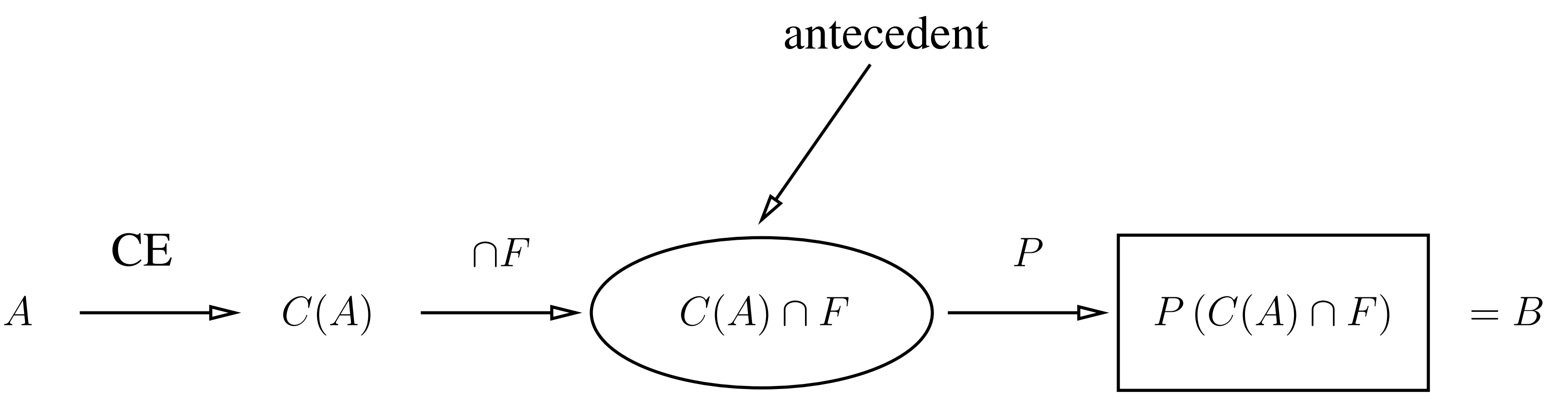
a and *b* are points
$y = f(x)$ is a curve



a and *b* are intervals
$y = f(x)$ is an interval-valued function

There is set of rules that make up the knowledge base of the system (describe it logically)

We have a set of if-then rules which we amalgamate using fuzzy reasoning so that we can derive an inference (aka decision signal).



This leads to the fuzzy set B.

$\underset{\underset{R_1}{\uparrow}}{C(A)} \underset{\underset{\circ}{\uparrow}}{\cap} \underset{\underset{R_2}{\uparrow}}{F(x, y)}$ known as the composition operator

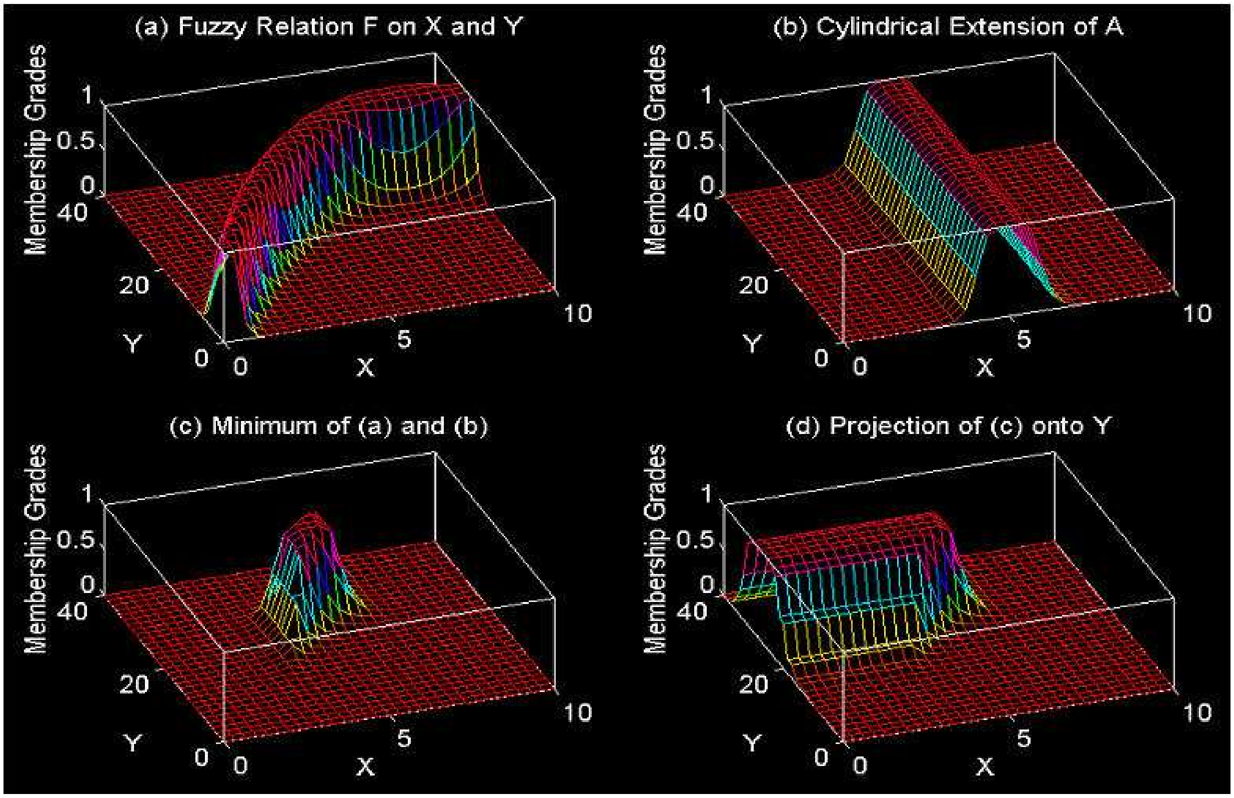
Definition: Fuzzy Reasoning

Fuzzy reasoning is basically the extension of the well known composition of elements and functions.

$$\underbrace{\mu_C(A)}_{C(A)} \cap F(x, y) = \min\left(\mu_{C(A)}(x), \mu_F(x, y)\right)$$

We have a cylindrical extension C to find the cylinder that will intersect with f to create the antecedent. P is the projection on the compliment universe of discourse. When we apply this to our antecedent we get B, our second fuzzy set. f is the set of knowledge about the rules between A and B. We need the fuzzy set A to be 3d to match f so that we can do the intersection which is why we apply C to it.

Graphical Illustration



a is a fuzzy set and $y = f(x)$ is a fuzzy relation a -¿ fuzzy relation f on x and y. if x is the temperature and y is the level of the air conditioner, then this is the result.

b -¿ cylindrical extension of a, basically $C(A)$

c -¿ this is just p this is the result of $C(A) \cap f$

d -¿ projection of (c) onto y, this is also just the max of the relationship in c and put it over y, also known as the consequent

- Projecting $\mu_C(A) \cap F(x, y)$ provides

$$
\begin{aligned}
B \equiv \mu_B(y) \\
&= \max_x \left[\min\left(\mu_{C(A)}(x), \mu_F(x, y)\right)\right] \\
&= \bigvee_x \left[\mu_{C(A)}(x) \wedge \mu_F(x, y)\right]
\end{aligned}
$$

- This is basically the max-min composition of two relations of A (unary relation) and F (binary relation)

$$B = A \circ F \qquad \text{Compositional rule of inference}$$

So B is the max over all x's of the min of the cylindrical extension of x and the relationship between x and y.

So its using the max-min composition rule

This is just the simplest case we are dealing with one variable in antecedent and one variable at consequent

Modus Ponens (MP)

- The rule of inference in conventional logic is *modus ponens*.
- MP leads to inference of truth of a proposition B from the truth of A and the implementation $A \rightarrow B$.

Example

- Rule: If tomato is red, then tomato is ripe ($A \rightarrow B$).
- Fact: Tomato is red (A).
- Conclusion: Tomato is ripe (B).

