Major Classes of Neural Networks

Outline

- Multi-Layer Perceptrons (MLPs)
- Radial Basis Function Network
- Kohonen's Self-Organizing Network
- Hopfield Network

Multi-Layer Perceptrons (MLPs)

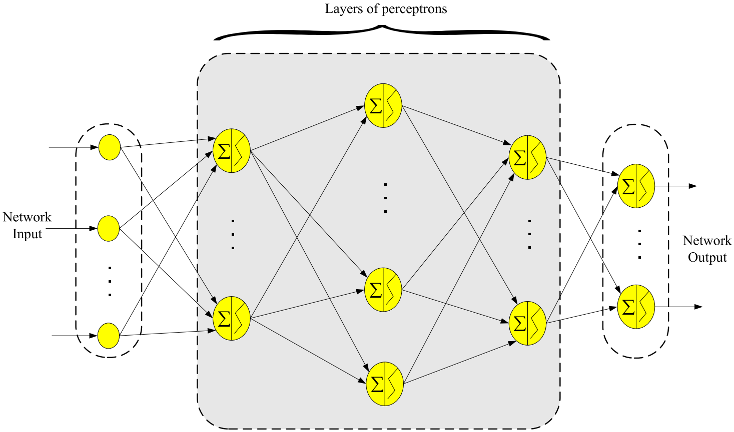
Background

- The perceptron lacks the important capability of recognizing patterns belonging to non-separable linear spaces.
- The madaline is restricted in dealing with complex functional mappings and multi-class pattern recognition problems.
- The multilayer architecture first proposed in the late sixties.

Background (cont.)

- MLP re-emerged as a solid connectionist model to solve a wide range of complex problems in the mid-eighties.
- This occurred following the reformulation of a powerful learning algorithm commonly called the Back Propagation Learning (BPL).
- It was later implemented to the multilayer perceptron topology with a great deal of success.

Schematic Representation of MLP Network



Backpropagation Learning Algorithm (BPL)

- The backpropagation learning algorithm is based on the **gradient descent technique** involving the **minimization of the network cumulative error**.

$$E(k) = \sum_{i=1}^{q} [t_i(k) - o_i(k)]^2$$

  - i represents i-th neuron of the output layer composed of a total number of q neurons.

- It is designed to **update the weights in the direction of the gradient descent of the cumulative error**.

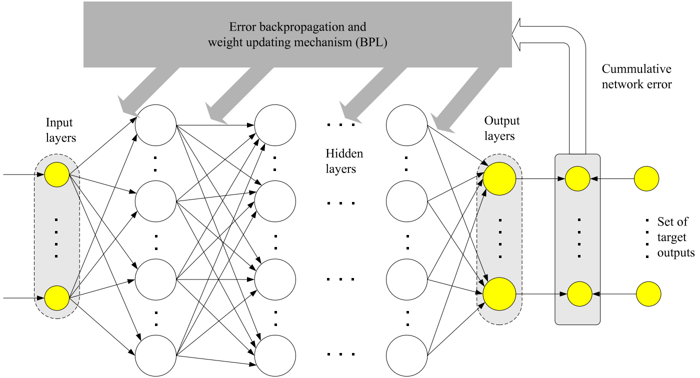
Backpropagation Learning Algorithm (cont.)

A Two-Stage Algorithm

1. First, patterns are presented to the network.
2. A feedback signal is then propagated backward with the main task of updating the weights of the layers connections according to the back-propagation learning algorithm.

BPL: Schematic Representation

- Schematic Representation of the MLP network illustrating the notion of error back-propagation



Backpropagation Learning Algorithm (cont.)

Objective Function

- Using the **sigmoid function** as the activation function for all the neurons of the network, we define E_c as

$$E_c = \sum_{k=1}^{n} E(k) = \frac{1}{2} \sum_{k=1}^{n} \sum_{i=1}^{q} [t_i(k) - o_i(k)]^2$$

Backpropagation Learning Algorithm (cont.)

- The formulation of the **optimization problem** can now be stated as **finding the set of the network weights** that minimizes E_c or $E(k)$.

Objective Function: Off-Line Training

$$min_w E_c = min_w \frac{1}{2} \sum_{k=1}^{n} \sum_{i=1}^{q} [t_i(k) - o_i(k)]^2$$

Objective Function: On-Line Training

$$min_w E(k) = min_w \frac{1}{2} \sum_{i=1}^{q} [t_i(k) - o_i(k)]^2$$

BPL: On-Line Training

- Objective Function: $min_w E(k) = min_w \frac{1}{2} \sum_{i=1}^{q} [t_i(k) - o_i(k)]^2$

Updating Rule for Connection Weights

$$\Delta w^{(l)} = -\eta \frac{\partial E(k)}{\partial w^l},$$

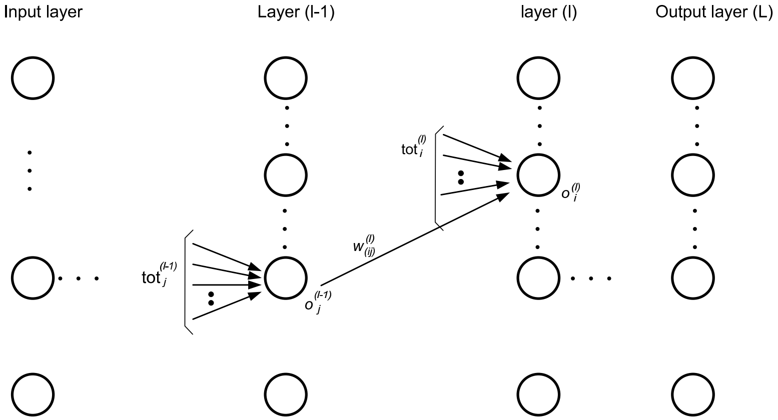
- l is layer (l-th) and η denotes the learning rate parameter,
- $\Delta w_{ij}^{(l)}$: the weight update for the connection linking the node j of layer $(l-1)$ to node i located at layer l.

