Chapter 5.
Present worth analysis

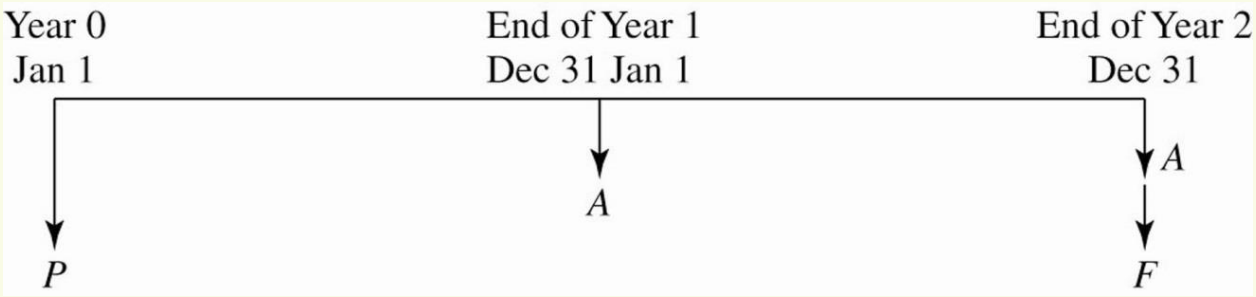
Using the various interest factors examined in chapter 3 and 4, we start to discuss the three main approaches of project evaluation: present worth, annual worth, and internal rate of return analysis. This chapter focuses on the present worth analysis, in which all cash flows are converted into the present worth before being analyzed.

1. Project evaluation

- In chapter 3 and 4, we have learned the concept of *equivalence* and a series of compound *interest factors*.
- Cash flows in decision making process are complicated.
  - The costs and benefits of a project occur at different times.
  - You may invest today to obtain profits in the future, or borrow money today and pay interests every year.
  - To compare projects, you need to convert cash flows at the same date, using the concept of equivalence and compound interest factor.
- We will learn several methods of comparing *projects*.
  - Present worth analysis (chapter 5)
  - Annual cash flow analysis (chapter 6)
  - Rate of return analysis (chapter 7 and 8)
  - Other techniques (chapter 9)

1.1. Basic assumptions (1)

- We will review a few simplifying assumptions for the analysis of project evaluation.
- End-of-period convention
  - All series of receipts and disbursements are assumed to occur at the *end* of the interest period.
  - Current decision is made at date *zero*, and the first stream starts at the end of period 1.
  - All formulas of various factors and the compound interest tables are based on the end-of-period convention.



1.1. Basic assumptions (2)

- Broad viewpoint of analysis
  - The decision is evaluated from the *total* firm's perspective, not from an individual department's view.
  - Depending on the situation, you can analyze from a narrow view.
  - An appropriate viewpoint should be carefully considered.
- Sunk costs
  - Past costs (or sunk costs) do not affect the decision of the present.
  - It is the *current* and *future* events that are important.
- Source of money
  - Where does the money for an investment come from?
  - By financing (or obtaining) money at an interest rate *i*.
  - The concept of interest is critical in investment.
- We ignore the effects of *taxes* and *inflation* until later chapters.

1.2. Rule of present worth analysis

- We compare several alternatives in the present worth analysis.
- Independent alternatives
  - The cash flows of each alternative are not related (e.g., buying a vacuum cleaner and a computer).
  - Choose all the projects which meet certain criteria.
- Mutually exclusive alternatives
  - We choose only one project out of many alternatives (e.g., renting a high-rise building or a townhouse).
  - Rank all the projects and pick the best one.
- Present worth analysis
  - Evaluate *mutually exclusive alternatives* at the *present* time.
  - Choose the alternative with a highest NPW (net present worth) = PW (benefit) – PW (cost).

Example 1-1. Present worth calculation

- A firm plans to buy a new machine for $200,000. The machine will save $50,000 in labor annually. It will require an annual maintenance cost of $9,000. The salvage value after 10 year of its service life is $10,000. Given an interest rate of 10%, do you recommend to buy this machine?