Modus Ponens (MP)

Crisp Case

Premise 1: fact	x is A
Premise 2: rule	If x is A, then y is B
Conclusion: consequence	y is B

Fuzzy Case: Fuzzy Reasoning

Premise 1: fact	x is A'
Premise 2: rule	If x is A, then y is B
Conclusion: consequence	y is B'

- A' could be close to A
- B' could be close to B
- A, B, A', and B' are fuzzy sets

Fuzzy Inferencing

- Let A, A', and B be fuzzy sets over X, X, and Y, respectively.
- Assume that the fuzzy implication $A \to B$ be expressed as fuzzy relation $R_{X \times Y}$.
- Then the fuzzy set B induced by (x is A') for the relation "x is A then y is B" is given by

$$\mu_{B'}(y) = \max_x \left[\min\left(\mu_{A'}(x), \mu_R(x, y)\right)\right]$$

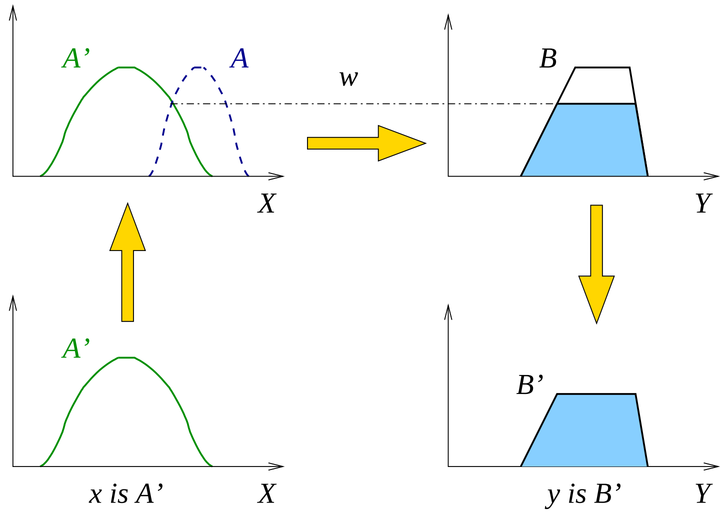
$$= \bigvee_x \left[\mu_{A'}(x) \wedge \mu_R(x, y)\right]$$

$$B' = A' \circ R = A' \circ (A \to B)$$

Case 1: Single Rule with Single Antecedent

$$\mu_{B'}(y) = \underbrace{\bigvee_x \left[\mu_{A'}(x) \wedge \mu_A(x)\right]}_{\text{degree of validity}=w} \wedge \mu_B(y) = w \wedge \mu_B(y)$$

Rule: if x is A then y is B
Fact: x is A'
Conclusion: y is B'



Using the associativity level we put together all the sets in the same universe of discourse, A and A'. B belongs to the compliment universe of discourse so it stays on its own. The t-norm is the linking value between A and A'. We call the degree of validity, the common region between A and A' and quantify it by the value given as level. This is then mapped to the compliment. The result of this gives us the membership function for B'.

The highest value of intersection of A and A' to get the weight, then map this onto the membershipt of B to get the consequent.

This is known as the mamdani representation for inferencing (aka min T-norm).

Case 2: Single Rule with Multiple Antecedents

If x is A and y is B, then z is C

Premise 1: fact	x is A' and y is B'
Premise 2: rule	If x is A and y is B, then z is C
Conclusion: consequence	z is C'

$$R : A \times B \to C \quad \Longrightarrow \quad R : A \times B \times C$$

$$R = \int_{X \times Y \times Z} \frac{\mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z)}{(x, y, z)}$$

Now we move on to multiple variables. For example we have temperature an humidity to deal with.

You do exactly the same thing (if you only have one rule) and just take the minimum value for them. This lowest value is called the firing strength.

Result: $C' : (A' \times B') \circ (A \times B \to C)$

$$\mu_{C'}(z) = \bigvee_{x,y} \underbrace{[\mu_{A'}(x) \wedge \mu_{B'}(y)]}_{(A' \times B')} \wedge \underbrace{[\mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z)]}_{(A \times B \to C)}$$

$$= \bigvee_{x,y} [\mu_{A'}(x) \wedge \mu_{B'}(y) \wedge \mu_A(x) \wedge \mu_B(y)] \wedge \mu_C(z)$$

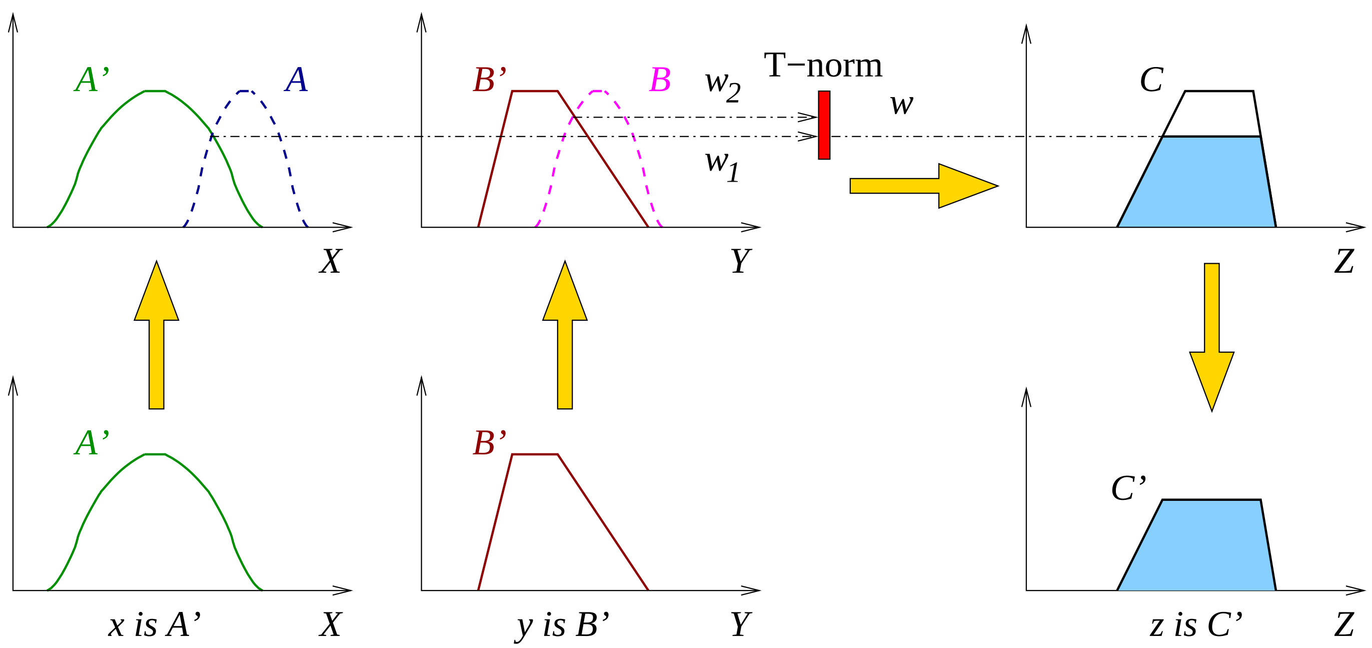
$$= \underbrace{\bigvee_{x,y} [\mu_{A'}(x) \wedge \mu_A(x)]}_{\text{degree of validity of } x \ (w_1)} \wedge \underbrace{\bigvee_{x,y} [\mu_{B'}(y) \wedge \mu_B(y)]}_{\text{degree of validity of } y \ (w_2)} \wedge \mu_C(z)$$

$$= \underbrace{(w_1 \wedge w_2)}_{\text{firing strength}} \wedge \mu_C(z)$$

General Case

In the case of n antecedents: $\mu_{C'}(z) = (w_1 \wedge w_2 \wedge \cdots \wedge w_n) \wedge \mu_C(z)$

Rule: If x is A and y is B, then z is C
Fact: x is A' and y is B'
Conclusion: z is C'



Case 3: Multiple Rules with Multiple Antecedents

Premise 1:	x is A' and y is B'
Premise 2:	If x is A_1 and y is B_1, then z is C_1, **else**
	If x is A_2 and y is B_2, then z is C_2
Consequence	z is C'

Let $R_1 : A_1 \times B_1 \to C_1$ and $R_2 : A_2 \times B_2 \to C_2$.
Since the max-min operator is distributive over Union, we get,

$$C' = \underbrace{(A' \times B')}_{\text{antecedents}} \circ \underbrace{(R_1 \cup R_2)}_{\text{rules}} = \underbrace{[(A' \times B') \circ R_1]}_{\substack{\text{rule 1 with}\\ \text{multiple}\\ \text{antecedents}}} \cup \underbrace{[(A' \times B') \circ R_2]}_{\substack{\text{rule 2 with}\\ \text{multiple}\\ \text{antecedents}}}$$

$$= C_1' \cup C_2', \text{ where } C_i' \text{ is the inferred consequent of rule } i$$

