MATH 239 Spring 2014: Assignment 10

Due: 3:00 PM, Monday July 28, 2014 in the dropboxes outside MC 4066

Last Name: First Name:

I.D. Number: Section:

Mark (For the marker only): /27

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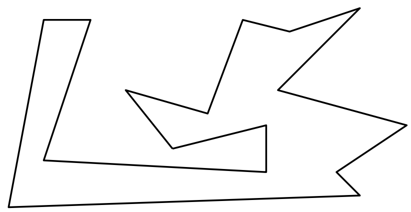
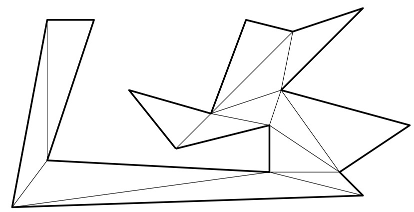
1. Recall from assignment 9 that a graph is *outerplanar* if it has a planar embedding where every vertex lies on the unbounded face.

   (a) {4 marks} Prove that every outerplanar graph is 3-colourable. For this question, you may assume (without proof) that every outerplanar graph has a vertex of degree at most 2.

(b) {3 marks} Martin is setting up a shop selling a combination of Peintings, Peigers and Peinkillers. Unlike some retailers, we accept PeiPal. The Peirimeter of the shop floor is shaped as a simple polygon with n sides. To catch non-Peiing customers, Martin decided to install some surveillance cameras in the shop. Prove that it is possible to place at most $\lfloor n/3 \rfloor$ cameras such that every point inside the shop can be tracked by some camera. (Assume that any camera has a 360-degree Peinoramic view. You may also assume that any simple polygon has a triangulation of its interior, as shown in an example below.)



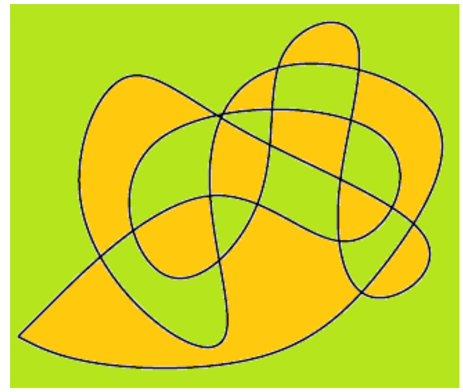
A polygon



A triangulation of the polygon

2. {4 marks} Let G be a simple connected planar graph with at least 2 vertices, and let G^* be the dual of a planar embedding of G. Prove that if G is isomorphic to G^*, then G is not bipartite.

3. {4 marks} Consider any closed curve on the plane that does not repeat any segments, but possibly crossing itself at several points. Prove that the faces are 2-colourable. An example is shown below. (Hint: Use question 3 from assignment 9.)

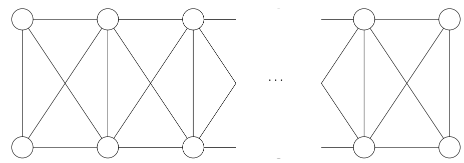


4. {4 marks} For each of the following parts, draw a non-bipartite graph that satisfy the conditions. Show the relevant matchings and covers.

   (a) The size of a maximum matching is strictly less than the size of a minimum cover.

   (b) The size of a maximum matching is equal to the size of a minimum cover.

5. {4 marks} For each $n \in \mathbb{N}$, let L_n be the graph with $2n$ vertices shown below. Determine the number of perfect matchings in L_n. (Hint: Let a_n be this number, and derive a recurrence relation for a_n.)



6. {4 marks} Two people play a game on a graph G by alternately selecting distinct vertices $v_1, v_2, \ldots$ forming a path. The last player who is able to select a vertex wins. Suppose G has no perfect matchings. Describe a winning strategy for the first player, and explain why this strategy works.