Example 1-2. Buying an apartment for renting

- You plan to buy an apartment for rental income. The annual income from the apartment is $20,000, and the annual expense is $2,000. The apartment can be sold for $100,000 at the end of 10 years. How much would you be willing to pay for it now, with required rate of return of 10%?

Example 1-3. Helping grandparents

- Your grandparents are asking you for advice on when they should start collecting their pension payments. They are now 60 years old and we assume they live until 80.
  - If they start to collect pension now, they will have $850 per month for the next 20 years.
  - If they wait until 65, they collect $1,200 per month for the next 15 years.
- If the annual interest rate is 6% compounded monthly, which plan has a higher PW?

Example 1-4. First-class travel for life (1)

- AAirpass
  - In 1981, American Airline introduced AAirpass which allows members to have unlimited first-class travels for life.
  - The price for couple was $400,000 in 1981, $600,000 in 1990, and $1 million in 1993. (It was discontinued in 1994.)
  - A total of 66 passes were sold.

Example 1-4. First-class travel for life (2)

- Would you buy this pass?
  - One member traveled to London 16 times in a month (worth $125,000).
  - Another member accumulated 11 million miles with the pass.
  - The program was considered "a huge disaster", and the company tried to sue some members for fraudulent behavior (LA Times, 2012).
- What is the present worth of the pass?
  - If you travel once a month and each travel costs $8,000, your annual benefit is about $100,000.
  - Suppose you purchase this pass at the age of 50 (rich enough to buy), and use it for 20 years (healthy enough to travel).
  - The present worth of this pass (with 5% interest rate) is

$$PW = 100{,}000(P/A, 5\%, 20) = 100{,}000(12.462) = \mathbf{\$1.2\ million}$$

2. Present worth with equal service lives

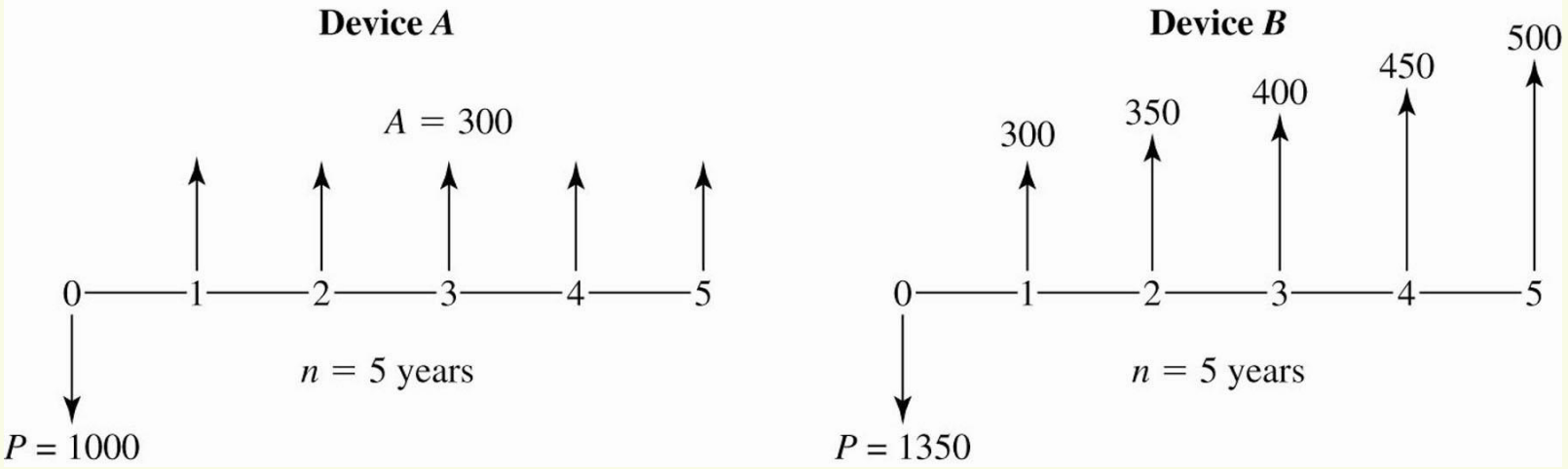
- In comparing present worth, careful consideration must be given to the time period covered by the analysis.
- Each alternative must be considered for the given period of time.
  - Any investment projects have a certain project life.
  - This period is called *analysis period*, *planning horizon* or *project life*.
  - In some industries with rapid technological changes, a short planning horizon (3 or 5 years) is often considered.
  - On the other hand, government agencies frequently use analysis period of 50 years or more.
- We will consider three different analysis-period situations.
  - The lives of each alternative are *equal*.
  - The lives of each alternative are *different*.
  - The period of analysis is *infinite*.

