

MATH 239 Spring 2014: Assignment 6
Due: 3:00 PM, Monday June 23, 2014 in the dropboxes outside MC 4066

Last Name:

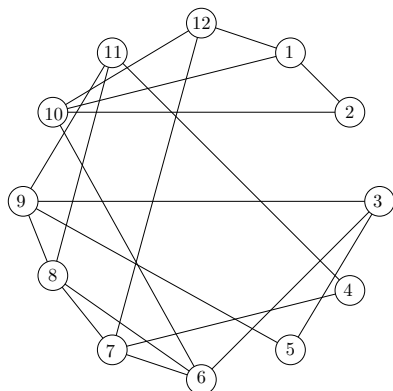
First Name:

I.D. Number:

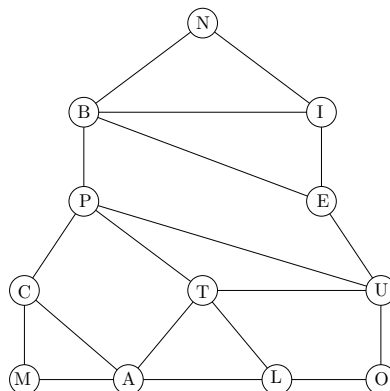
Section:

Mark (For the marker only): /28

1. {3 marks} The following two graphs G and H are isomorphic. Find an isomorphism. (You do not need to prove that your mapping is an isomorphism.)



Graph G



Graph H

2. {3 marks} Draw all non-isomorphic graphs with 5 vertices where the degree of each vertex is even.

3. {3 marks} A group of 3141 software engineers gathered to change a lightbulb. Some of them have 13 friends within the group, some have 33 friends, and the rest have 37 friends. Using graph theory, prove that this group of software engineers does not exist (and it's not just because this is a hardware problem). Note that we assume that friendship is a symmetric relationship, i.e. if A is a friend of B, then B is a friend of A. In other words, we are assuming Facebook's model of friendship rather than Google+'s model of friendship. (Nobody uses Google+ anyway.)
4. {4 marks} Married couple Mario and Peach invited 3 other couples to the castle on the mountain for a cake party (and it's no lie). During the party, some handshaking took place with the restriction that a person cannot shake hands with themselves nor with their own spouse. After all the handshaking was done, Peach went around to ask the 7 others in the party how many people they shook hands with, and she received a different answer from everyone. How many hands did Mario shake? How many hands did Peach shake?

5. {3 marks} Let $k \in \mathbb{N}$. Prove that if G is a k -regular bipartite graph with bipartition (A, B) , then $|A| = |B|$.
6. For integers $n \geq k \geq 1$, let $S_{n,k}$ be the graph whose vertices are all the k -subsets of $[n]$. There is an edge between two vertices A, B in the graph if and only if the two sets intersect in $k - 1$ elements, i.e. $|A \cap B| = k - 1$.
- (a) {2 marks} Draw $S_{5,1}$ and $S_{4,2}$.
- (b) {2 marks} Prove that $S_{n,k}$ is regular by determining the degree of each vertex.

(c) {2 marks} Determine the total number of edges in $S_{n,k}$.

(d) {3 marks} Prove that when $n > k \geq 2$, $S_{n,k}$ is not bipartite.

(e) {3 marks} Prove that for any $n \geq 2$, $S_{n,k}$ is isomorphic to $S_{n,n-k}$.