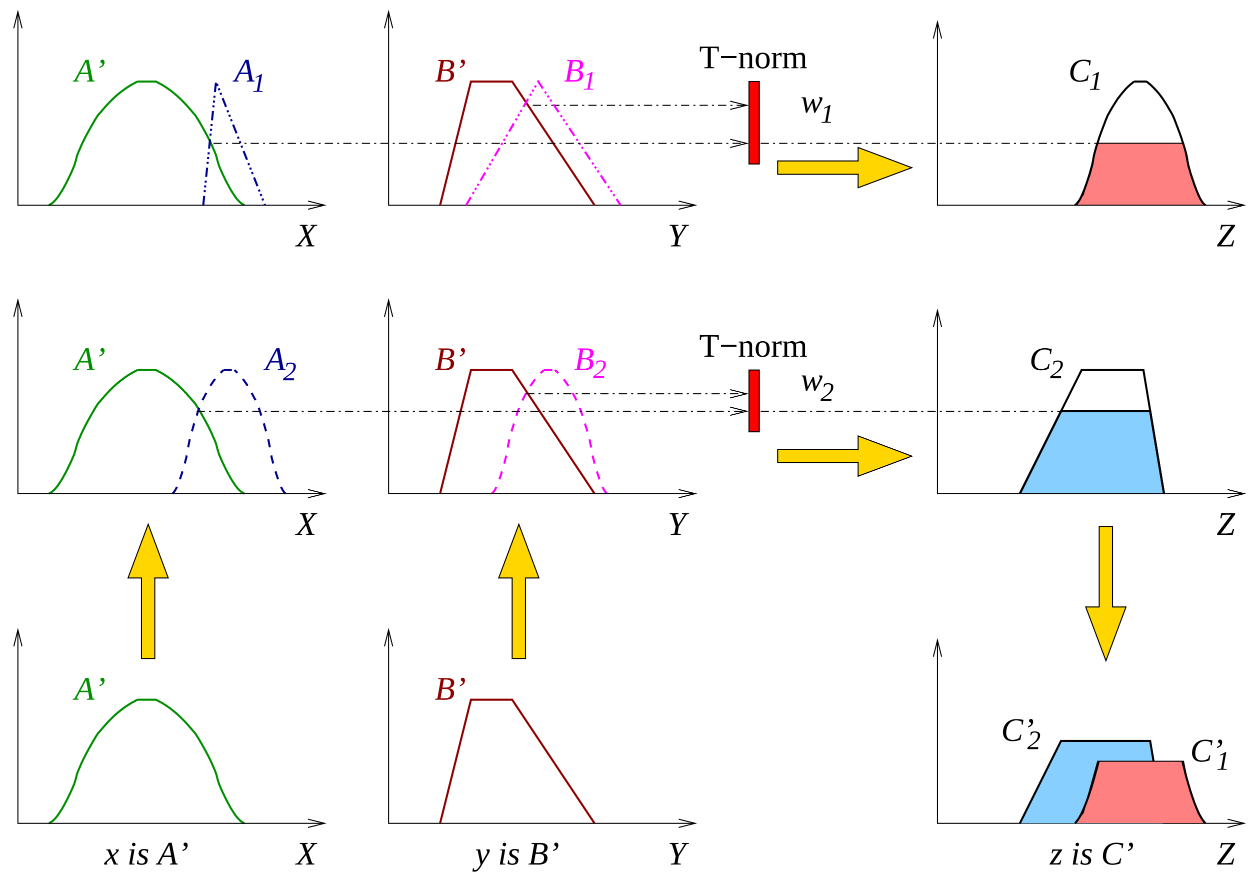
As shown previously, for $C_i' = (A' \times B') \circ R_i$,

$$\mu_{C_i'}(z) = \bigvee_x [\mu_{A'}(x) \wedge \mu_{A_i}(x)] \wedge \bigvee_y [\mu_{B'}(y) \wedge \mu_{B_i}(y)] \wedge \mu_{C_i}(z)$$

Premise one is basically just the context and premise 2 is basically the knowledge base.

For R_1 we have the consequence or C_1 and the same for R_2

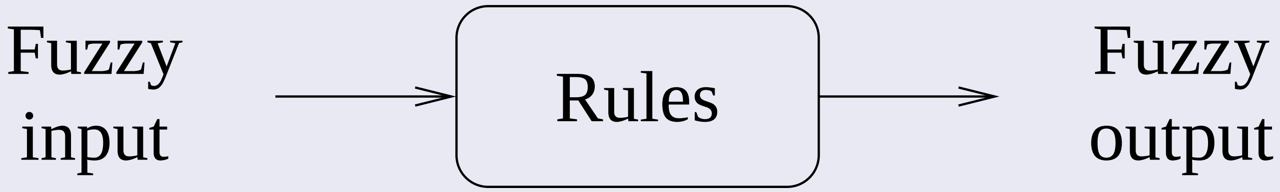
Graphical Illustration



Here we have the first rule in green and blue, the second in red and pink, and the end union in light blue and pink

A Typical Fuzzy System

Fuzzy Reasoning



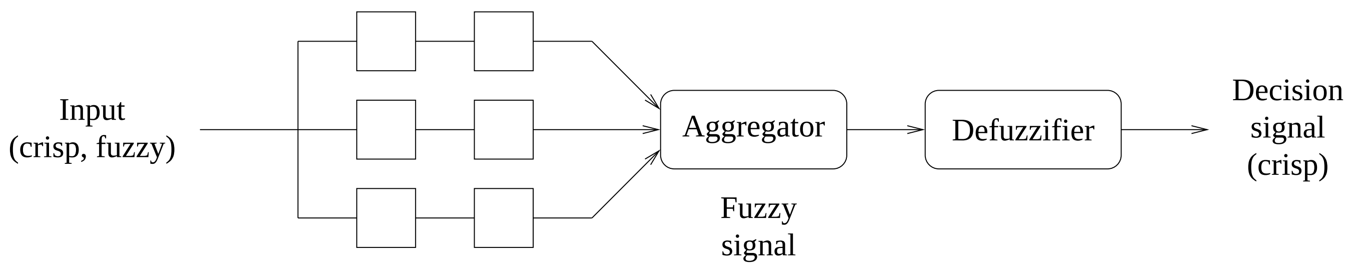
Required Steps for Extended Modus Ponens (Fuzzy Reasoning)

1. Obtain degree of compatibility
   - Compare the known facts with the antecedents of the fuzzy rules $\rightarrow$ degree of compatibility
2. Find the firing strength which combines the degree of compatibility using fuzzy “and” or “or”
   - This indicates the degree at which the antecedent part of the rule is satisfied
3. Qualified consequent
   - Apply the firing strength to the consequent membership function of a rule to generate a qualified consequent membership function
4. Overall output MF aggregates all qualified consequent MF’s to obtain the overall MF

Here MF means membership function

Fuzzy Inference System (FIS)

- The basic structure of any FIS is made of:
  1. Rule base
  2. Database of rules
  3. Reasoning mechanism (fuzzification, defuzzification)



Usually our input is a crisp value. When the input is fuzzy were are good to go, but when its crisp we need to fuzzify it (not lying, thats what its called).

Fuzzification

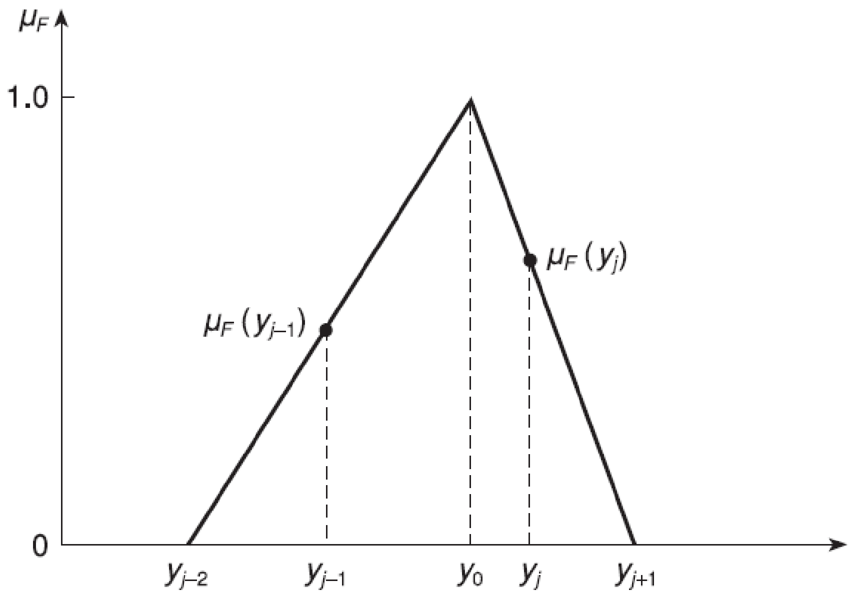
- Fuzzification refers to the representation of a crisp value by a membership function.
- It is needed prior to applying the composition (CRI), when the data (measurements) are crisp values, as common in control applications.
- It may be argued that the process of fuzzification amounts to giving up the accuracy of crisp data.
- This is not so in general. The reason is, a measured piece of data may not be known to be 100% accurate.

Different Fuzzification Methods

- Discrete case of fuzzification
- Continuous case of fuzzification
  - Singleton method
  - Triangular function method
  - Gaussian function method

Discrete Case of Fuzzification

- In the case of discrete membership functions, the crisp quantity y0 may not correspond to one of the discrete points of the membership function of the fuzzy variable Y (or fuzzy state Y_j).
- Suppose that the crisp data value y_0 falls between the discrete values y_{j-1} and y_j of the membership function.



Discrete Case of Fuzzification (cont.)

- Assigning a membership grade of 1 for y_0, the membership grades of the fuzzified quantity F at y_{j-1} and y_j are determined through linear interpolation as $\frac{y_{j-1}-y_{j-2}}{y_0-y_{j-2}}$, $\frac{y_{j+1}-y_j}{y_{j+1}-y_0}$, respectively.
- Accordingly, the discrete membership function of the fuzzified quantity F is given by:

$$F = \{\frac{\frac{y_{j-1}-y_{j-2}}{y_0-y_{j-2}}}{y_{j-1}}, \frac{\frac{y_{j+1}-y_j}{y_{j+1}-y_0}}{y_j}\}$$

- The approach can be extended to include more than two discrete points, thereby providing a wider membership function (greater fuzziness).