Example 2-1. PW with equal service lives (1)

- A firm will select one of two devices to reduce costs. Both devices have five years of service life with no salvage value.
  - Device A costs $1,000 and can be expected to result in $300 savings annually.
  - Device B costs $1,350 and provides cost savings of $300 in the first year with an increase of $50 annually from the second year.
- With an interest rate of 7%, which device should the firm purchase?

  - The analysis period is the same for both devices (5 years), and we choose the one which has a higher present worth.
  - Drawing a cash flow is important to correctly evaluate the values.

EXAMPLE 2-1. PW WITH EQUAL SERVICE LIVES (2)



Example 2-2. PW with equal service lives

- Wayne County will build an aqueduct. It can be built at a reduced size now for $300 million and be enlarged 25 years later for an additional $350 million. An alternative is to construct the full-sized aqueduct now for $400 million. Both alternatives would provide service for 50 years. At 6% interest, which alternative should be selected?

Example 2-3. PW with equal service lives

- A firm is trying to decide which of two weighing scales it should install. The benefits and costs of each are summarized in the following table. If both scales have a 6 year service life and the interest rate is 8%, which scale should be selected?

Alternatives	Cost	Annual benefit	Salvage value
Atlas scale	$2,000	$450	$100
Tom Thumb scale	$3,000	$600	$700

3. Present worth analysis with unequal lives

- What if two (or more) alternatives have different service lives?
  - Do you want to buy a cheap, but short-lived machine?
  - Or, do you want to buy an expensive, but long-lived machine?
- We use the *least common multiple* to equalize the service life.
  - Assuming the service will be needed repeatedly, we consider a common time period using the least common multiple (LCM).
  - If option A has twice the life of option B, we assume option B is purchased twice to make the total service life equal.
- What if the service lives are 7 years and 13 years, respectively?
  - The LCM is 91 years! How many times do you repeat?
  - Analysis period: Consider a specific usage life and estimate the terminal values (or salvage values) for all alternatives.
  - For unequal service lives, we better use the *annual worth* method.

Example 3-1. Unequal service lives

- An agent is considering the purchase of some new equipment, and two manufacturers have provided the following quotations. Assuming a 7% interest rate, which one should be selected?

Manufacturer	Cost	Useful life	Salvage value
Speedy	$1,500	5 years	$200
Allied	$1,600	10 years	$325

