MATH 239 Spring 2014: Assignment 4

Due: 3:00 PM, Monday June 9, 2014 in the dropboxes outside MC 4066

Last Name: First Name:

I.D. Number: Section:

Mark (For the marker only): /28

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1. {6 marks} For each of the following expressions representing sets of binary strings, determine if it is ambiguous or unambiguous. If it is ambiguous, provide an example that can be decomposed in two different ways. If it is unambiguous, determine the generating series (as a simplified rational expression) with respect to the lengths of strings.

   (a) $(\{1\}^*\{001, 0001\}^*)^*$

   (b) $\{10001, 001, 00110\}^*$

   (c) $\{00\}^*(\{11\}\{111\}^*\{0\}\{000\}^*)^*\{1, 11, 111\}$

2. {9 marks} For each of the following sets of binary strings, determine an unambiguous expression which generates every string in that set (no justification required).

   (a) The set of binary strings where the length of each block of 0's is divisible by 3 and the length of each block of 1's is divisible by 4.

   (b) The set of binary strings where each block of 1's of odd length is followed by a block of 0's of length at least 3.

   (c) The set of binary strings which contain 000111 as a substring.
   (Note: An obvious wrong answer is $\{0,1\}^*(000111)\{0,1\}^*$.)

3. {5 marks} On Martin's new game show "It Peis to Play", there is an unlimited supply of gold and silver coins buried inside a box of sand. As a contestant, you dig up one coin at a time, and Martin will offer you \$3 for each gold coin and \$1 for each silver coin. You may stop at any time and keep your winnings. However, if you dig up 3 gold coins in a row or 4 silver coins in a row, the game is over and you lose everything. (For this question, you may represent your answers as coefficients of simplified rational expressions.)

   (a) For some $n \in \mathbb{N}$, how many ways can you win exactly $\$n$ from Martin and walk away with your winnings?

   (b) For some $n \in \mathbb{N}$, suppose you have won $\$n$, but decided to be greedy and then lost everything on the next dig. How many ways can this happen?

4. {4 marks} Let S be the set of binary strings where a block of 0's cannot be followed by a block of 1's of greater length. For example, 111110010001110 is in S, but 100011110010 is not. Prove that the generating series for S with respect to the lengths of the strings is

$$\Phi_S(x) = \frac{1+x}{1-x-2x^2+x^3}.$$

(Hint: You may want to consider using the set $T = \{01, 0011, 000111, 00001111, \ldots\}$.)

5. {4 marks} Let T be the set of all binary strings that do not have 0001 as a substring. Determine a recursive definition for T. Briefly explain why your definition is correct and unambiguous. Use this to find the generating series for T with respect to the lengths of the strings.