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By filling out the names above, the group members acknowledge that a) they have jointly authored this submission, b) this work represents their original work, c) that they have not been provided with nor examined another person's assignment, either electronically or in hard copy, and d) that this work has not been previously submitted for academic credit.			

LAB 2. SECOND-ORDER SYSTEM IDENTIFICATION AND ANALYSIS

ASSIGNED DATA

For easily referencing it, the Assigned Data has been placed at the start of this document.

On your pre-lab and post-lab submissions, always include this page at the beginning of the document.

Select your lab session:		☐ morning lab; ☒ afternoon lab; ☒ Tue; ☐ Wed; ☐ Thu
CourseBook Group Number		45
Assigned plant number (*Plant* ¿=*CourseBook GroupNumber* ¿		45
Plant parameters:	a	2
	b	5
	T	100
	K_i	330

The green highlighting present in this document is meant to draw your attention to things that need to be done.

Section 4.1

T_p(s)	c_{max} (V)	c_{ss}(V)	T_r(s)	T_s(s)
8.22E-003	1.48	1	3.01E-003	3.364E-002

Table 1: Response specifications - experimental values

Figure 1: Second order underdamped

T_p(s)	c_{max} (V)	c_{ss}(V)	OS(%)	ζ	ω_n(rad/s)
8.22E-003	1.48	1	48	0.2275	435.68

Table 2: System identification data and results

Calculations for table 2:

Using the formula for overshoot

$$
\begin{aligned}
OS &= e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \\
0.48 &= e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \\
\ln 0.48 &= \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \\
\ln^2 0.48 &= \frac{\zeta^2\pi^2}{1-\zeta^2} \\
\ln^2 0.48 \times (1-\zeta^2) &= \zeta^2\pi^2 \\
\ln^2 0.48 &= \zeta^2 \times (\pi^2 + \ln^2 0.48) \\
\zeta &= \sqrt{\frac{\ln^2 0.48}{\pi^2 + \ln^2 0.48}} \\
\zeta &= 0.2275
\end{aligned}
$$

Using the formula for peak time

$$
\begin{aligned}
T_p &= \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \\
\omega_n &= \frac{\pi}{T_p\sqrt{1-\zeta^2}} \\
\omega_n &= \frac{\pi}{0.00822\sqrt{1-\zeta^2}} \\
\omega_n &= \frac{382.93}{\sqrt{1-\zeta^2}} \\
\omega_n &= \frac{382.93}{\sqrt{1-0.2275}} \\
\omega_n &= 435.68
\end{aligned}
$$

Discussing Table 2

Prelab values: $\zeta = 0.2462$ $\omega = 406.20$

Experimental values: $\zeta = 0.2275$ $\omega = 435.68$

The experimental values differed from the theoretical values calculated in the prelab by around 8%. Errors may have stemmed from inaccurate graph reading when recording experimental values, or by small rounding errors during mathematical calculations for experimental and theoretical values.

Section 4.2

Figure 2: Open-loop step response with K=1

K_p	T_p(s)	y_{max}(V)	y_{ss}(V)	OS(%)
1	8.2E-003	1.48	1	48
1.3	8.17E-003	1.98	1.33	49
1.66	8.27E-003	2.47	1.66	49
2	8.17E-003	2.93	2	47

Table 3: Open-loop step response values for Kp=variable

Discussing Table 3

Peak Value: This increased with Kp. This fits control theory as the peak value is the highest oscillation above the steady state value, since the steady state value is increasing without ζ changing this value should rise as well.

Steady state: As Kp increases the steady state value increases to match the Kp value. This fits with control theory as the output for G(S) has a steady state value of 1, experimentally verified. For the open loop system the output should be equal to $G(S) \times K \times K_p$ (as they are all in series) and since K stayed at 1 for all experiments the output increase was proportional to Kp.

Overshoot and Peak time: These values did not change much. This fits control theory as the equations for overshoot and peak time are proportional to ζ and ω_n which do not vary with with K or Kp in the open-loop system.

