ECE 457B Assignment 2

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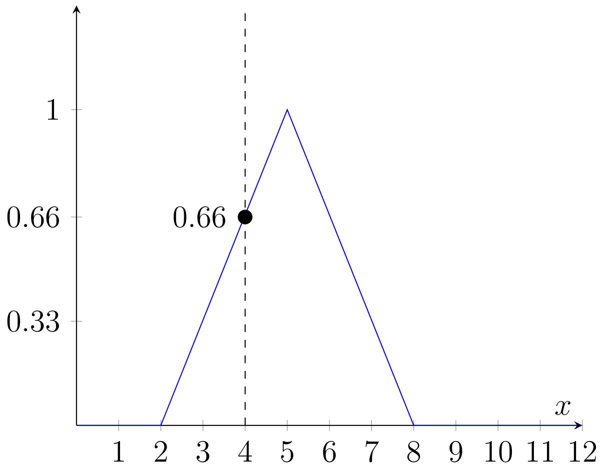
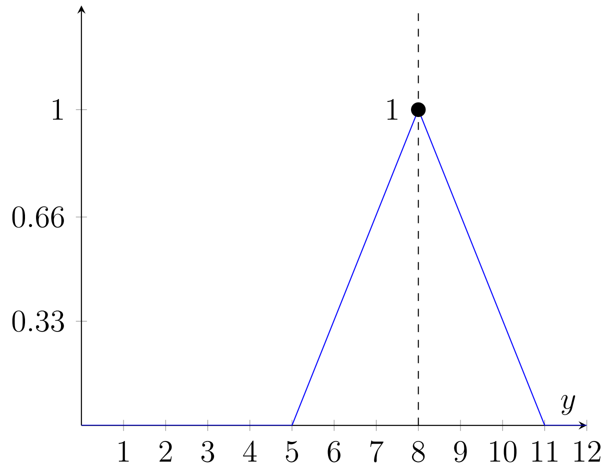
Question 1