BPL: On-Line Training (cont.)

Updating Rule for Connection Weights

- o_j^{l-1}: the output of the neuron j at layer $l-1$, the one located just before layer l,
- tot_i^l: the sum of all signals reaching node i at hidden layer l coming from previous layer $l-1$·

Illustration of Interconnection Between Layers of MLP



Interconnection Weights Updating Rules

- $\Delta w^{(l)} = \Delta w_{ij}^{(l)} = -\eta[\frac{\partial E(k)}{\partial o_i^{(l)}}][\frac{\partial o_i^{(l)}}{\partial tot_i^{(l)}}][\frac{\partial tot_i^{(l)}}{\partial w_{ij}^{(l)}}]$
- For the case where the layer (l) is the output layer (L):
  $\Delta w_{ij}^{(L)} = \eta[t_i - o_i^{(L)}][f'(tot)_i^{(L)}]o_j^{(L-1)}; \quad f'(tot)_i^{(l)} = \frac{\partial f(tot_i^{(l)})}{\partial tot_i^{(l)}}$
- By denoting $\delta_i^{(L)} = [t_i - o_i^{(L)}][f'(tot)_i^{(L)}]$ as being the **error signal** of the i-th node of the output layer, the weight update at layer (L) is as follows: $\Delta w_{ij}^{(L)} = \eta \delta_i^{(L)} o_j^{(L-1)}$
- In the case where f is the sigmoid function, the error signal becomes expressed as:
  $\delta_i^L = [(t_i - o_i^{(L)})o_i^{(L)}(1 - o_i^{(L)})]$

Interconnection Weights Updating Rules (cont.)

- Propagating the error backward now, and for the case where (l) represents a hidden layer ($l < L$), the expression of $\Delta w_{ij}^{(l)}$ becomes given by: $\Delta w_{ij}^{(l)} = \eta \delta_i^{(l)} o_j^{(l-1)}$, where $\delta_i^{(l)} = f'(tot)_i^{(l)} \sum_{p=1}^{n_l} \delta_p^{l+1} w_{pi}^{l+1}$.
- Again when f is taken as the sigmoid function, $\delta_i^{(l)}$ becomes expressed as: $\delta_i^{(l)} = o_i^{(l)}(1 - o_i^{(l)}) \sum_{p=1}^{n_l} \delta_p^{l+1} w_{pi}^{l+1}$.

Updating Rules: Off-Line Training

- The weight update rule:

$$\Delta w^{(l)} = -\eta \frac{\partial E_c}{\partial w^l}.$$

- All previous steps outlined for developing the on-line update rules are reproduced here with the exception that $E(k)$ becomes replaced with E_c.

- In both cases though, once the network weights have reached steady state values, the training algorithm is said to converge.

Required Steps for Backpropagation Learning Algorithm

- Step 1: Initialize weights and thresholds to small random values.
- Step 2: Choose an input-output pattern from the training input-output data set $(x(k), t(k))$.
- Step 3: Propagate the k-th signal forward through the network and compute the output values for all i neurons at every layer (l) using $o_i^l(k) = f(\sum_{p=0}^{n_{l-1}} w_{ip}^l o_p^{l-1})$.
- Step 4: Compute the total error value $E = E(k) + E$ and the error signal $\delta_i^{(L)}$ using formulae $\delta_i^{(L)} = [t_i - o_i^{(L)}][f'(tot)_i^{(L)}]$.

Required Steps for BPL (cont.)

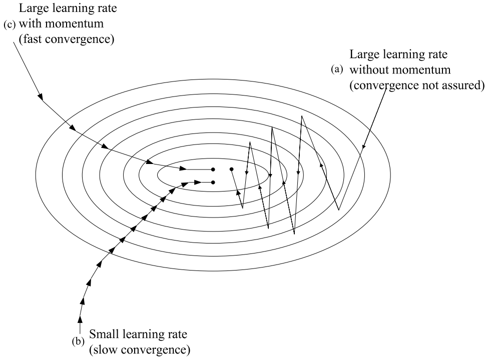
- Step 5: Update the weights according to $\Delta w_{ij}^{(l)} = \quad \eta \delta_i^{(l)} o_j^{(l-1)}$, for $l = L, \cdots, 1$ using $\delta_i^{(L)} = [t_i - o_i^{(L)}][f'(tot)_i^{(L)}]$ and proceeding backward using $\delta_i^{(l)} = o_i^l(1 - o_i^l) \sum_{p=1}^{n_l} \delta_p^{l+1} w_{pi}^{l+1}$ for $l < L\cdot$
- Step 6: Repeat the process starting from step 2 using another exemplar. Once all exemplars have been used, we then reach what is known as one epoch training.
- Step 7: Check if the cumulative error E in the output layer has become less than a predetermined value. If so we say the network has been trained. If not, repeat the whole process for one more epoch.