Fuzzification: Singleton Method

- Consider a crisp measurement y_0 of a fuzzy variable Y.
- It is known that the measurement y_0 is perfectly accurate.
- y_0 may be represented by a fuzzy quantity F with the singleton membership function

$$\mu_F(y) = \delta(y - y_0) = \begin{cases} 1 & \text{when } y = y_0 \\ 0 & \text{elsewhere} \end{cases}$$

- Since the measured data are not perfectly accurate, a more appropriate method of using fuzzy singleton to fuzzify a crisp value is given now.

Fuzzification: Singleton Method (cont.)

- Suppose that a crisp measurement y_0 is made of a fuzzy variable Y. Let Y can take n fuzzy states $Y_1, Y_2, \cdots, Y_n$.
- Since $Y = Y_1\ OR\ Y_2\ OR\ \cdots\ OR\ Y_n$, the membership function of Y is given as the union of the membership functions of the individual fuzzy states:

$$\mu_Y(y) = max_{j=1}^{n} \mu_{Y_j}(y)$$

- The membership function of the fuzzified quantity F is given according to the extended singleton method by a set of fuzzy singletons. For state j:

$$\mu_F(y) = \mu_{Y_j}(y)\delta(y - y_0)$$

Fuzzification: Triangular Function Method

- A triangular membership function may be used to represent the fuzzified quantity for each fuzzy state.
- A triangular membership function (continuous case) may be expressed as

$$\mu_A(y) = \begin{cases} 1 - \frac{|y-y_0|}{s} & \text{for } |y - y_0| \leq s \\ 0 & \text{elsewhere} \end{cases}$$

  - where y_0 shows the peak point and s denotes the base length (support set).

Fuzzification: Triangular Function Method (cont.)

- Again $Y = Y_1 \ OR \ Y_2 \ OR \ \cdots \ OR \ Y_n \cdot$
- The membership function of the fuzzified quantity F is such that, for state j,

$$\mu_F(y) = \mu_{Y_j}(y_0)\mu_{A_j}(y),$$

  - where

$$\mu_{A_j}(y) = \begin{cases} 1 - \frac{|y-y_0|}{s_j} & \text{for } |y - y_0| \leq s_j \\ 0 & \text{elsewhere} \end{cases}$$

    - s_j : base length for state $j \cdot$

Fuzzification: Gaussian Function Method

- A Gaussian membership function may be used to represent the fuzzified quantity for each fuzzy state.
- A Gaussian membership function (continuous case) may be expressed as

$$\mu_A(y) = exp(\frac{y - y_0}{s})^2$$

- The smaller the s the sharper (or less fuzzy) the membership function.

Fuzzification: Gaussian Function Method (cont.)

- Again $Y = Y_1\ OR\ Y_2\ OR\ \cdots\ OR\ Y_n$.
- The membership function of the fuzzified quantity F is such that, for state j,

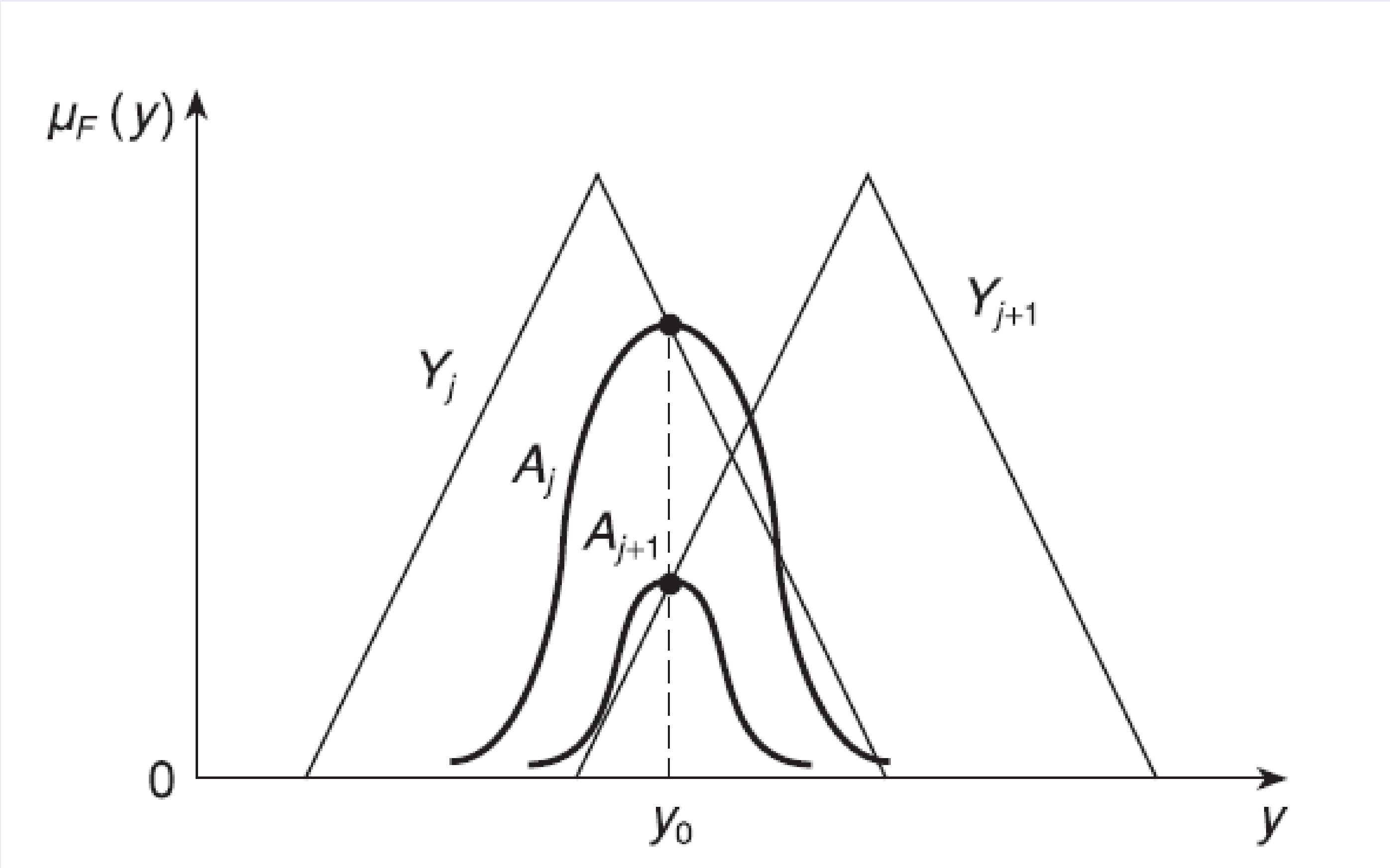
$$\mu_F(y) = \mu_{Y_j}(y_0)\mu_{A_j}(y),$$

  - where

$$\mu_{A_j}(y) = exp(\frac{y - y_0}{s_j})^2$$

Fuzzification: Gaussian Function Method (cont.)

Graphical Representation



Defuzzification

- Usually, the decision (control action) of a fuzzy logic controller is a fuzzy value and is represented by a membership function.
- Because low-level control actions are typically crisp, the control inference must be defuzzified for physical purposes such as actuation.
- Methods of defuzzification:
  - Centroid method
  - Mean of maxima method
  - Threshold methods

These were very briefly covered and will not be in the final as much (singleton might be).

Defuzzification

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- Methods of defuzzification:
    - Centroid method
    - Mean of maxima method
    - Threshold methods

The centroid method is the most difficult but it makes the most sense. Basically the center of an object best represents it. The maxima method is the mean of the maxes (I think, his accent messed it up).

Centroid Method (Center of Gravity)

- Suppose that the membership function of a control inference is $\mu_C(c)$, and its support set is given by : $S = c|\mu_C(c) > 0\cdot$
- The centroid method of defuzzification is expressed as:

Continuous Case

$$\hat{c} = \frac{\int_{c \in S} c\mu_C(c)dc}{\int_{c \in S} \mu_C(c)dc}$$

Discrete Case

$$\hat{c} = \frac{\sum_{c_i \in S} c_i \mu_C(c_i)}{\sum_{c_i \in S} \mu_C(c_i)}$$

I think this works by breaking the signal up into chunks, finding the area and weighting it using the local max for that area. We sum up all of these and divide by the sum of the weights (normalize it).

Mean of Maxima Method

- If the membership function of the control inference is unimodal, the control value at the peak membership grade is chosen as the defuzzified control action.

$$\hat{c} = c_{max} \text{ such that } \mu_C(c_{max}) = \mu_C(c)$$

- If the control membership function is multi-modal, the mean of the control values at these peak points, weighted by the corresponding membership grades, is used as the defuzzified value.

$$c_i \text{ such that } \mu_C(c_i)\Delta = \mu_i = max_{c \in S}\mu_C(c), \quad i = 1, 2, \cdots, P$$

  - Then

$$\hat{c} = \frac{\sum_{i=1}^{P} \mu_i c_i}{\sum_{i=1}^{P} \mu_i}, \quad p : \text{total number of modes (peaks)}$$

You find the interval of the max values and take the mean over that interval. When it is not over some interval we weight each peak value using its membership grades (denoted by μ) and find the average of it.