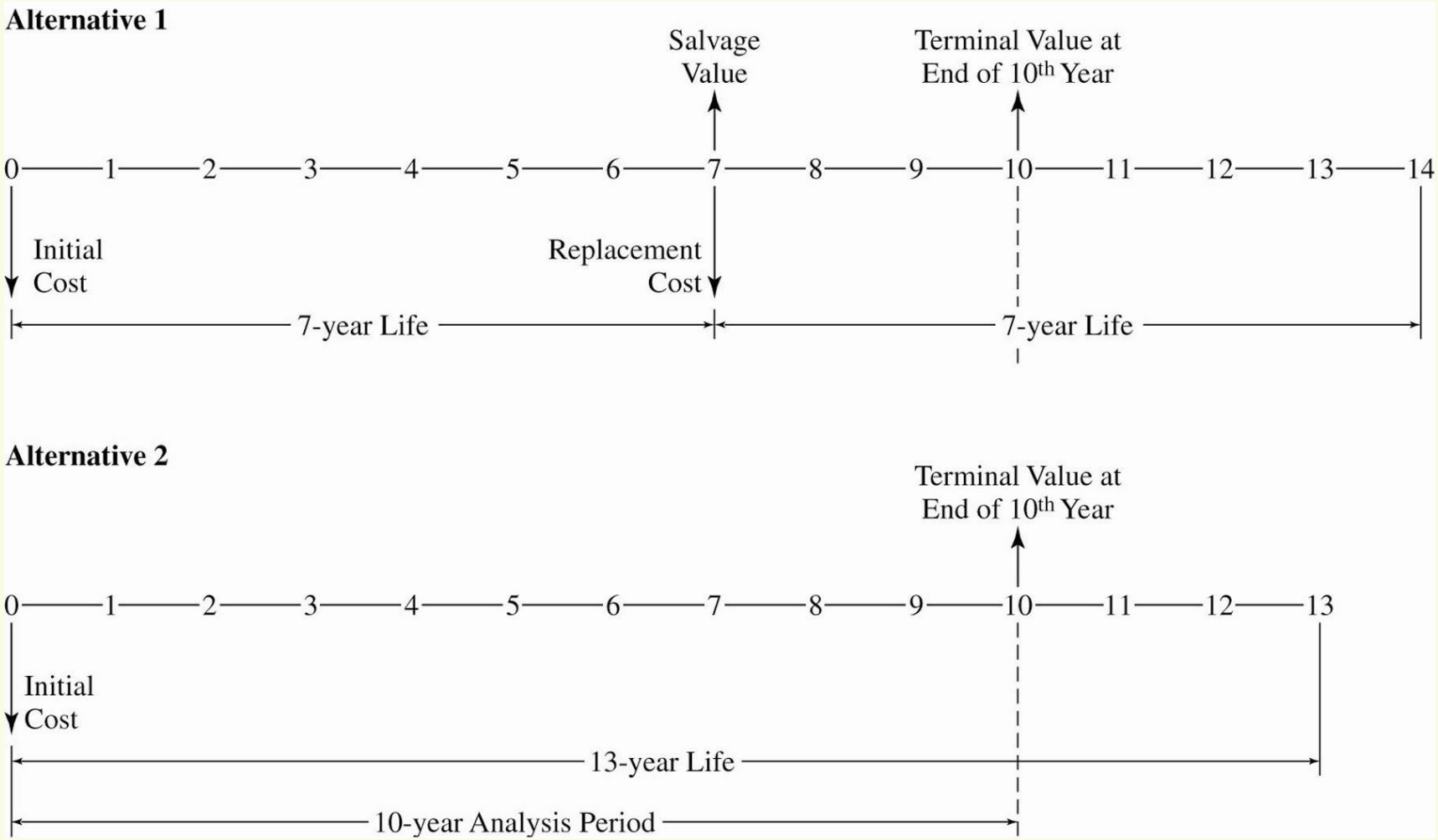
Example 3-2. Unequal service lives (1)

- A diesel manufacturer is considering two alternative machines. It uses an interest rate of 8% and wants to use the PW method to compare these alternatives over a 10-year period.

	Alt. 1	Alt. 2
Initial cost	$50,000	$75,000
Salvage value	$10,000	$12,000
Useful life (years)	7	13
Market value at the end of 10-year period	$20,000	$15,000

- This is the question of analysis period.
- Note the market value estimates at the end of the analysis period.

Example 3-2. Unequal service lives (2)



Example 3-2. Unequal service lives (3)

- If we don't use the analysis period approach, we have to consider the 91-year period of LCM.
  - Alternative 1 is replaced 13 times; alternative 2 is replaced 7 times.
  - The present worth approach in this case is very tedious.