Momentum

- The gradient descent requires by nature infinitesimal differentiation steps.
- For small values of the learning parameter η, this leads most often to a very slow convergence rate of the algorithm.
- Larger learning parameters have been known to lead to unwanted oscillations in the weight space.
- To avoid these issues, the concept of momentum has been introduced.

Momentum (cont.)

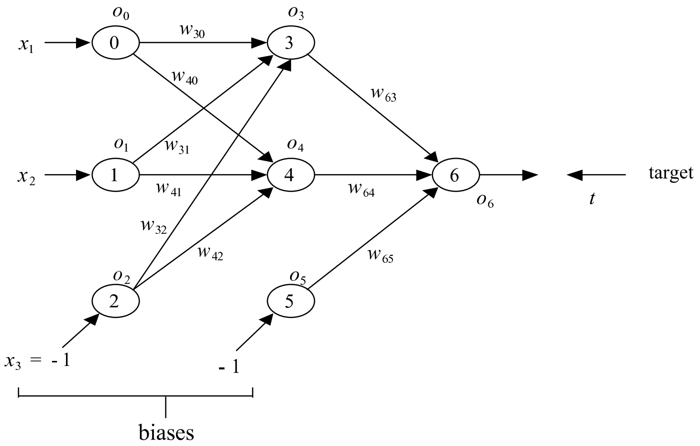
The modified weight update formulae including momentum term given as: $\Delta w^{(l)}(t+1) = -\eta \frac{\partial E_c(t)}{\partial w^l} + \gamma \Delta w^l(t)$.



Example 1

- To illustrate this powerful algorithm, we apply it for the training of the following network shown in the next page.
- x : training patterns, and t : output data
  $x^{(1)} = (0.3, 0.4), \;\; t(1) = 0.88$
  $x^{(2)} = (0.1, 0.6), \;\; t(2) = 0.82$
  $x^{(3)} = (0.9, 0.4), \;\; t(3) = 0.57$
- Biases: -1
- Sigmoid activation function: $f(tot) = \frac{1}{1+e^{-\lambda tot}}$, using $\lambda = 1$, then $f'(tot) = f(tot)(1 - f(tot))$.

Example 1: Structure of the Network



Example 1: Training Loop (1)

- Step (1) Initialization
  - Initialize the weights to 0.2, set learning rate to $\eta = 0.2$; set maximum tolerable error to $E_{max} = 0.01$ (i.e. 1% error), set $E = 0$ and $k = 1$.
- Step (2) - Apply input pattern
  - Apply the 1^{st} input pattern to the input layer. $x^{(1)} = (0.3, 0.4)$, $t(1) = 0.88$, then,

    $o_0 = x_1 = 0.3$; $o_1 = x_2 = 0.4$; $o_2 = x_3 = -1$;

Example 1: Training Loop (1)

- Step (3) - Forward propagation
  - Propagate the signal forward through the network

$$o_3 = f(w_{30}o_0 + w_{31}o_1 + w_{32}o_2) = 0.485$$

$$o_4 = f(w_{40}o_0 + w_{41}o_1 + w_{42}o_2) = 0.485$$

$$o_5 = -1$$

$$o_6 = f(w_{63}o_3 + w_{64}o_4 + w_{65}o_5) = 0.4985$$

Example 1: Training Loop (1)

- Step (4) - Output error measure
  - Compute the error value E

$$E = \frac{1}{2}(t - o_6)^2 + E = 0.0728$$

  - Compute the error signal δ_6 of the output layer

$$\begin{aligned}\delta_6 &= f'(tot_6)(t - o_6) \\ &= o_6(1 - o_6)(t - o_6) \\ &= 0.0945\end{aligned}$$

Example 1: Training Loop (1)

Step (5) - Error back-propagation

- Third layer weight updates:

$$\Delta w_{63} = \eta \delta_6 o_3 = 0.0093 \qquad w_{63}^{new} = w_{63}^{old} + \Delta w_{63} = 0.2093$$

$$\Delta w_{64} = \eta \delta_6 o_4 = 0.0093 \qquad w_{64}^{new} = w_{64}^{old} + \Delta w_{64} = 0.2093$$

$$\Delta w_{65} = \eta \delta_6 o_5 = 0.0191 \qquad w_{65}^{new} = w_{65}^{old} + \Delta w_{65} = 0.1809$$

- Second layer error signals:

$$\delta_3 = f_3'(tot_3) \sum_{i=6}^{6} w_{i3}\delta_i = o_3(1 - o_3) w_{63}\delta_6 = 0.0048$$

$$\delta_4 = f_4'(tot_4) \sum_{i=6}^{6} w_{i4}\delta_i = o_4(1 - o_4) w_{64}\delta_6 = 0.0048$$

Example 1: Training Loop (1)

Step (5) - Error back-propagation (cont.)