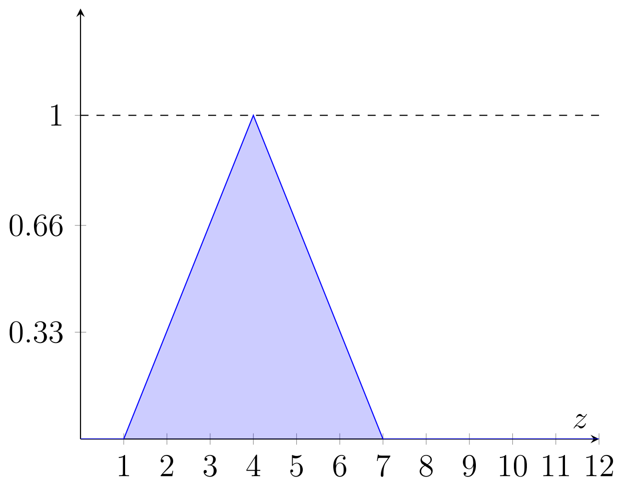


μ_{A_1}

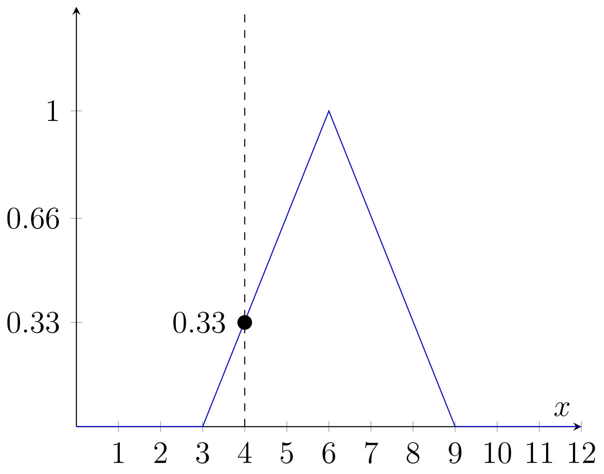


μ_{B_1}

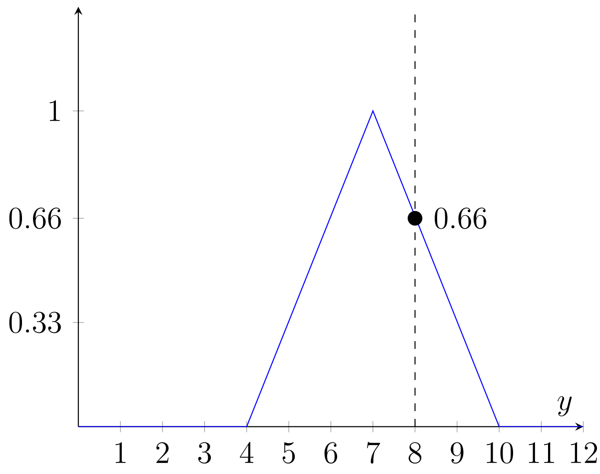
The strength of these two rules is 0.66 and the resulting and the cut on μ_{C_1} will occur at the max, which is 1.



μ_{C_1}

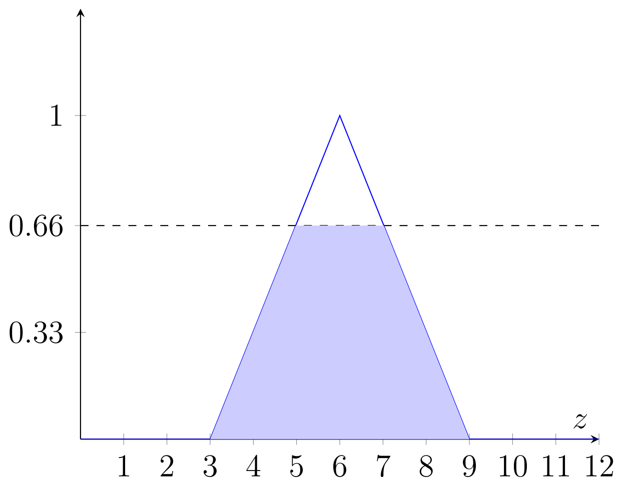


μ_{A_2}

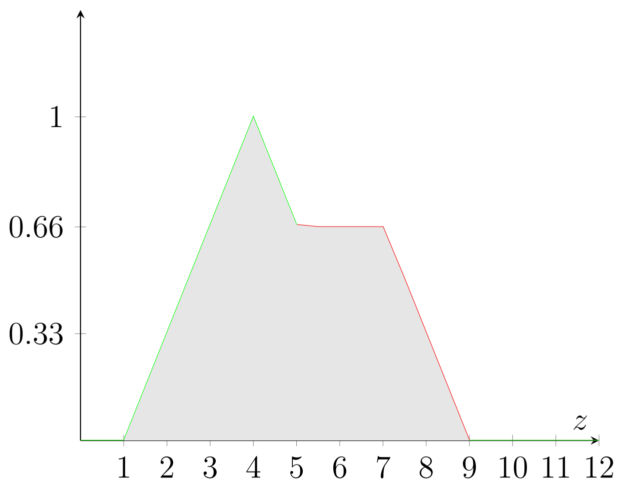


μ_{B_2}

The strength of these two rules is 0.33 and the resulting and the cut on μ_{C_1} will occur at the max, which is 0.66.



μ_{C_2}



output

In this instance we have only one maximum at 4 so the mean of maxima is 4, giving us the defuzzified output at t_1.

When calculating for the centroid of area a slightly different answer occurs.

$$
\begin{aligned}
c &= \frac{\int c\mu_c(c)\delta c}{\int \mu_c(c)\delta c} \\
&= \frac{\int_1^5 \frac{x-1}{3} x\delta x + \int_5^7 0.66x\delta x + \int_7^9 \frac{9-x}{3} x\delta x}{\int_1^5 \frac{x-1}{3}\delta x + \int_5^7 0.66\delta x + \int_7^9 \frac{9-x}{3}\delta x} \\
&= \frac{22.8089}{4.6533} \\
&= 4.9016
\end{aligned}
$$

The centroid of area is slightly higher because the mean of maxima disregards the values outside of its single peak. This is also why the centroid of area returns a more accurate result.