Mean of Maxima Method

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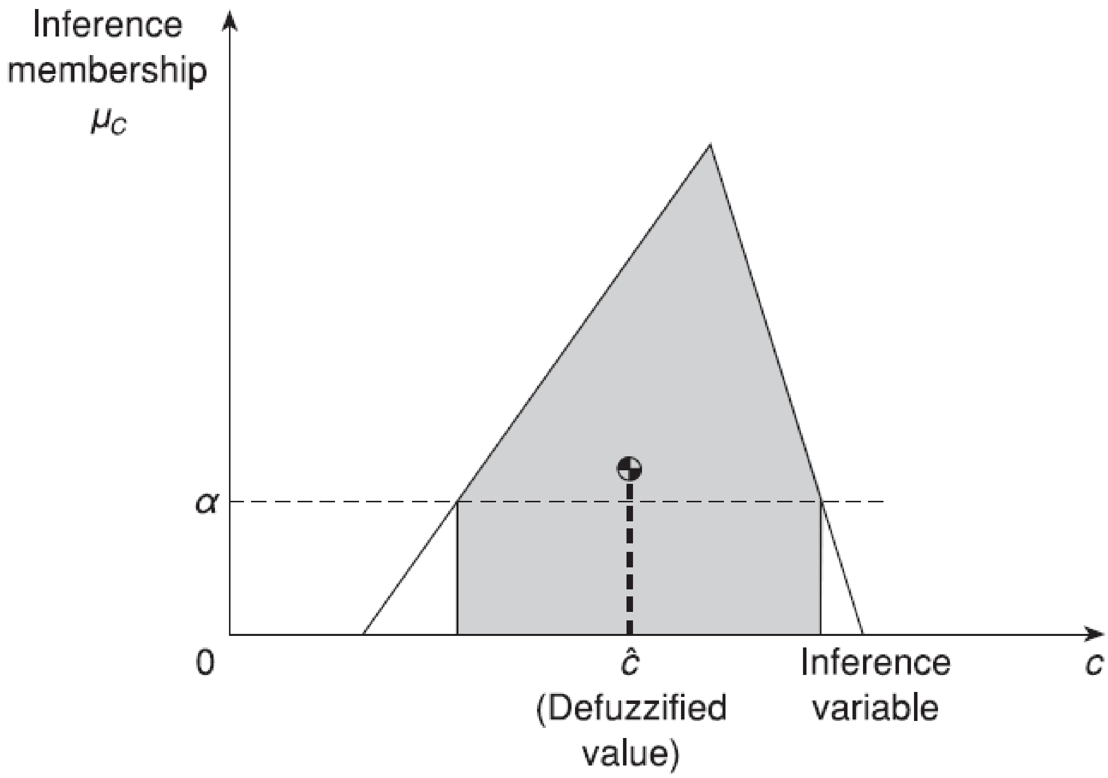
Threshold Methods

- Sometimes, it may be desirable to leave out the boundaries of the control inference membership function.
- Only the main core of the control inference is used, not excessively diluting or desensitizing the defuzzified value.
- The corresponding procedures of defuzzification are known as threshold methods.
- The formulae remain the same as given before. However, we use an α-cut of the control inference set not the entire support set.

Threshold Methods: Graphical Illustration



This is basically the point at which the area on the left of this line is equal to the area right of the line. Its a value on the x axis. This is the *bisector of area defuzzified.*

The smallest of maximum is the smallest input value for all maximum output values, and vice versa for largest of maximum.

Defuzzification: Other Methods

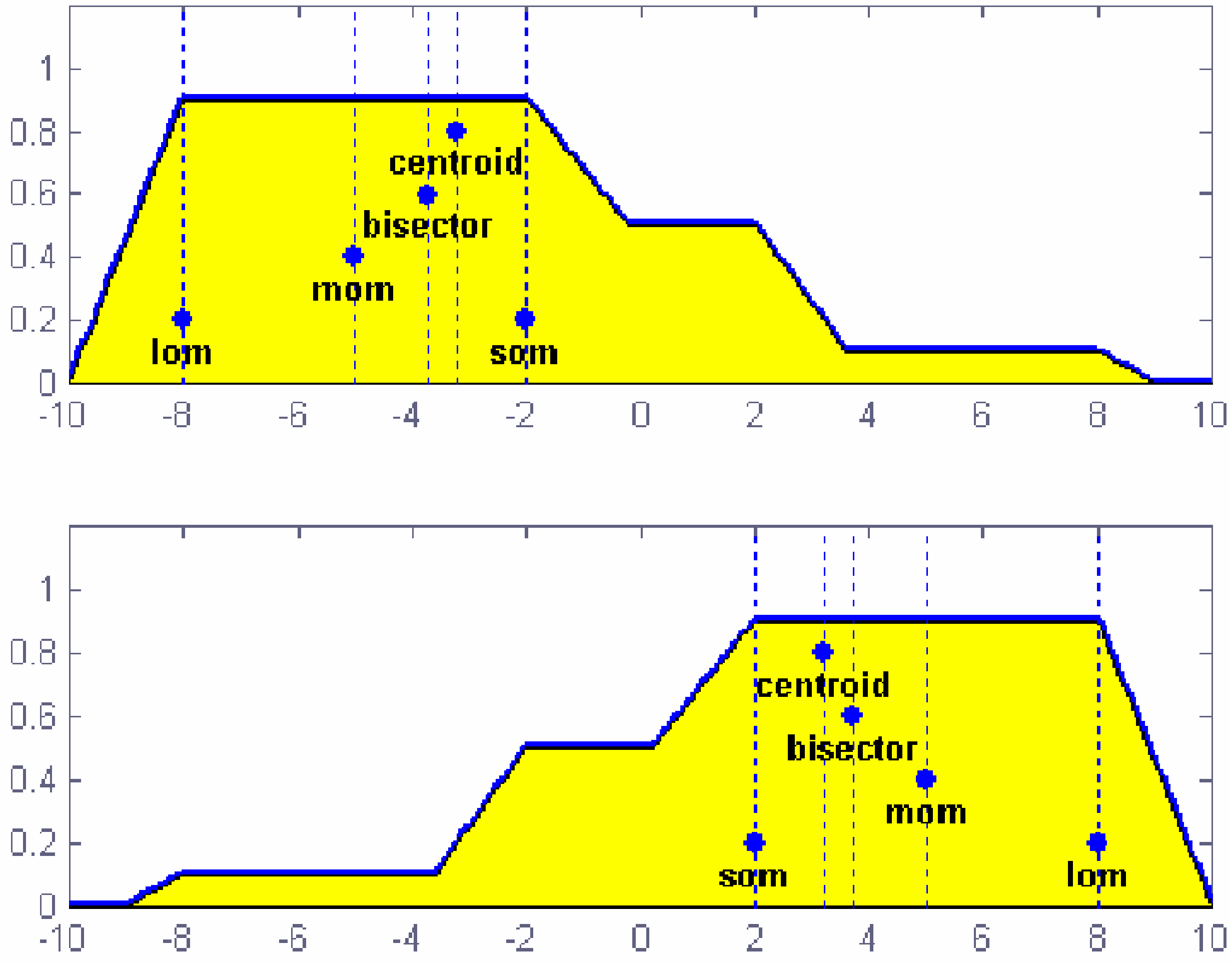
Defuzzification

Bisector of Area	$\Rightarrow$	c_{BOA} such that, $\int_{\alpha}^{c_{\text{BOA}}} \mu_C(c)\, dc = \int_{c_{\text{BOA}}}^{\beta} \mu_C(c)\, dc$, with $\alpha = \min\{c\}$ and $\beta = \max\{c\}$
Smallest of Maximum	$\Rightarrow$	min of max μ_C
Largest of Maximum	$\Rightarrow$	max of max μ_C

Note, the first image is incorrect, only look at second. Also the dots dont mean anything, its only the lines that matter.

For this class we are going to say that all defuzzifiers are equal (there is not mathematical proof for this) so use whichever one you want and be consistent.

Defuzzification Methods: Graphical Example



The mamdani fuzzy model was proposed in 70's but people didnt really use it untill 80's at which point the sugeno model was introduced. Both models are still used today.

Different Inferencing Systems

- Different inferencing procedures have been used in the literature.
- The main difference among them is the aggregation and the defuzzification.

Fuzzy Inference Systems

1. Mamdani fuzzy model
2. Sugeno fuzzy model
3. Tsukomoto fuzzy model

In this image each column is a variable set (with the final column being the antecedent) and each row is a rule (the the final row being the context).

We map the context onto each of the rules. Here we use the max product operation (though this is a choice where you can use your own function). We get the intersections and use the T-norm on them. The two intersections are represented by the horizontal arrows in the image (they are the max intersections). The T-norm in this case is the min function. In the image they are using the max-min but we are using the max-product so we multiply the antecedent by the T-norm. We repeat this with the second rule. These two rules overlapping are the consequent.

Different Inferencing Systems

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Fuzzy Inference Systems

1. Mamdani fuzzy model
2. Sugeno fuzzy model
3. Tsukomoto fuzzy model

Here we have x as the antecedent and y as the consequent. This is a simplistic system. Remember, if you are not told anything about it the compositional rule is max-min. If we are told it is mamdani we use the max-product.

Different Inferencing Systems

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- The main difference among them is the aggregation and the defuzzification.