Example 3-3. Magazine subscription

- A weekly business magazine offers a one-year subscription for $60 and a three-year subscription for $160. If you plan to read the magazine for at least the next three years and the interest rate is 6%, which subscription would you buy?

4. Present worth analysis with infinite life

- In government analyses, services or projects sometimes must be maintained for an infinite period (e.g. bridges, pipelines, etc.).
  - In these situations, the present worth analysis would have an infinite number of periods.
  - We call this analysis *capitalized cost*.
- Capitalized cost
  - The present sum that is required to provide the service indefinitely.
  - In general, a sum of money (often called "*endowment*") is set aside, and interest from the fund is withdrawn regularly forever.
  - Only interest from the fund is used, not the principal, so the principal stays forever.
  - Capitalized cost formula

$$P = A/i$$

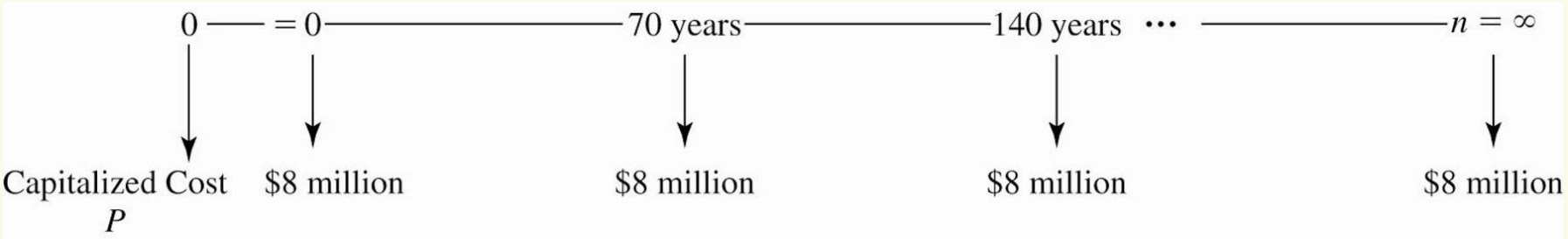
Example 4-1. Cash for life

- OLG (Ontario Lottery and Gaming) has a lottery game, called “Cash for Life.” A winner receives $1,000 a week for life. What is the present worth of this lottery if the interest rate is 10%?

- If the payment is only for 20 years, what would be the present worth?

Example 4-2. Capitalized cost (1)

- A city plans a pipeline to transport water. The pipeline will cost $8 million with an expected life of 70 years. The city expects it will need to keep the water line in service indefinitely. What is the capitalized cost, assuming 7% interest rate?



- Method 1: Annualize the cost that starts from now.

Example 4-2. Capitalized cost (2)

- Method 2: Annualize the cost that starts 70 years from now.

- Method 3: Make the 70-year as a single period

Example 4-3. Trust fund

- A trust fund is to be established for three purposes:
  - To provide $750,000 for the construction and $250,000 for the initial equipment of a small engineering laboratory;
  - To pay the annual operating cost $150,000 of the laboratory;
  - To pay for $100,000 of replacement equipment every 4 years, beginning four years from now.
- At 6% interest rate, how much is required in the trust fund?

5. Extensions of present worth analysis

- We can solve other variations of problems using the PW method.
- Multiple alternatives
  - If there are multiple mutually exclusive projects, we solve the PW of each project and choose the one with the largest value.
  - Note that we always have an option of "do nothing."
- Irregular cash flows
  - The pattern of cash flows can be irregular.
  - We can use the techniques learned in chapter 4, or use Excel.
- Bond pricing
  - What is a bond? How to calculate the price of a bond?
  - The cash flows from a bond are peculiar, and we can use the PW approach to calculate the bond's price.

Example 5-1. Multiple alternatives (1)

- An investor hired a consulting firm to analyze what he might do with a small parcel of land that can be bought for $30,000. The consultant suggested four alternatives. Assuming 10% interest rate, what should the investor do?