Figure 3: Closed-loop step response with K=1

K_p	T_p(s)	y_{max}(V)	y_{ss}(V)	e_{ss}	OS(%)
1	5.6E-003	0.82	0.5	0.5	64
1.3	5.2E-003	0.93	0.57	0.565	63
1.66	4.92E-003	1.03	0.64	0.615	62
2	4.5E-003	1.12	0.68	0.667	65

Table 4: Closed-loop step response values for Kp=variable

Discussing Table 4

Peak Value and Steady State value: These values are increasing with Kp. Due to control theory our system output with the increase of Kp from 1 is:

$$Y(S) = \frac{K_p H(S)}{1 + K_p H(S)}$$

As Kp increases in the above equation the output will also increase is what causes the increase in the peak value and the steady state value.

Overshoot: The overshoot does not change much. This is due to the steady state and peak value increasing by the same ratio. We can also see that this value will not change because it is related to ζ which has not changed.

Peak Time: The peak time is decreasing as Kp increases. This is due to the peak time being inversely proportional to the natural frequency. In this system the natural frequency was proportional to Kp, so when Kp increases the natural frequency increases which causes the peak time to decrease.

Zeta and Omega Calculations

The new transfer function for the system is:

$$
\begin{aligned}
H(S) &= \frac{K_i bTKK_p}{s^2 + aTs + K_i bT + K_i bTK_pK} \\
K_p &= \mathtt{1 and K} = \mathtt{1} \\
&= \frac{K_i bT}{s^2 + aTs + 2K_i bT} \\
&= \frac{1}{2} \times \frac{2K_i bT}{s^2 + aTs + 2K_i bT}
\end{aligned}
$$

This is a standard second order system with a coefficient. Since coefficients carry over during a Laplace transform we can use the equations for a standard second order system with the coefficient.

Based on the definition of standard second order system:

$$
\begin{aligned}
\omega_n^2 &= 2K_i bTKK_p \\
\omega_n &= \sqrt{2K_i bTKK_p} \\
&= \sqrt{2 * 330 * 5 * 100} \\
&= \sqrt{2 * 330 * 5 * 100} \\
&= 574.45
\end{aligned}
$$

$$
\begin{aligned}
aT &= 2\zeta\omega_n \\
\zeta &= \frac{aT}{2\omega_n} \\
&= \frac{2 * 100}{2 * 574.45} \\
&= 0.35
\end{aligned}
$$

Using the formula for overshoot

$$
\begin{aligned}
OS &= \frac{1}{2} e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \\
\zeta &= \sqrt{\frac{\ln^2 \frac{OS}{2}}{\pi^2 + \ln^2 \frac{OS}{2}}} \\
\zeta &= \sqrt{\frac{\ln^2 \frac{0.64}{2}}{\pi^2 + \ln^2 \frac{0.64}{2}}} \\
\zeta &= 0.34
\end{aligned}
$$

Since the peak time formula is found by taking the sinusoidal function is highest, it is not influenced by the added coefficient so we can use the same function for calculating ω_n. Using the formula for peak time

$$\begin{aligned}
T_p &= \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \\
\omega_n &= \frac{\pi}{T_p\sqrt{1-\zeta^2}} \\
\omega_n &= \frac{\pi}{0.0056\sqrt{1-0.34^2}} \\
\omega_n &= 596.76
\end{aligned}$$

Experimental Open-loop: $\zeta = 0.2275$ $\omega_n = 435.68$

Experimental Closed-loop: $\zeta = 0.34$ $\omega_n = 596.76$

Theoretical Open-loop: $\zeta = 0.2462$ $\omega = 406.20$

Theoretical Closed-loop: $\zeta = 0.35$ $\omega_n = 575.45$

The dampening ratio increased when the loop was closed due to the stabilizing nature of a closed loop. The output is dampened by the subtraction of itself from the input. The transfer function multiplied to natural frequency by 2 which resulted in it increasing by a factor of $\sqrt{2}$.

Adjusted value is k=1.04

	Low-freq gain (dB)	Experimental bw (rad/s)	Theor or sim bw (rad/s)	Error (%)
Open-loop	0.5	589	603	2.4
Closed-loop	-5.8	850	869	2.2

Table 5: Bandwidth (bw) measurement results

Closing the loop dramatically increased the measured bandwidth. Bandwidth is defined as the range of frequencies over which the output is is relatively close to the input. We want to be able to change bandwidth in order to lessen the amount of disturbance.