Fuzzy Inference Systems

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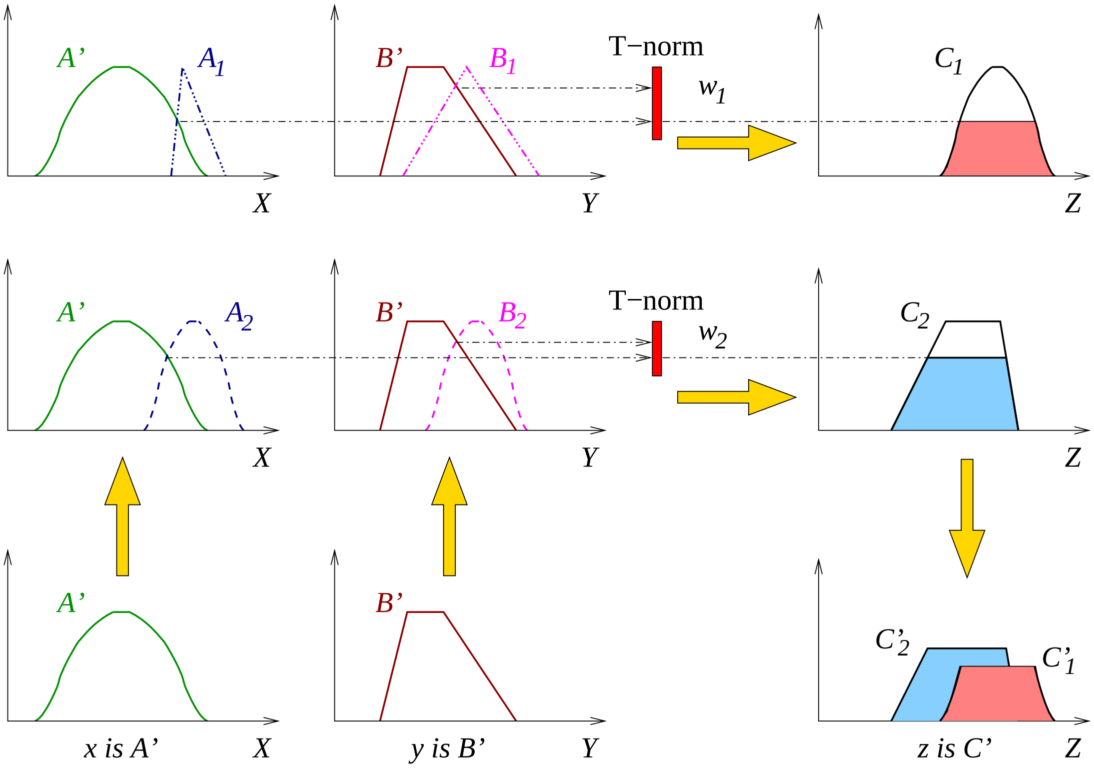
Mamdani Fuzzy Model

- (max-min operator) for aggregation part.



Here we have three membership functions for small, medium, and large. One for each rule. We then have two graphs one for each of our fuzzy sets x and y.

We are using the mamdani which means we use the max-min inferencing mechanism. The defuzzifier is the centroid of area.

X goes from -10 to 10 and y goes from 0 to 10. Start with the value at -10 for x. The T-norm is equal to 1 at x equals -9.8 (since -10 is on the edge). At -9.8 the medium function has no intersection and neigther does the large function. So these values have a T-norm of 0 at 9.8. Where the T-norm is 1 we take the full membership function (for the small function), for the others you taking nothing. Then you sum these and still get the full membership function for small. Find the centroid of area and that is your output. This is the output for when x is equal to -9.8. You gotta keep going for all values of x. So for $x \in (-10, -6)$ the output is the centroid of the small membership function.

So we get a new x map it onto all three functions to get the intersection point take this intersection point and clip the membership function, sum all membership functions, take the centroid of area and then map that onto our output graph. Rinse and repeat for all unique areas of x.

Example: Mamdani Fuzzy Model

Example

- Let $x \in [-10, 10]$ and $y \in [0, 10]$ be two linguistic variables representing the antecedent and consequent of a fuzzy inference system, respectively.
- Rules

  If x is small then y is small

  If x is medium then y is medium

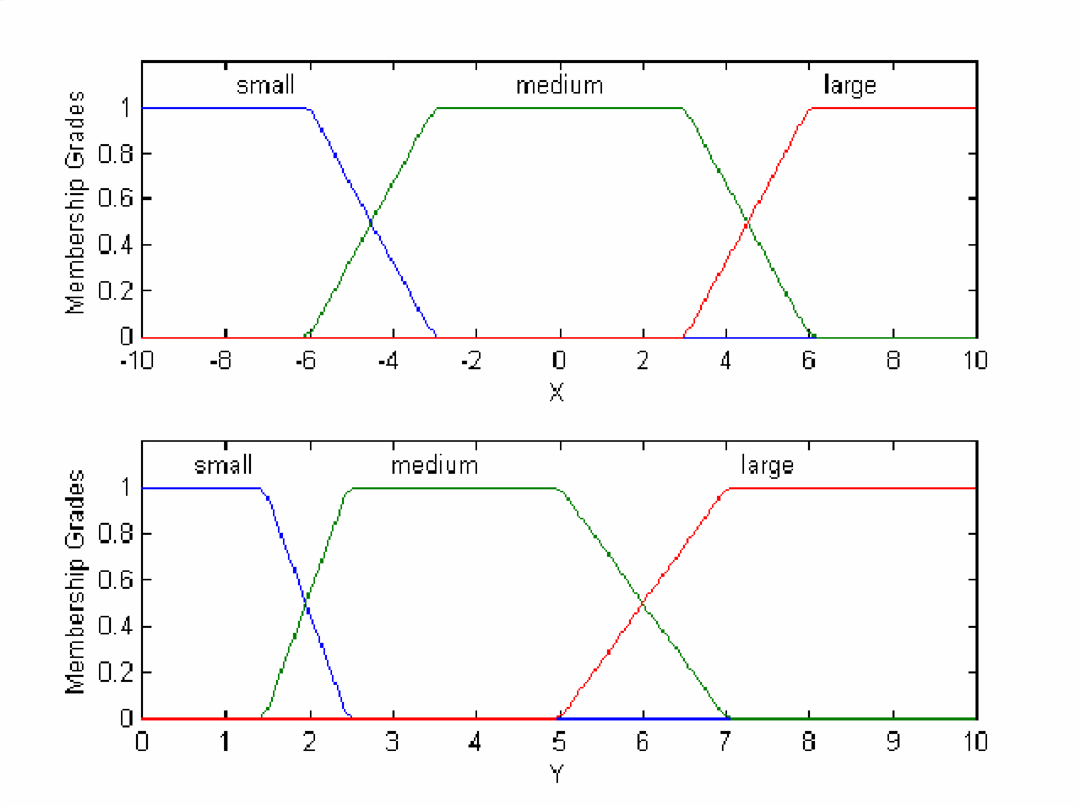
  If x is large then y is large

- Obtain the output of this system using the Mamdani fuzzy inferencing model. Use the center of area as defuzzification operator.

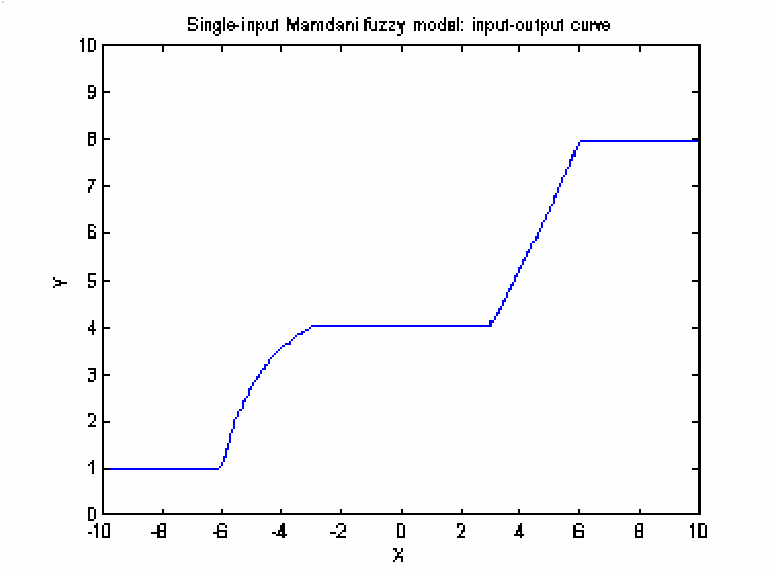
The end result is this funky as shit graph. Its called the input output curve, its what the controller uses. We take our context as x and look it up on the graph. Basically it maps inpt x to output y.

We frequently use the Max-Min where we take the min of the t-norm and aggregate by taking the max. We can also take the Max-Product where we multiply by the T-norm (as described in the previous page) called the **Larson Produce**

Membership Functions for Input and Output Variables



Input-Output Curve



- The advantage of this curve is that we can directly calculate the value of output y for a given value of x.
- Hence, in practice the system will be faster.

We can technically go up farther in order but they are very rarely used. We really only use zero and first order systems.

