MATH 213, SPRING 2013

ASSIGNMENT NO. 6

1. Many vehicles are inherently unstable, and can't be piloted by a human being without the aid of automatic control. Examples include some swept-wing aircraft, and Segway human transporters.

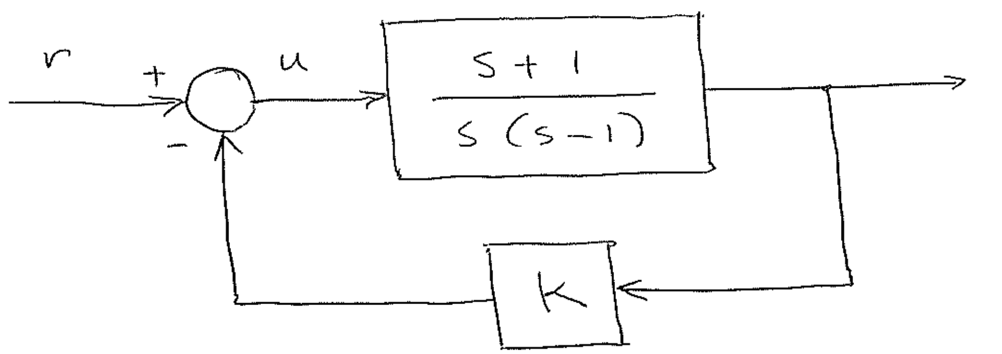
Suppose that one aspect of the dynamics of such a vehicle is represented by the following transfer function:

$$u \longrightarrow \boxed{\frac{s+1}{s(s-1)}} \longrightarrow y$$

a) Is this system BIBO stable?

b) Compute the system's step response.

Suppose that an "autopilot" is implemented by feeding back a measurement of the output:



$$Y(s) = \frac{s+1}{s(s-1)} U(s)$$

$$U(s) = R(s) - K Y(s)$$

(c) Eliminate $U(s)$ from the above equations and find a transfer function from $R(s)$ to $Y(s)$ (by solving for $Y(s)$ in terms of $R(s)$).

(d) Find the poles of the transfer function when $K = 1$. Is the transfer function stable? Find its step response.

e) Set $K = 2$. Is the transfer function stable?

f) The ability of a human pilot to control a vehicle sometimes depends on the natural frequency and damping ratio associated with a pair of complex poles. With $K = 2$, what are the values of ω_n and ζ for the poles of this system? Find the system's step response.

g) The block in the feedback path is replaced by one having the following transfer function:

$$K \frac{s}{s + 0.5}$$

Find the final value of the step response of a system having the above transfer function.

Such a system is called a "washout" circuit, because of this steady-state value of its response to steps.

h) It may be desired that

$\frac{Y(s)}{R(s)}$ retain the pole at the origin

that $\frac{Y(s)}{U(s)}$ has. For instance,

$R(s)$ may be a rudder position, and $Y(s)$ may be a "yaw" angle, which should continue to change under a constant, nonzero rudder input — at a steady rate.

Find the transfer function

$\frac{Y(s)}{R(s)}$ with the washout circuit

in the feedback path. For $K=2$, find the poles of the transfer function. Find its step response.