- Second layer weight updates:

$$\Delta w_{30} = \eta\delta_3 o_0 = 0.00028586 \quad w_{30}^{new} = w_{30}^{old} + \Delta w_{30} = 0.2003$$

$$\Delta w_{31} = \eta\delta_3 o_1 = 0.00038115 \quad w_{31}^{new} = w_{31}^{old} + \Delta w_{31} = 0.2004$$

$$\Delta w_{32} = \eta\delta_3 o_2 = -0.00095288 \quad w_{32}^{new} = w_{32}^{old} + \Delta w_{32} = 0.199$$

$$\Delta w_{40} = \eta\delta_4 o_0 = 0.00028586 \quad w_{40}^{new} = w_{40}^{old} + \Delta w_{40} = 0.2003$$

$$\Delta w_{41} = \eta\delta_4 o_1 = 0.00038115 \quad w_{41}^{new} = w_{41}^{old} + \Delta w_{41} = 0.2004$$

$$\Delta w_{42} = \eta\delta_4 o_2 = -0.00095288 \quad w_{42}^{new} = w_{42}^{old} + \Delta w_{42} = 0.199$$

Example 1: Training Loop (2)

- Step (2) - Apply the 2^{nd} input pattern $x^{(2)} = (0.1, 0.6)$, $t(2) = 0.82$, then, $o_0 = 0.1$; $o_1 = 0.6$; $o_2 = -1$;
- Step (3) - Forward propagation

$$o_3 = f(w_{30}o_0 + w_{31}o_1 + w_{32}o_2) = 0.4853$$

$$o_4 = f(w_{40}o_0 + w_{41}o_1 + w_{42}o_2) = 0.4853$$

$$o_5 = -1$$

$$o_6 = f(w_{63}o_3 + w_{64}o_4 + w_{65}o_5) = 0.5055$$

- Step (4) - Output error measure

$$E = \frac{1}{2}(t - o_6)^2 + E = 0.1222$$

$$= o_6(1 - o_6)(t - o_6) = 0.0786$$

Training Loop - Loop (2)

Step (5) - Error back-propagation

- Third layer weight updates:

$$\Delta w_{63} = \eta \delta_6 o_3 = 0.0076 \qquad w_{63}^{new} = w_{63}^{old} + \Delta w_{63} = 0.2169$$

$$\Delta w_{64} = \eta \delta_6 o_4 = 0.0076 \qquad w_{64}^{new} = w_{64}^{old} + \Delta w_{64} = 0.2169$$

$$\Delta w_{65} = \eta \delta_6 o_5 = 0.0157 \qquad w_{65}^{new} = w_{65}^{old} + \Delta w_{65} = 0.1652$$

- Second layer error signals:

$$\delta_3 = f_3'(tot_3) \sum_{i=6}^{6} w_{i3}\delta_i = o_3(1 - o_3) w_{63}\delta_6 = 0.0041$$

$$\delta_4 = f_4'(tot_4) \sum_{i=6}^{6} w_{i4}\delta_i = o_4(1 - o_4) w_{64}\delta_6 = 0.0041$$

Example 1: Training Loop (2)

Step (5) - Error back-propagation (cont.)

- Second layer weight updates:

$$\Delta w_{30} = \eta\delta_3 o_0 = 0.000082169 \quad w_{30}^{new} = w_{30}^{old} + \Delta w_{30} = 0.2004$$

$$\Delta w_{31} = \eta\delta_3 o_1 = 0.00049302 \quad w_{31}^{new} = w_{31}^{old} + \Delta w_{31} = 0.2009$$

$$\Delta w_{32} = \eta\delta_3 o_2 = -0.00082169 \quad w_{32}^{new} = w_{32}^{old} + \Delta w_{32} = 0.1982$$

$$\Delta w_{40} = \eta\delta_4 o_0 = 0.000082169 \quad w_{40}^{new} = w_{40}^{old} + \Delta w_{40} = 0.2004$$

$$\Delta w_{41} = \eta\delta_4 o_1 = 0.00049302 \quad w_{41}^{new} = w_{41}^{old} + \Delta w_{41} = 0.2009$$

$$\Delta w_{42} = \eta\delta_4 o_2 = -0.00082169 \quad w_{42}^{new} = w_{42}^{old} + \Delta w_{42} = 0.1982$$

Example 1: Training Loop (3)

- Step (2) - Apply the 2^{nd} input pattern $x^{(3)} = (0.9, 0.4)$, $t(3) = 0.57$, then, $o_0 = 0.9$; $o_1 = 0.4$; $o_2 = -1$;
- Step (3) - Forward propagation

$$o_3 = f(w_{30}o_0 + w_{31}o_1 + w_{32}o_2) = 0.5156$$

$$o_4 = f(w_{40}o_0 + w_{41}o_1 + w_{42}o_2) = 0.5156$$

$$o_5 = -1$$

$$o_6 = f(w_{63}o_3 + w_{64}o_4 + w_{65}o_5) = 0.5146$$

- Step (4) - Output error measure

$$E = \frac{1}{2}(t - o_6)^2 + E = 0.1237$$
$$= o_6(1 - o_6)(t - o_6) = 0.0138$$

Example 1: Training Loop (3)

Step (5) - Error back-propagation

- Third layer weight updates:

$$\Delta w_{63} = \eta \delta_6 o_3 = 0.0014 \qquad w_{63}^{new} = w_{63}^{old} + \Delta w_{63} = 0.2183$$

$$\Delta w_{64} = \eta \delta_6 o_4 = 0.0014 \qquad w_{64}^{new} = w_{64}^{old} + \Delta w_{64} = 0.2183$$

$$\Delta w_{65} = \eta \delta_6 o_5 = -0.0028 \qquad w_{65}^{new} = w_{65}^{old} + \Delta w_{65} = 0.1624$$

- Second layer error signals:

$$\delta_3 = f_3'(tot_3) \sum_{i=6}^{6} w_{i3}\delta_i = o_3(1 - o_3) w_{63}\delta_6 = 0.00074948$$

$$\delta_4 = f_4'(tot_4) \sum_{i=6}^{6} w_{i4}\delta_i = o_4(1 - o_4) w_{64}\delta_6 = 0.00074948$$

Example 1: Training Loop (3)

Step (5) - Error back-propagation (cont.)