Question 2

a)

i)

Classical:

f\K	$1e+3$	$1e+4$	$1e+5$	$5e+5$	$1e+6$	$5e+6$	$1e+7$
100	1	0.8	0.5	0.2	0	0.2	0.8
200	1	0.8	0.5	0.2	0	0.2	0.8
500	1	0.8	0.5	0.2	0.2	0.2	0.8
800	1	0.8	0.5	0.5	0.5	0.5	0.8
1000	1	0.8	0.5	0.8	1	0.8	0.8
2000	1	0.8	0.5	0.8	0.8	0.8	0.8
5000	1	0.8	0.5	0.2	0.2	0.2	0.8

ii)

Mamdani:

f\K	$1e+3$	$1e+4$	$1e+5$	$5e+5$	$1e+6$	$5e+6$	$1e+7$
100	0	0	0	0	0	0	0
200	0	0	0	0	0	0	0
500	0	0.2	0.2	0.2	0.2	0.2	0.2
800	0	0.2	0.5	0.5	0.5	0.5	0.2
1000	0	0.2	0.5	0.8	1	0.8	0.2
2000	0	0.2	0.5	0.8	0.8	0.8	0.2
5000	0	0.2	0.2	0.2	0.2	0.2	0.2

iii)

Product:

f\K	$1e+3$	$1e+4$	$1e+5$	$5e+5$	$1e+6$	$5e+6$	$1e+7$
100	0	0.0	0.0	0.0	0	0.0	0.0
200	0	0.0	0.0	0.0	0	0.0	0.0
500	0.0	0.04	0.1	0.16	0.2	0.16	0.04
800	0.0	0.1	0.25	0.4	0.5	0.4	0.1
1000	0	0.2	0.5	0.8	1	0.8	0.2
2000	0.0	0.16	0.4	0.64	0.8	0.64	0.16
5000	0.0	0.04	0.1	0.16	0.2	0.16	0.04

b)

$$\begin{aligned}
R &= \begin{bmatrix} 1 & 0.8 & 0.5 & 0.2 & 0 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0.2 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.5 & 0.5 & 0.5 & 0.8 \\ 1 & 0.8 & 0.5 & 0.8 & 1 & 0.8 & 0.8 \\ 1 & 0.8 & 0.5 & 0.8 & 0.8 & 0.8 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0.2 & 0.2 & 0.8 \end{bmatrix} \\
K' &= \begin{bmatrix} 0 \\ 0.8 \\ 0.2 \end{bmatrix} \\
f_1 &= R \circ K' \\
&= \max_{rows}\left(\min_{cols}(K', R)\right) \\
&= \max_{cols}\left(\min_{cols}\left(\begin{bmatrix} 0 \\ 0.8 \\ 0.2 \end{bmatrix}, \begin{bmatrix} 1 & 0.8 & 0.5 & 0.2 & 0 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0.2 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.5 & 0.5 & 0.5 & 0.8 \\ 1 & 0.8 & 0.5 & 0.8 & 1 & 0.8 & 0.8 \\ 1 & 0.8 & 0.5 & 0.8 & 0.8 & 0.8 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0.2 & 0.2 & 0.8 \end{bmatrix}\right)\right) \\
&= \max_{cols}\left(\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.8 & 0.5 & 0.2 & 0 & 0.2 & 0.8 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}\right) \\
&= \begin{bmatrix} 0.8 & 0.8 & 0.5 & 0.2 & 0.2 & 0.2 & 0.8 \end{bmatrix}
\end{aligned}$$

Question 3

$$\begin{aligned}
T_{300} &= \left\{\frac{0}{LW} + \frac{1}{HG}\right\} \\
M_{800} &= \left\{\frac{0}{SM} + \frac{1}{LG}\right\} \\
P_{1.3} &= \left\{\frac{0.2}{FR} + \frac{0.8}{NR}\right\}
\end{aligned}$$

Rule 1: If T is LW and P is FR then F is IN: $\left\{\frac{0}{LW} + \frac{0.2}{FR}\right\} = \frac{0}{IN}$