Here we have y as a function of x instead of what we had earlier with rather binary values. The values in these equations are tuned overtime. Usually they are found through other AI procedures, for now we are going to just pretend we are given them. If we aren't given the parameters ask for them.

Sugeno Fuzzy Model

- The consequent in the Sugeno fuzzy model is a function of the antecedent.

$$\textit{If } x \textit{ is } A_1 \textit{ and } y \textit{ is } B_1 \textit{ then } z = f_1(x, y)$$

$$\textit{If } x \textit{ is } A_2 \textit{ and } y \textit{ is } B_2 \textit{ then } z = f_2(x, y)$$

where f_i, $i = 1, 2, \ldots$, is a crisp polynomial function in its arguments.

Note: this is just and example, ignore that the large graph is in the wrong location.

The little x is a context value (really should be x_0). We map this value to each one of the rules and get the minimum value between the membership function and the context. This then returns the firing strength, denoted w_i.

The consequent is the summation of the multiplication of the firing strength multiplied by the membership functions divided by the sum of the firing strengths. Here x is crisp. So the ouput is crisp so we don't need any defuzzification.

Sugeno Fuzzy Model

Sugeno Zero Order Model: $f_i(x, y) = \text{constant}$

Sugeno First Order Model: $f_i(x, y) = a_i x + b_i y + c_i$

$\Rightarrow$ The overall output (aggregation) is obtained through a weighted average.

Example 1: Sugeno Fuzzy Model

If x is small then $y = 0.1x + 6.4$

If x is medium then $y = 0.5x + 4$

If x is large then $y = x - 2$

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Example 1: Sugeno Fuzzy Model

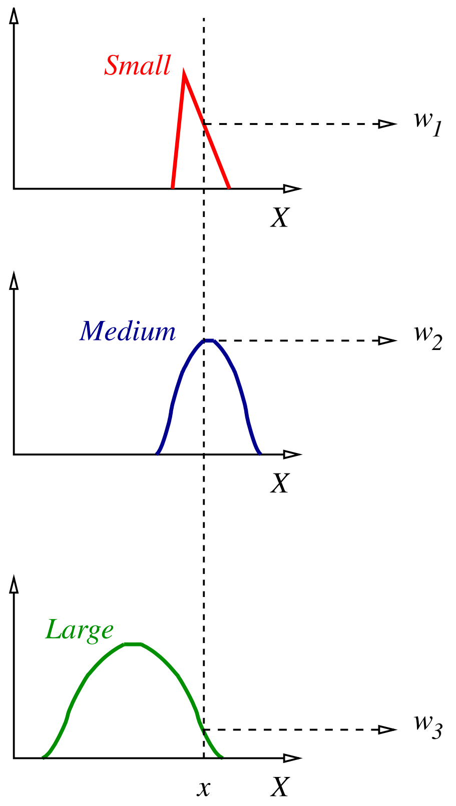
If x is small then $y = 0.1x + 6.4$

If x is medium then $y = 0.5x + 4$

If x is large then $y = x - 2$

Here we have two antecedents we are dealing with.

Example 1: Graphical Solution



$$y_1 = 0.1x + 6.4$$

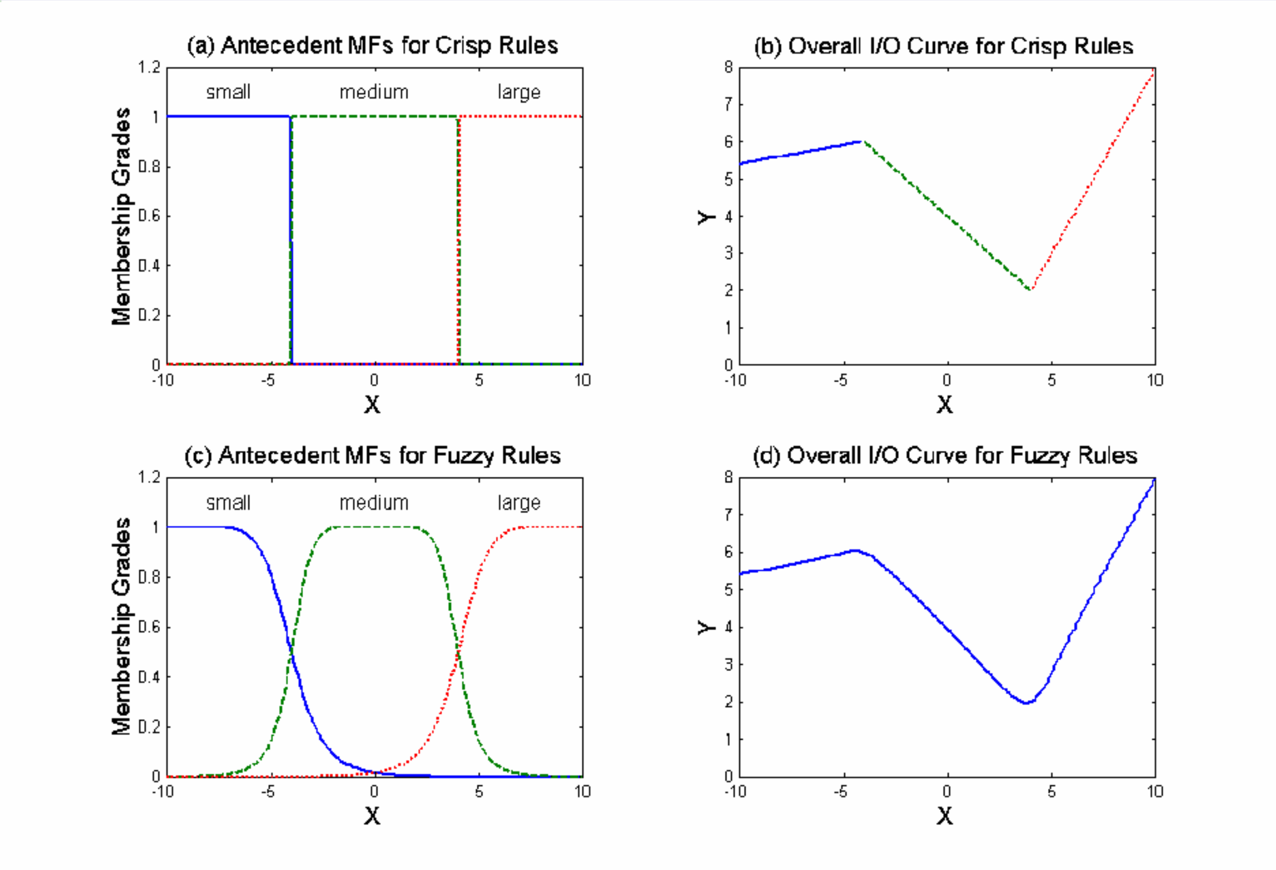
$$y_2 = 0.5x + 4$$

$$y_3 = x - 2$$

$$y = \frac{w_1 y_1 + w_2 y_2 + w_3 y_3}{w_1 + w_2 + w_3}$$

Example 1 (cont.)

Crisp Rules vs Fuzzy Rules



Fuzzy Associative Memory - this is not in the slides but it should be so here goes.