- Second layer weight updates:

$\Delta w_{30} = \eta\delta_3 o_0 = 0.00013491 \quad w_{30}^{new} = w_{30}^{old} + \Delta w_{30}{=}0.2005$

$\Delta w_{31} = \eta\delta_3 o_1 = 0.000059958 \quad w_{31}^{new} = w_{31}^{old} + \Delta w_{31}{=}0.2009$

$\Delta w_{32} = \eta\delta_3 o_2 = -0.0001499 \quad w_{32}^{new} = w_{32}^{old} + \Delta w_{32}{=}0.1981$

$\Delta w_{40} = \eta\delta_4 o_0 = 0.00013491 \quad w_{40}^{new} = w_{40}^{old} + \Delta w_{40}{=}0.2005$

$\Delta w_{41} = \eta\delta_4 o_1 = 0.000059958 \quad w_{41}^{new} = w_{41}^{old} + \Delta w_{41}{=}0.2009$

$\Delta w_{42} = \eta\delta_4 o_2 = -0.0001499 \quad w_{42}^{new} = w_{42}^{old} + \Delta w_{42}{=}0.1981$

Example 1: Final Decision

- Step (6) - One epoch looping

  The training patterns have been cycled one epoch.

- Step (7) - Total error checking

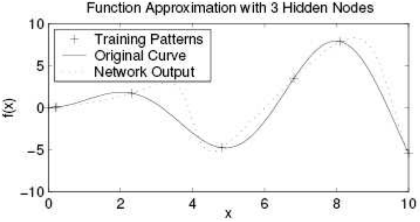
  $E = 0.1237$ and $E_{max} = 0.01$, which means that we have to continue with the next epoch by cycling the training data again.

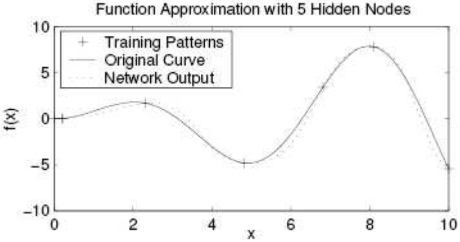
Example 2

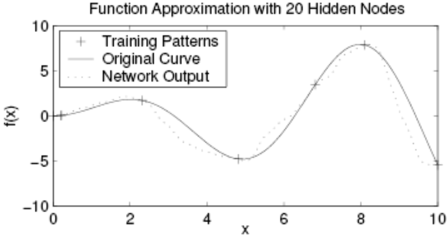
Effect of Hidden Nodes on Function Approximation

- Consider this function $f(x) = x\sin(x)$
- Six input/output samples were selected from the range $[0, 10]$ of the variable x
- The first run was made for a network with 3 hidden nodes
- Another run was made for a network with 5 and 20 nodes, respectively.

Example 2: Different Hidden Nodes







Example 2: Remarks

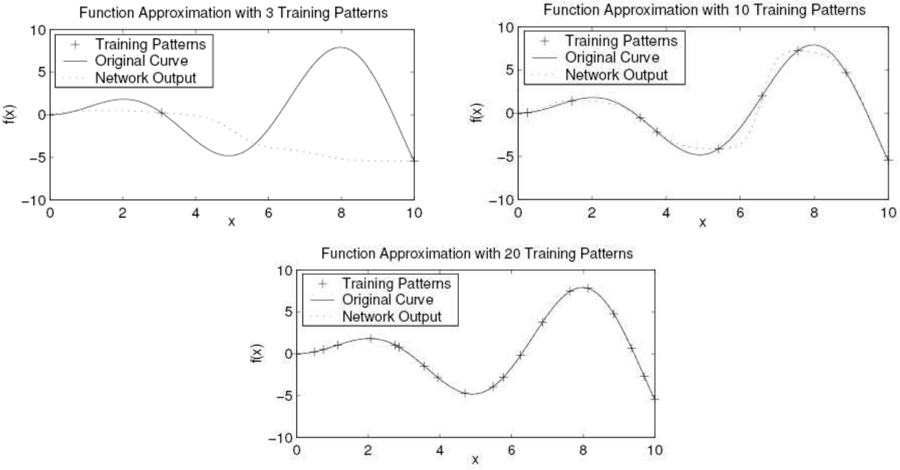
- A higher number of nodes is not always better. It may overtrain the network.
- This happens when the network starts to memorize the patterns instead of interpolating between them.
- A smaller number of nodes was not able to approximate faithfully the function given the nonlinearities induced by the network was not enough to interpolate well in between the samples.
- It seems here that this network (with five nodes) was able to interpolate quite well the nonlinear behavior of the curve.

Example 3

Effect of Training Patterns on Function Approximation

- Consider this function $f(x) = x \sin(x)$
- Assume a network with a fixed number of nodes (taken as five here), but with a variable number of training patterns
- The first run was made for a network with 3 three samples
- Another run was made for a network with 10 and 20 samples, respectively.