Rule 2: If T is HG then F is RD: $\left\{\frac{1}{HG}\right\} = \frac{1}{RD}$

Rule 3: If M is SM and P is NR then F is MN: $\left\{\frac{0}{SM} + \frac{0.8}{NR}\right\} = \frac{0}{MN}$

Rule 4: If M is LG and P is FR then F is IN: $\left\{\frac{1}{LG} + \frac{0.2}{FR}\right\} = \frac{0.2}{IN}$

Rule 5: If P is NR then F is RD: $\left\{\frac{0.8}{NR}\right\} = \frac{0.8}{RD}$

Then $F = \left\{\frac{1}{RD} + \frac{0.2}{IN}\right\}$

When these is applied you get a membership function of:

$$f = \begin{cases} 0.8, & \texttt{if } x \leq 5.4 \\ (5-x)/2, & \texttt{if } 5.4 < x \leq 7 \\ 0, & \texttt{if } 7 < x \leq 12 \\ (x-12)/2, & \texttt{if } 12 < x \leq 12.4 \\ 0.2, & \texttt{if } x < 12 \end{cases} \tag{1}$$

Applying the centroid of area gives a value of 6.23

This value seems to make sense as it falls roughly in the middle. Since the temperature is very high you might want to reduce the fuel rate, but the mass is very large so you might want to increase the temperature.

Question 4

a)

Input 1:

$$\begin{aligned} r &= \sum_i w_i x_i \\ &= (1)(1) + (-2)(-1) + (0)(0) + (-1)(0.5) \\ &= 2.5 \\ \Delta w &= \eta(t_1 - r)x_1 \\ &= 0.1(-1-2.5)\begin{bmatrix}1 & -2 & 0 & -1\end{bmatrix} \\ &= \begin{bmatrix}-0.35 & 0.7 & 0 & -0.35\end{bmatrix} \\ w^{(2)} &= w^{(1)} - \Delta w \\ &= \begin{bmatrix}1 & -1 & 0.5 & -1\end{bmatrix} - \begin{bmatrix}-0.35 & 0.7 & 0 & -0.35\end{bmatrix} \\ &= \begin{bmatrix}1.35 & -1.7 & 0.5 & 1.35\end{bmatrix} \end{aligned}$$

Input 2:

$$
\begin{aligned}
r &= (1.35)(0) + (-1.7)(1.5) + (0.5)(-0.5) + (1.35)(-1) \\
&= -4.15 \\
\Delta w &= 0.1(-1 + 4.15)\begin{bmatrix}0 & 1.5 & -0.5 & -1\end{bmatrix} \\
&= \begin{bmatrix}0 & 0.47 & -0.16 & -0.315\end{bmatrix} \\
w^{(3)} &= \begin{bmatrix}1.35 & -1.7 & 0.5 & 1.35\end{bmatrix} - \begin{bmatrix}0 & 0.47 & -0.16 & -0.315\end{bmatrix} \\
&= \begin{bmatrix}1.35 & -2.17 & 0.66 & 1.665\end{bmatrix}
\end{aligned}
$$

Input 3:

$$
\begin{aligned}
r &= (1.35)(-1) + (-2.7)(1) + (0.66)(0.5) + (1.665)(-1) \\
&= 5.385 \\
\Delta w &= 0.1(1 + 5.385)\begin{bmatrix}-1 & 1 & 0.5 & -1\end{bmatrix} \\
&= \begin{bmatrix}-0.6385 & 0.6385 & 0.31925 & -0.6385\end{bmatrix} \\
w^{(4)} &= \begin{bmatrix}1.35 & -2.17 & 0.66 & 1.665\end{bmatrix} - \begin{bmatrix}-0.6385 & 0.6385 & 0.31925 & -0.6385\end{bmatrix} \\
&= \begin{bmatrix}1.9885 & 3.3385 & 0.34075 & 2.3035\end{bmatrix}
\end{aligned}
$$

b)