If x is A_1 and y is B_1 then z is C_1

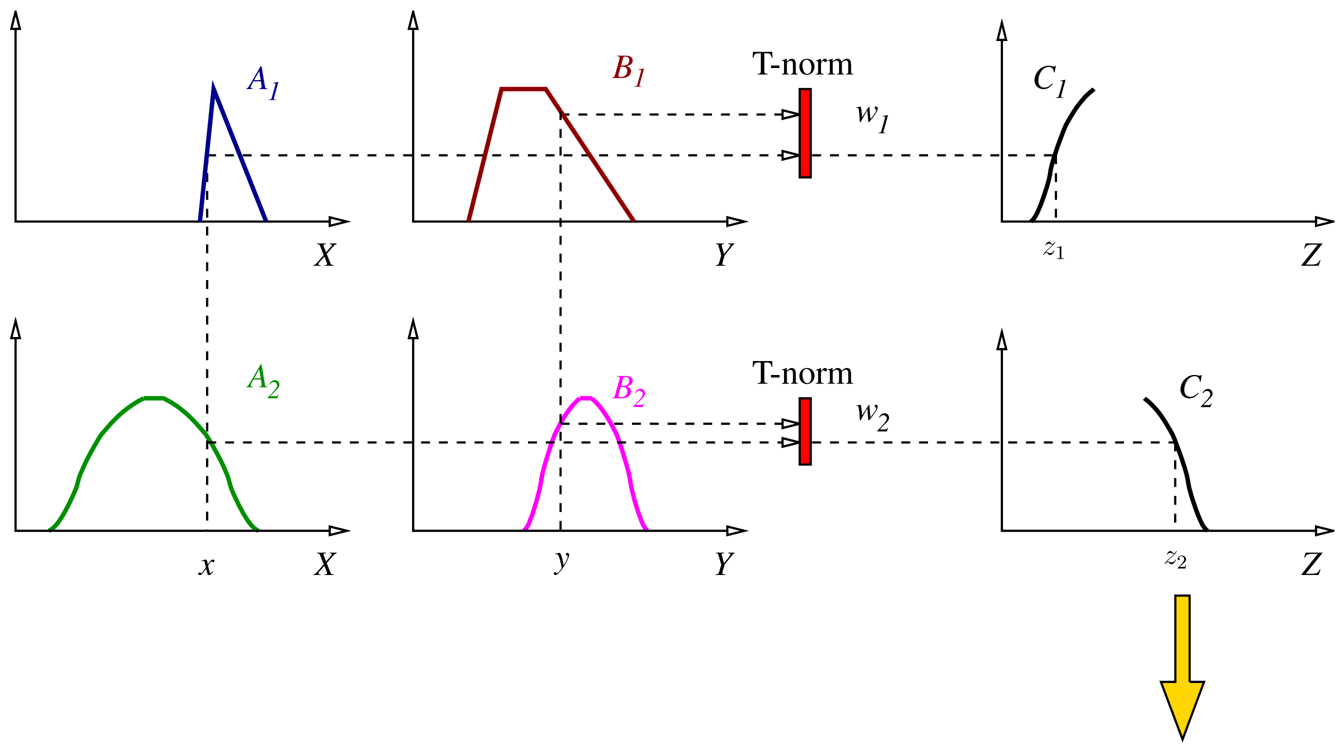
If x is A_2 and y is B_2 then z is C_2

If x is A_5 and y is B_3 then z is C_15

This shows that x has five rules and y has 3 rules so we need a knowledge base of 15 rules. The table of rules that can build built out of the combinations of A's and B's and resulting C's is called the FAM (fuzzy associative memory). This can also be thought of as the knowledge of the system. It should be accompanied with a compositional rule of inference (if one is not provided assume that it is max-min). A defuzzifier should be listed as well, if not it is the centroid of area.

Tsukomoto Fuzzy Model

The output membership function is a monotonic mapping (f^{-1} exists).



$$z = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2}$$

Example: Tsukomoto Inference Model

Rules

If x is small then y is C_1

If x is medium then y is C_2

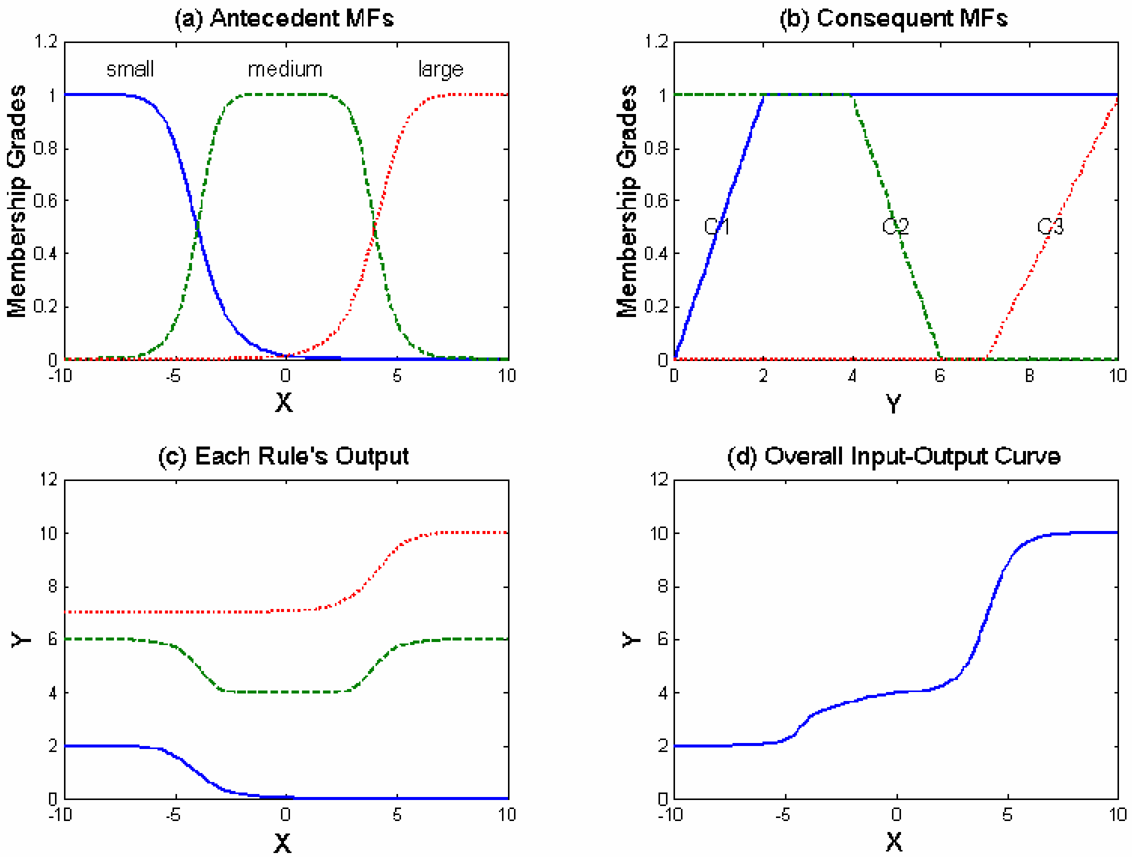
If x is large then y is C_3

where C_1, C_2, and C_3, are the consequent membership function variables.

This is an example from the textbook. Here we have two inputs H and T and one output C.

Before entering the controller we need to fuzzify the input data since it is crisp. Similarly the controller has to defuzzify the C value before sending it to the air conditioner.

Example: Input and Output Variables and Input-Output Curve



Case Study 1: Room Comfort Control System

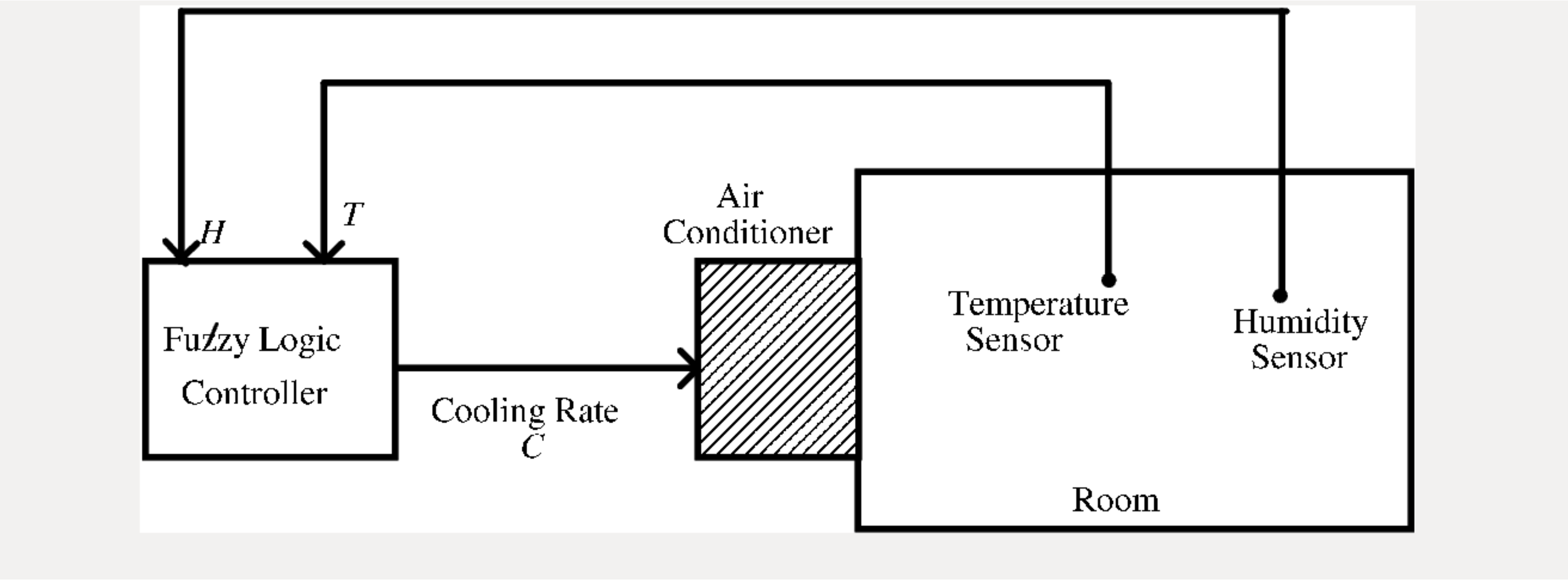
Both the inputs have different universes of discourse. Which is important to note, if you forget this it will fuck your day.

Room Comfort Control System

Problem Statement

- The temperature (T) and humidity (H) are the process variables that are measured.
- These sensor signals are provided to the fuzzy logic controller.
- The fuzzy logic controller determines the cooling rate (C) that should be generated by the air conditioning unit.
- The objective is to maintain a particular comfort level inside the room.

Structure of Room Comfort Control System



We have two rules for temperature and 2 for humitidiy. From here we build our FAM based on these rules

Input Variables

- Temperature level (T): There are two fuzzy states (HG, LW), which denote high and low, respectively, with the corresponding membership functions.
- Humidity level (H): There exist two other fuzzy states (HG, LW) with associated membership functions.
- The membership functions of T are quite different from those of H, even though the same nomenclature is used.

Here we see the mamdani rules using the max min values. We have been given one pair of values to look at.