Example 3: Different Samples



Example 3: Remarks

- The first run with three samples was not able to provide a good mach with the original curve.
- This can be explained by the fact that the three patterns, in the case of a nonlinear function such as this, are not able to reproduce the relatively high nonlinearities of the function.
- A higher number of training points provided better results.
- The best result was obtained for the case of 20 training patterns. This is due to the fact that a network with five hidden nodes interpolates extremely well in between close training patterns.

Applications of MLP

- Multilayer perceptrons are currently among the most used connectionist models.
- This stems from the relative ease for training and implementing, either in hardware or software forms.

Applications

- Signal processing
- Pattern recognition
- Financial market prediction
- Weather forecasting
- Signal compression

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Limitations of MLP

- Among the well-known problems that may hinder the generalization or approximation capabilities of MLP is the one related to the convergence behavior of the connection weights during the learning stage.
- In fact, the gradient descent based algorithm used to update the network weights may never converge to the global minima.
- This is particularly true in the case of highly nonlinear behavior of the system being approximated by the network.

Limitations of MLP

- Many remedies have been proposed to tackle this issue either by retraining the network a number of times or by using optimization techniques such as those based on:
    - Genetic algorithms,
    - Simulated annealing.

MLP NN: Case Study

Function Estimation (Regression)

MLP NN: Case Study

- Use a feedforward backpropagation neural network that contains a single hidden layer.
- Each of hidden nodes has an activation function of the logistic form.
- Investigate the outcome of the neural network for the following mapping.

$$f(x) = exp(-x^2), \quad x \in [0\ 2]$$

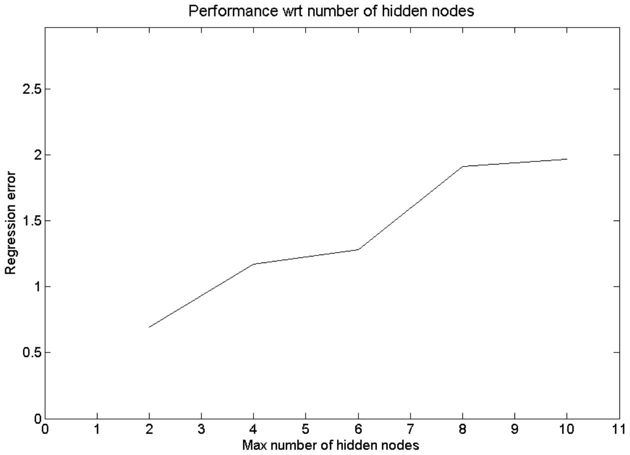
- Experiment with different number of training samples and hidden layer nodes

MLP NN: Case Study

Experiment 1: Vary Number of Hidden Nodes

- Uniformly pick six sample points from [0 2], use half of them for training and the rest for testing
- Evaluate regression performance increasing the number of hidden nodes
- Use sum of regression error (i.e. $\sum_{i \in \text{test samples}} (Output(i) - True\_output(i))$) as performance measure
- Repeat each test 20 times and compute average results, compensating for potential local minima

MLP NN: Case Study

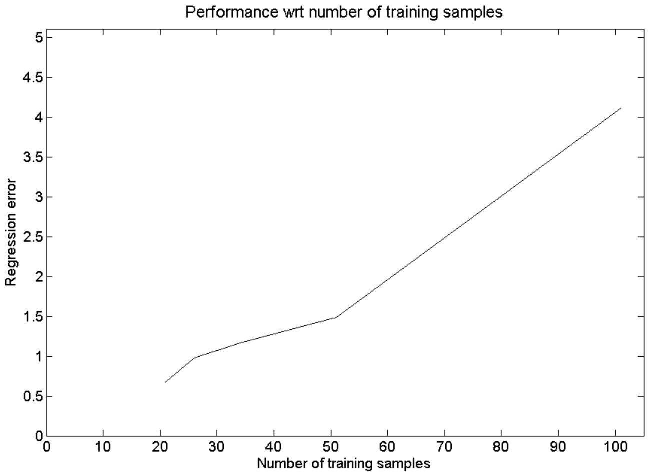


MLP NN: Case Study

Experiment 2: Vary Number of Training Samples

- Construct neural network using three hidden nodes
- Uniformly pick sample points from [0 2], increasing their number for each test
- Use half of sample data points for training and the rest for testing
- Use the same performance measure as experiment 1, i.e. sum of regression error
- Repeat each test 50 times and compute average results

MLP NN: Case Study

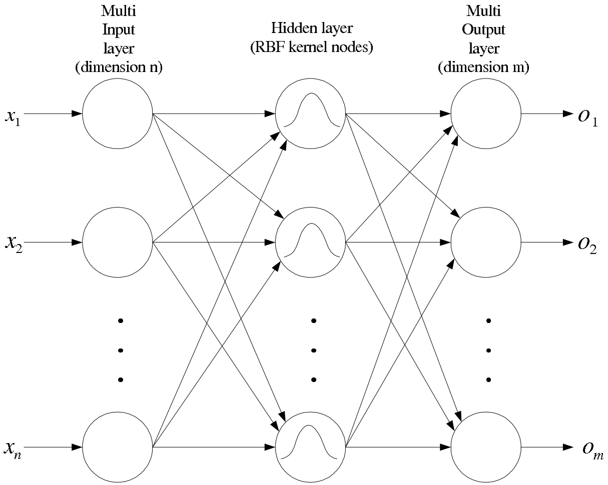


Radial Basis Function Network

Topology

- Radial basis function network (RBFN) represent a special category of the **feedforward** neural networks architecture.
- Early researchers have developed this connectionist model for **mapping nonlinear behavior of static processes** and for **function approximation purposes**.
- The basic RBFN structure consists of **an input layer**, **a single hidden layer** with **radial activation function** and **an output layer**.

