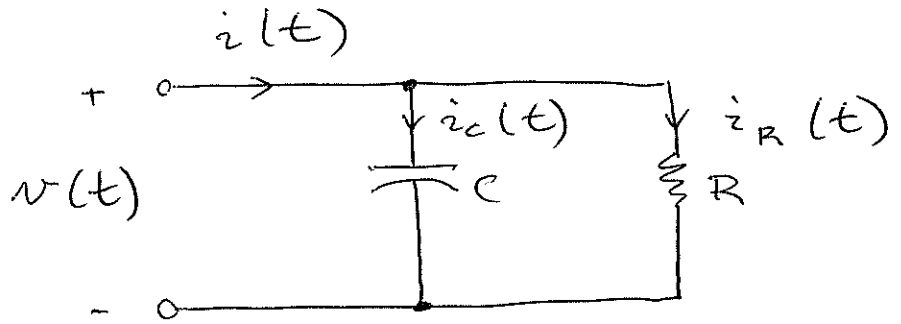
MATH 213, ASSIGNMENT #1

1. Carbon occurs primarily in the form of three isotopes: C12 and C13 – which are both stable; and C14, which decays radioactively into N14 with a half-life of approximately 5,730 years.

In a living organism, about 1.0×10^{-10} % of carbon atoms are C14. But when the organism dies, it takes in no more carbon, and its C14 decays.

Write down a differential equation and an initial condition describing the evolution of the percentage of carbon atoms that are C14 after the death of an organism. This is the basis of C14 dating.

2. Consider the following electrical circuit:



Recall that for the capacitor,

$$i_c(t) = C \frac{d}{dt} v(t) .$$

Suppose that $R = 1\,M\Omega$, $C = 0.1\,\mu F$, and $v(0) = 5\,V$. Find $v(t)$.

3. Decaying exponentials are sometimes written $e^{-t/\tau}$, where τ is called the time constant. How many time constants does it take for $e^{-t/\tau}$ to decay to 0.05? 0.02?

Consider a tangent to $e^{-t/\tau}$ at $t = 0$. Where does this tangent intersect the t axis?

4. Solve the following differential equations and sketch the solutions:

a) $\frac{d^2y}{dt^2} - 7\frac{d}{dt}y + 12y = 0; \; y(0)=1, \dot{y}(0)=0;$

b) $\frac{d^2y}{dt^2} - 3\dot{y} + 2y = 0; \; y(0) = 2, \dot{y}(0)=1$

5. Find a solution of

$$\frac{d^2y}{dt^2} + y = 0; \quad y(0) = 0, \; \dot{y}(0) = 1.$$