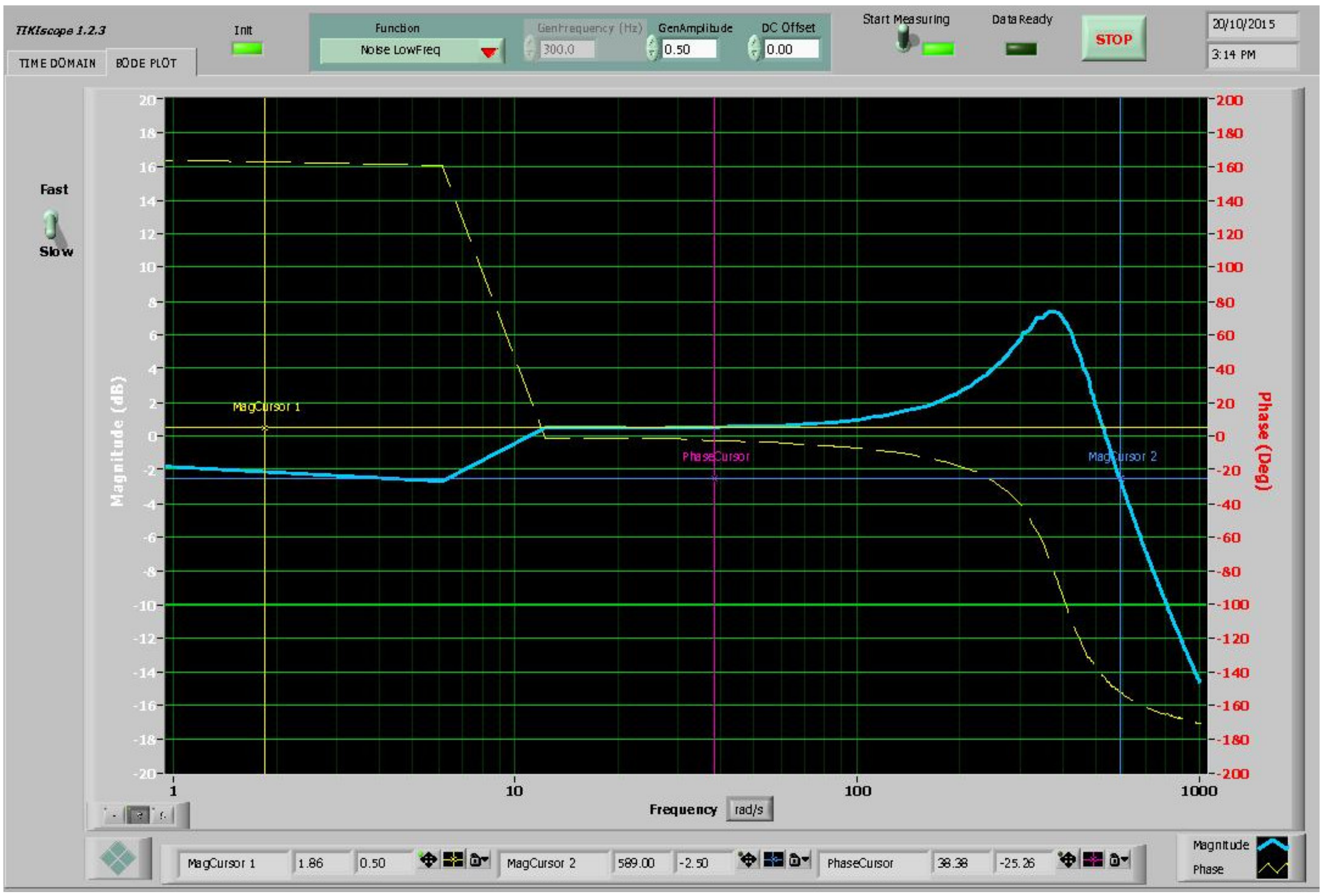

Figure 4: Open-loop frequency response

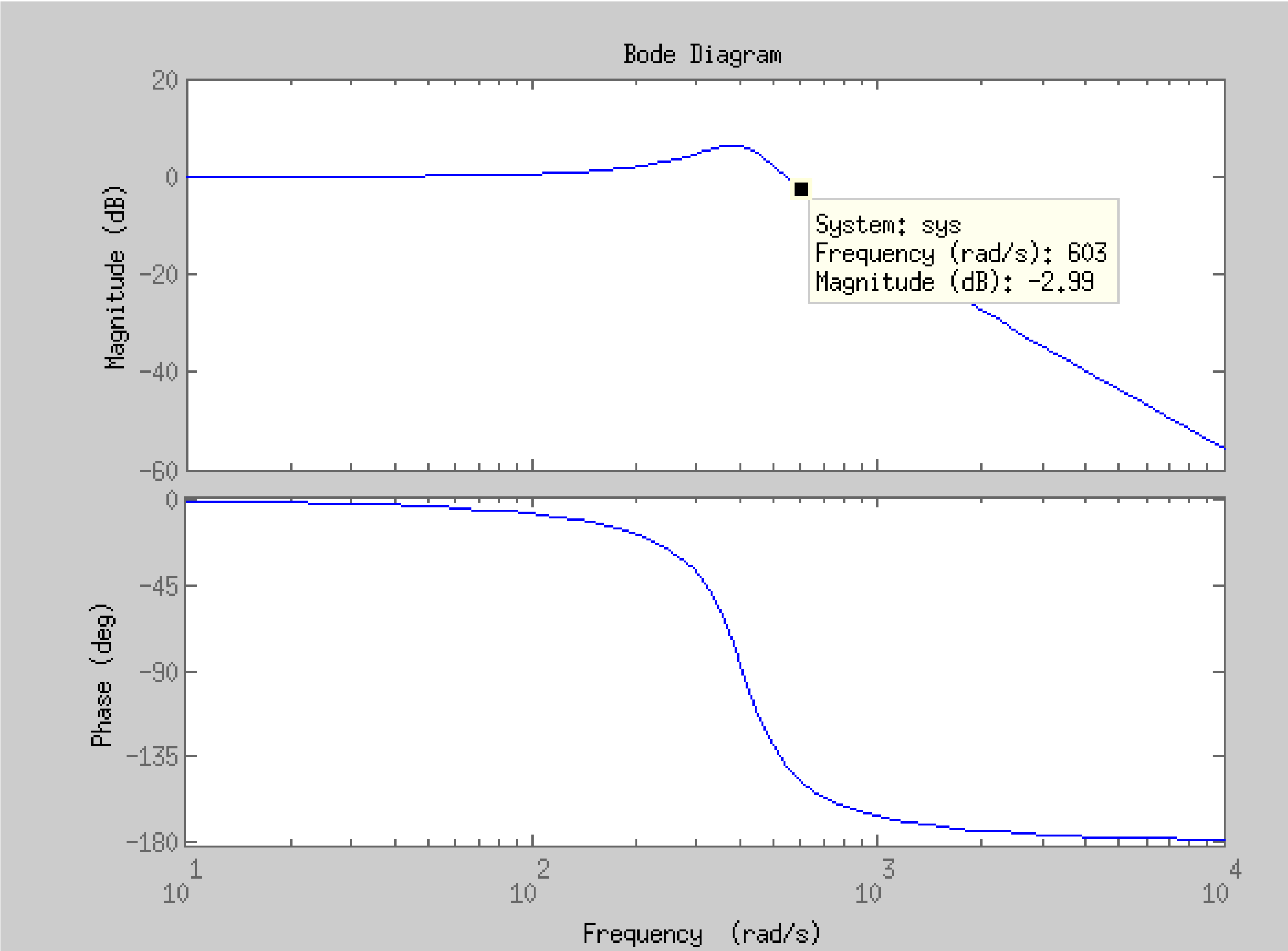

Figure 5: Matlab open-loop frequency response

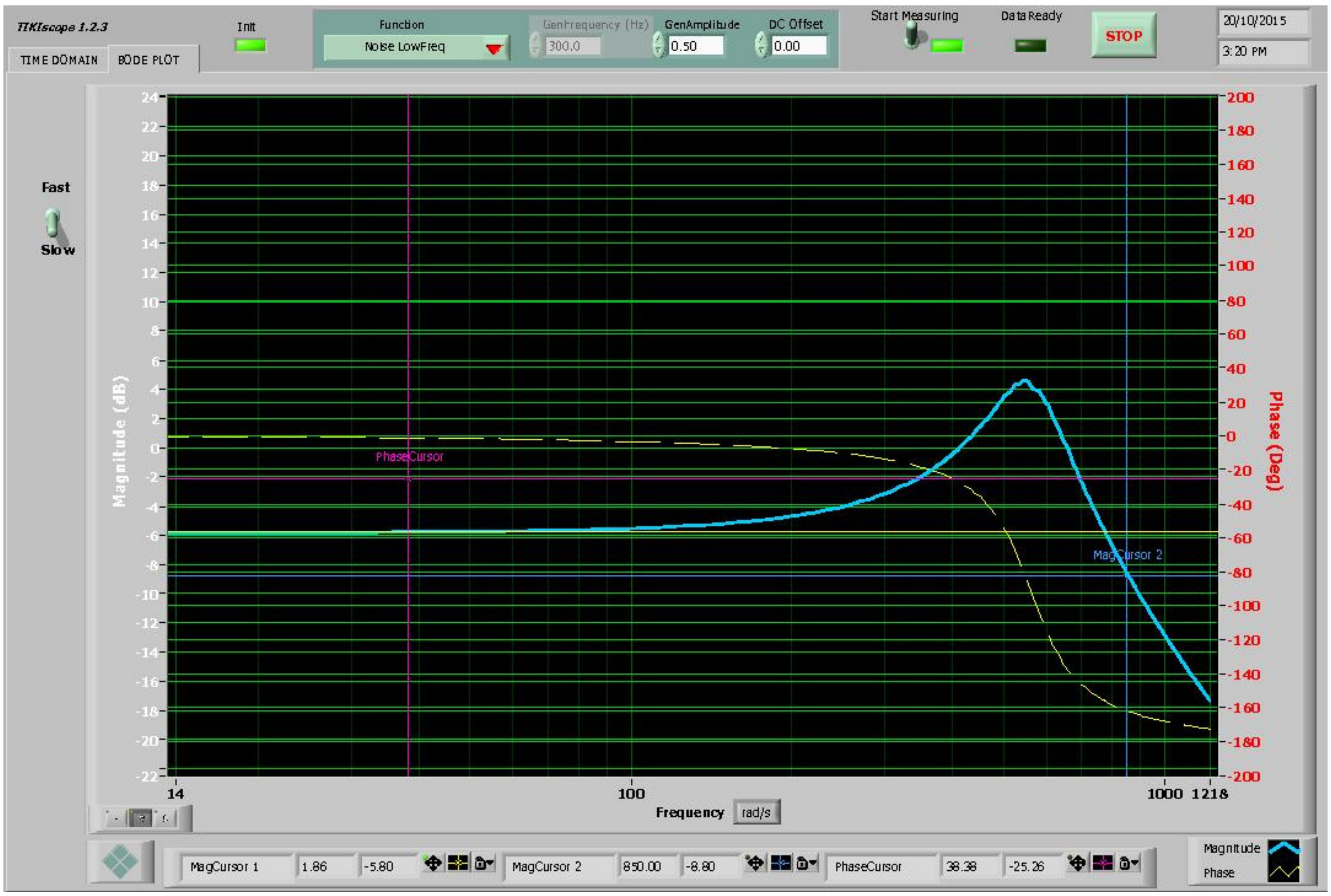