Topology: Graphical Representation



Topology (cont.)

- The network structure uses **nonlinear transformations** in its hidden layer (typical transfer functions for hidden functions are Gaussian curves).
- However, it uses **linear transformations** between the hidden and output layers.
- The rationale behind this is that input spaces, cast nonlinearly into high-dimensional domains, are more likely to be linearly separable than those cast into low-dimensional ones.

Topology (cont.)

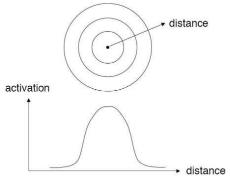
- Unlike most FF neural networks, the connection weights between the input layer and the neuron units of the hidden layer for an RBFN are all equal to **unity**.
- The nonlinear transformations at the hidden layer level have the main characteristics of being symmetrical.
- They also attain their maximum at the function center, and generate positive values that are rapidly decreasing with the distance from the center.

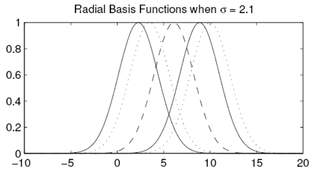
Topology (cont.)

- As such they produce radially activation signals that are bounded and localized.

Parameters of Each activation Function

- The center
- The width





Topology (cont.)

- For an optimal performance of the network, the hidden layer nodes should span the training data input space.
- Too sparse or too overlapping functions may cause the degradation of the network performance.

Radial Function or Kernel Function

- In general the form taken by an RBF function is given as:

$$g_i(x) = r_i\left(\frac{\| x - v_i \|}{\sigma_i}\right)$$

  - where x is the input vector,
  - v_i is the vector denoting the center of the radial function g_i,
  - σ_i is width parameter.

Famous Radial Functions

- The Gaussian kernel function is the most widely used form of RBF given by:

$$g_i(x) = exp(\frac{- \parallel x - v_i \parallel^2}{2\sigma_i^2})$$

- The logistic function has also been used as a possible RBF candidate:

$$g_i(x) = \frac{1}{1 + exp(\frac{\|x - v_i\|^2}{\sigma_i^2})}$$

Output of an RBF Network

- A typical output of an RBF network having n units in the hidden layer and r output units is given by:

$$o_j(x) = \sum_{i=1}^{n} w_{ij} g_i(x), \; j = 1, \cdots, r \cdot$$

  - where w_{ij} is the connection weight between the i-th receptive field unit and the j-th output,
  - g_i is the i-th receptive field unit (radial function).

Learning Algorithm

Two-Stage Learning Strategy

- At first, an unsupervised clustering algorithm is used to extract the parameters of the radial basis functions, namely the width and the centers.
- This is followed by the computation of the weights of the connections between the output nodes and the kernel functions using a supervised least mean square algorithm.

Learning Algorithm: Hybrid Approach

- The standard technique used to train an RBF network is the **hybrid approach**.

Hybrid Approach

- Step 1: Train the RBF layer to get the adaptation of centers and scaling parameters using the **unsupervised training**.
- Step 2: Adapt the weights of the output layer using **supervised training algorithm**.

Learning Algorithm: Step 1

- To determine the centers for the RBF networks, typically **unsupervised** training procedures of **clustering** are used:
  - K-means method,
  - "Maximum likelihood estimate" technique,
  - Self-organizing map method.
- This step is very important in the training of RBFN, as the accurate knowledge of v_i and σ_i has a major impact on the performance of the network.

Learning Algorithm: Step 2

- Once the centers and the widths of radial basis functions are obtained, the next stage of the training begins.
- To update the weights between the hidden layer and the output layer, the supervised learning based techniques such as are used:
  - Least-squares method,
  - Gradient method.
- Because the weights exist only between the hidden layer and the output layer, it is easy to compute the weight matrix for the RBFN.

Learning Algorithm: Step 2 (cont.)

- In the case where the RBFN is used for interpolation purposes, we can use the **inverse** or **pseudo-inverse method** to calculate the **weight matrix**.
- If we use Gaussian kernel as the radial basis functions and there are n input data, we have:

$$G = [\{g_{ij}\}],$$

where

$$g_{ij} = exp(\frac{- \parallel x_i - v_j \parallel^2}{2\sigma_j^2}), \;\; i, j = 1, \cdots, n$$

Learning Algorithm: Step 2 (cont.)

- Now we have:

$$D = GW$$

where D is the desired output of the training data.

- If G^{-1} exists, we get:

$$W = G^{-1}D$$

- In practice however, G may be ill-conditioned (close to singularity) or may even be a non-square matrix (if the number of radial basis functions is less than the number of training data) then W is expressed as:

$$W = G^{+}D$$

Learning Algorithm: Step 2 (cont.)

- We had:

$$W = G^+ D,$$

  - where G^+ denotes the pseudo-inverse matrix of G, which can be defined as

$$G^+ = (G^T G)^{-1} G^T$$

- Once the weight matrix has been obtained, all elements of the RBFN are now determined and the network could operate on the task it has been designed for.

Learning Algorithm: Step 2 (cont.)