	Alternatives	Total investment	Annual benefit	Terminal value at the end of 20 yrs
A	Do nothing	0	0	0
B	Vegetable market	50,000	5,100	30,000
C	Gas station	95,000	10,500	30,000
D	Small motel	350,000	36,000	150,000

Example 5-1. Multiple alternatives (2)

- Alternative A: Do nothing
- Alternative B: Vegetable market
- Alternative C: Gas station
- Alternative D: Small motel

Example 5-2. Choosing a stock (1)

- An investor would like to choose one stock out of six firms whose risks are about the same. He plans to keep the stock for 4 years and he requires a 10% minimum attractive rate of return. Which stock should the investor choose?

Common stock	Price per share	Dividend per share	Estimated price at the end of 4 yrs.
Western House	$23.5	$1.25	$32
Find Foods	$45	$4.5	$45
Mobile Motors	$30.8	0	$42
Spar Products	$12	0	$20
Canada Tire	$33.3	$2.0	$40
Wine Products	$52.5	$3.0	$60

Example 5-2. Choosing a stock (2)

- To compute the present worth of benefits, we need
  - $(P/A, 10\%, 4) = 3.170$ for the present worth of annual dividends.
  - $(P/F, 10\%, 4) = 0.683$ for the present worth of the future sale

Common stock	PW (C)	PW (B) Dividend per share	PW/share	PW/$1
Western House	23.5	1.25(3.17) + 32(0.683) = 25.8	**$2.32**	0.09
Find Foods	45	4.5(3.17) + 45(0.683) = 45	$0	0
Mobile Motors	30.8	0(3.17) + 42(0.683) = 28.69	–$2.11	–0.07
Spar Products	12	0(3.17) + 20(0.683) = 13.66	$1.66	**0.14**
Canada Tire	33.3	2(3.17) + 40(0.683) = 33.66	$0.36	0.01
Wine Products	52.5	3(3.17) + 60(0.683) = 50.49	–$2.01	–0.04

- Western House has the largest PW per share. Should you choose it?
- No, you should choose Spar Products. Why?

5.1. Sources of capital: stock vs. bond

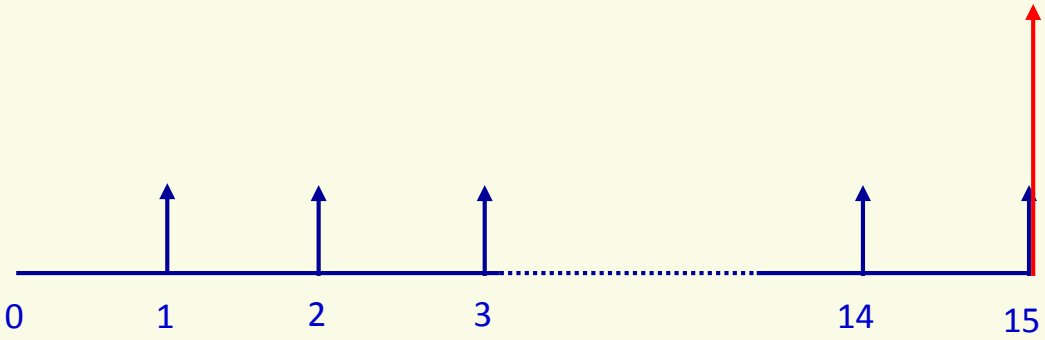
- Stock: *equity* stake issued often by company
  - An issuer sells the ownership of the company.
  - The holder of stocks owns a part of the company.
  - An issuer may pay annual *dividends* if there exist profits.
  - From an investment point of view, stocks are very risky (but with a high return).
- Bond: *debt securities* issued by government or company
  - A bond is a *loan* in the form of a security.
  - An issuer promises to pay the holder
    - Principal (face value) at the end of the maturity date, and
    - Interest (coupon rate) at every period.
  - From an investment point of view, bonds are less risky (with less return) than stocks.