Figure 6: Closed-loop frequency response

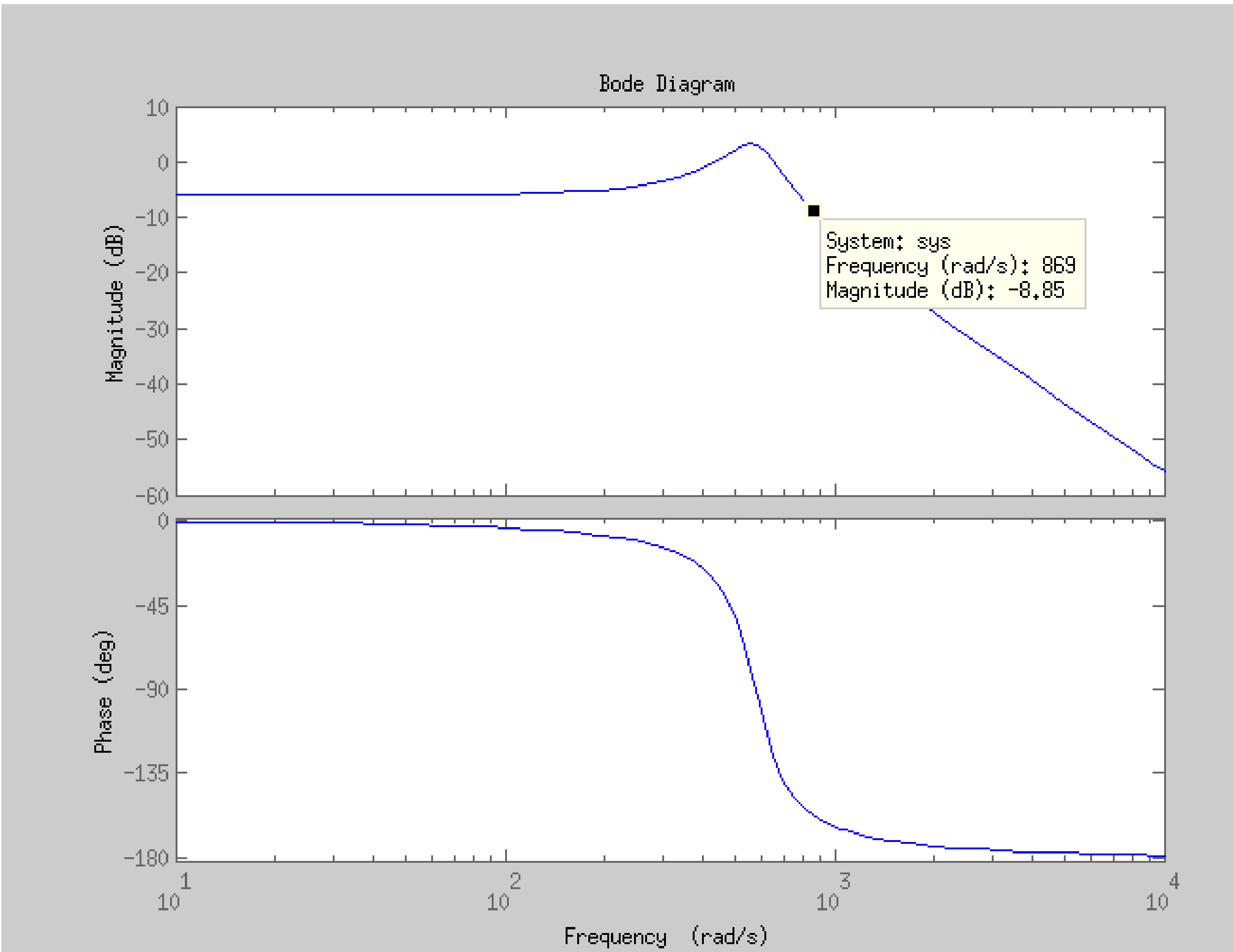

Figure 7: Matlab closed-loop frequency response

Figure 8: Frequency response response with disturbance

Section 4.3

The equation for the system with input at D(S) is:

As shown in Figure 8 frequencies below 336.51 rad/s show little disturbance and frequencies after that show wild changes in response to the disturbances.

In the open-loop system there is no disturbance rejection since nothing is piped back in. The disturbance is directly added to our output. Due to the closed nature of the closed-loop system there is disturbance rejection by subtracting the output from the input.