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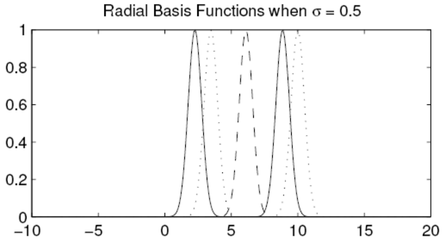
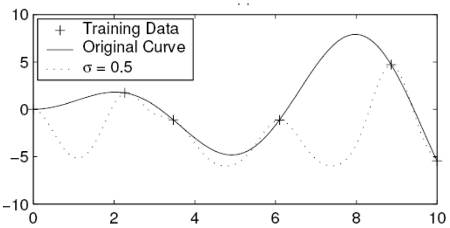
Example

Approximation of Function $f(x)$ Using an RBFN

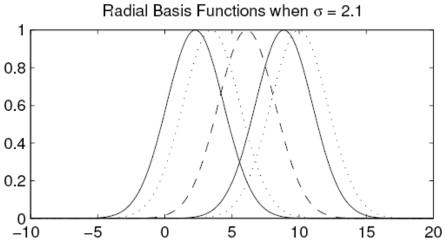
- We use here the same function as the one used in the MLP section, $f(x) = x\sin(x)$.
- The RBF network is composed here of five radial functions.
- Each radial function has its center at a training input data.
- Three width parameters are used here: 0.5, 2.1, and 8.5.
- The results of simulation show that the width of the function plays a major importance.

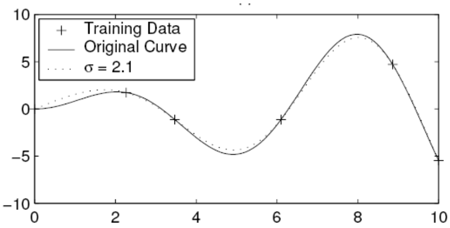
Example: Function Approximation with Gaussian Kernels ($\sigma = 0.5$)



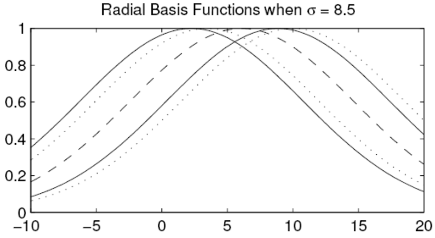


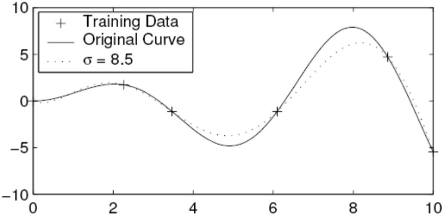
Example: Function Approximation with Gaussian Kernels ($\sigma = 2.1$)



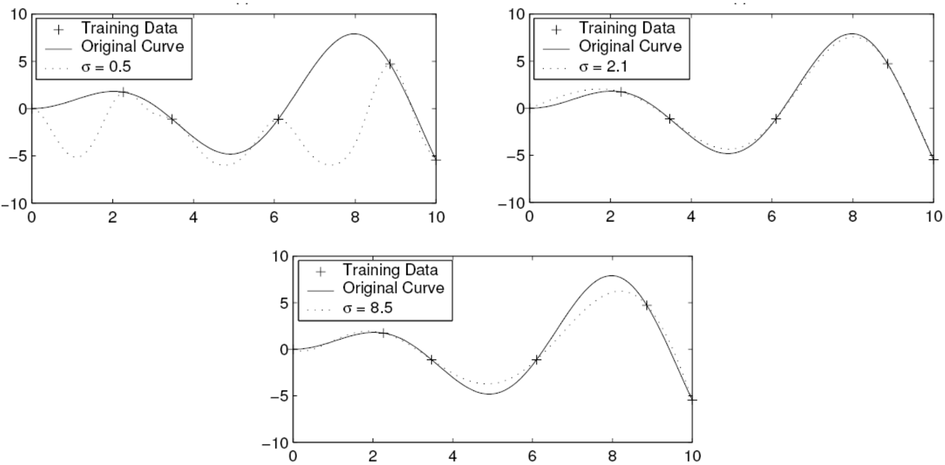


Example: Function Approximation with Gaussian Kernels ($\sigma = 8.5$)





Example: Comparison



Example: Remarks

- A smaller width value 0.5 doesn't seem to provide for a good interpolation of the function in between sample data.
- A width value 2.1 provides a better result and the approximation by RBF is close to the original curve.
  - This particular width value seems to provide the network with the adequate interpolation property.
- A larger width value 8.5 seems to be inadequate for this particular case, given that a lot of information is being lost when the ranges of the radial functions are further away from the original range of the function.

Advantages/Disadvantages

- Unsupervised learning stage of an RBFN is not an easy task.
- RBF trains faster than a MLP.
- Another advantage that is claimed is that the hidden layer is easier to interpret than the hidden layer in an MLP.
- Although the RBF is quick to train, when training is finished and it is being used it is slower than a MLP, so where speed is a factor a MLP may be more appropriate.

Applications

- Known to have **universal approximation capabilities**, **good local structures** and **efficient training algorithms**, RBFN have been often used for nonlinear mapping of complex processes and for solving a wide range of **classification problems**.
- They have been used as well for control systems, audio and video signals processing, and pattern recognition.

Applications (cont.)

- They have also been recently used for **chaotic time series prediction**, with particular application to weather and power load forecasting.

- Generally, RBF networks have an undesirably high number of hidden nodes, but the dimension of the space can be reduced by careful planning of the network.