Code is also attached in q4.py

```
# Training data
x1 = [1, -2, 0, -1]
x2 = 0, 1.5, -0.5, -1
x3 = [-1, 1, 0.5, -1]
inputs = [x1, x2, x3]
# Labels for training data
t1 = -1
t2 = -1
t3 = 1
targets = [t1, t2, t3]
# Initial weights
weights = [1, -1, 0, 0.5]
# Learning rate
n = 0.1
# Maximum number of iterations
maxIterations = 50
# Thresholds for determining if a steady state has been achieved
threshold = 0.1

def calcR(i):
    r = 0
    for j in range(0, len(inputs[i])):
            r += inputs[i][j] * weights[j]
    return r

iters = 0
while iters < maxIterations: # loop for each epoch
    output = 0
    isSteady = True
    for i in range(0, len(inputs)): #loop for each piece of training data
        r = calcR(i)
```

```
        for data in range(0, len(weights)):
            # Calculate delta
            delta =  n * (targets[i] - r) * inputs[i][data]
            # Check if delta is close enough to 0
            isSteady = isSteady and (abs(delta) < threshold)
            # Update weights
            weights[data] = weights[data] + delta
    if isSteady: # stop looping if steady state has been achieved
        break
    iters += 1
if iters == maxIterations - 1:
    print 'Did not converge'
else:
    print 'Converged in {0} epochs'.format(iters)
    print weights
```

The output for this was that the training data was linearly separable in 6 epochs and weights $\begin{bmatrix}-0.36 & 0.17 & 0.68 & 0.34\end{bmatrix}$ (rounded to 2 digits).

c)

$$\begin{aligned}
u &= \begin{bmatrix}-1 & -1 & 0 & 0.5\end{bmatrix} \\
w &= \begin{bmatrix}-0.36 & 0.17 & 0.68 & 0.34\end{bmatrix} \\
r &= (-1)(-0.6) + (-1)(0.17) + (0)(0.68) + (0.5)(0.34) \\
&= 0.6 \\
signum(r) &= 1
\end{aligned}$$

So u belongs in class C1.

Question 5

For brevity's sake we will abbreviate:

$$r^{(k)} = \sum_i w_i x_i^{(k)} + \theta \tag{2}$$

This makes the new error function:

$$\begin{aligned}
E(w) &= \sum_i \left(t^{(k)} - S(r^{(k)})\right)^2 \\
&= \sum_i \left(t^{(k)} - \frac{1}{1 + e^{r^{(k)}}}\right)^2
\end{aligned}$$

$$
\begin{aligned}
\nabla_w E(w) &= \frac{\delta E(w)}{\delta w} \\
&= \frac{\delta \left(t^{(k)} - \frac{1}{1+e^{r^{(k)}}} \right)^2}{\delta w} \\
&= \frac{\delta \left(t^{(k)} - \frac{1}{1+e^{r^{(k)}}} \right)^2}{\delta r^{(k)}} \frac{\delta r^{(k)}}{\delta w} \\
&= 2 \left(t^{(k)} - \frac{1}{1+e^{r^{(k)}}} \right) \frac{\delta \left(t^{(k)} - \frac{1}{1+e^{r^{(k)}}} \right)}{\delta r^{(k)}} \frac{\delta r^{(k)}}{\delta w} \\
&= 2 \left(t^{(k)} - \frac{1}{1+e^{r^{(k)}}} \right) \left(0 - \frac{\delta \left(\frac{1}{1+e^{r^{(k)}}} \right)}{\delta r^{(k)}} \right) \frac{\delta r^{(k)}}{\delta w} \\
&= 2 \left(t^{(k)} - \frac{1}{1+e^{r^{(k)}}} \right) \left(\frac{e^{r^{(k)}}}{(1+e^{r^{(k)}})^2} \right) \frac{\delta r^{(k)}}{\delta w} \\
&= 2 \left(t^{(k)} - \frac{1}{1+e^{r^{(k)}}} \right) \left(\frac{e^{r^{(k)}}}{(1+e^{r^{(k)}})^2} \right) x_i \\
\Delta w &= -\eta \nabla_w E(w) \\
&= -\eta \left(t^{(k)} - \frac{1}{1+e^{r^{(k)}}} \right) \left(\frac{e^{r^{(k)}}}{(1+e^{r^{(k)}})^2} \right) x_i
\end{aligned}
$$