5.1. Case: When to use bonds?

- In 1998, tobacco companies agreed to pay state governments in the United States about $250 billion over 25 years.
  - This is to settle claims arising from the health effects of smoking.
  - Several states were desperate to plug the budget deficits.
  - They wanted money now, instead of waiting for the payment to dribble in year by year.
- What did they do?
- They issued "tobacco bonds."
  - The states sold bonds to investors and pay them interest out of the tobacco settlement payments the states would be receiving.
  - State governments got the money up front (sale price of the bonds), and investors got ongoing income from the bond.
- Bond is a way of *borrowing money*, for immediate use.

EXAMPLE 5-3. VALUE OF BONDS

- What is the value of a bond maturing in 15 years with a face value of $5,000 and a coupon rate of 7%, given an interest rate of 10%?

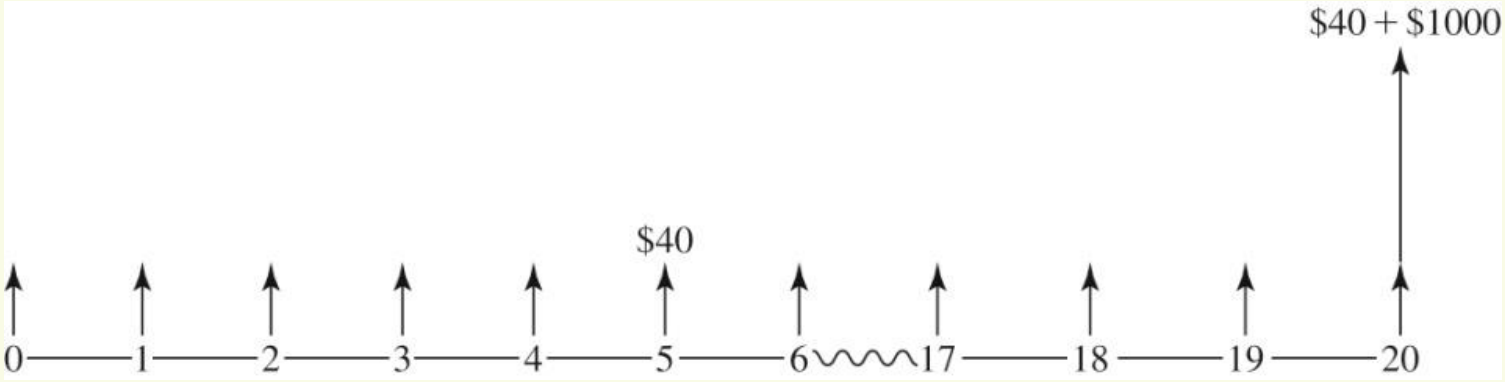


Example 5-4. Bond pricing (1)

- A 15-year municipal bond was issued 5 years ago. Its coupon rate is 8%, coupon interest payments are made *semi-annually*, and its face value is $1,000. What should be the bond's price if the market interest rate is 12%?

  - The first 5 years are past, and there are 20 more semiannual payments (i.e., 10 years).
  - The coupon rate of 8% is annual. Thus, the semiannual amount of coupon value is $40 (= 4% × 1,000).
  - At the end of 20 periods, the bond holder is paid the face value of $1,000.
  - The bond's price is the PW of the cash flows: coupon value every 6 month and face value at the end of 20 periods.

EXAMPLE 5-4. BOND PRICING (2)



Example 5-5. Issuing bonds

- A provincial government wants to raise money by issuing 10,000 bonds. The bond's coupon rate is 8% with *quarterly* payments. Its face value is $2,000 with the maturity after 10 years. If the market interest rate is 12%, how much will the government raise?

Summary of chapter 5

- Decision rule under present worth analysis
- PW with equal service life
- PW with unequal service life
  - Least common multiple method
  - Analysis period method
- Capitalized cost (infinite service life)
  - Government fund or endowment
- Value of bonds

- Selective end-of-chapter problems
  - 5, 11, 16, 29, 37, 42, 45, 58, 74, 89