Group Members			
Name	Userid	Name	Userid
Lara Janecka	20460089 lajaneck	Tyler Babran	20457511 tbabran
By filling out the names above, the group members acknowledge that a) they have jointly authored this submission, b) this work represents their original work, c) that they have not been provided with nor examined another person's assignment, either electronically or in hard copy, and d) that this work has not been previously submitted for academic credit.			

LAB 4. PROPORTIONAL-INTEGRAL-DERIVATIVE CONTROLLER ANALYSIS

ASSIGNED DATA

For easily referencing it, the Assigned Data has been placed at the start of this document. To determine your assigned data, read Section 2 of this prelab.

On your pre-lab and post-lab submissions, always include this page at the beginning of the document.

Select your lab session:	☐ morning lab; ☒ afternoon lab; ☒ Tue; ☐ Wed; ☐ Thu	
GroupNumber		
Assigned plant formula	GroupNumber - rounddown(GroupNumber/24.5, 0) * 24 (valid Excel formula syntax)	
Assigned plant number:	21	
P(s) parameters	On ACS-13005	$K_1 = 10$
	On ACS-13008	$a = 1$
		$B = 2.5$
		$T_1 = 100$
	On ACS-13007	$K_2 = 5$
	On ACS-13006	$T_2 = 10$

a)

$$
\begin{aligned}
\mathrm{M(S)} &= \frac{\frac{K_1 b T_1}{s(s+aT_1)}}{1+\frac{K_1 b T_1}{s(s+aT_1)}} \\
&= \frac{K_1 b T_1}{s(s+aT_1)} \times \frac{s(s+aT_1)}{s(s+aT_1)+K_1 b T_1} \\
&= \frac{K_1 b T_1}{s(s+aT_1)+K_1 b T_1} \\
&= \frac{K_1 b T_1}{s^2+saT_1+K_1 b T_1}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{N(S)} &= \frac{\frac{K_2 T_2}{s}}{1+\frac{K_2 T_2}{s}} \\
&= \frac{K_2 T_2}{s+K_2 T_2} \\
&= \frac{1}{\frac{s}{K_2 T_2}+1}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{P(S)} &= \mathrm{M(S)} \times \mathrm{N(S)} \\
&= \frac{K_1 b T_1}{s^2+saT_1+K_1 b T_1} \times \frac{K_2 T_2}{s+K_2 T_2} \\
&= \frac{bK_1 T_1 K_2 T_2}{(s+K_2 T_2)(s^2+saT_1+K_1 b T_1)}
\end{aligned}
$$

b)

$$
\begin{aligned}
\mathrm{H(S)} &= \frac{K_p \times \mathrm{P(S)}}{1+K_p \mathrm{P(S)}} \\
&= \frac{K_p b K_1 T_1 K_2 T_2}{(s+K_2 T_2)(s^2+saT_1+K_1 b T_1)+2bK_1 T_1 K_2 T_2} \\
&= \frac{K_p b K_1 T_1 K_2 T_2}{s^3+(aT_1+K_2 T_2)s^2+(K_1 b T_1+K_2 T_2 a T_1)s+2K_2 T_2 K_1 b T_1} \\
&= \frac{125000}{s^3+(150)s^2+(7500)s+250000}
\end{aligned}
$$

Output Parameter	**Symbol**	**Value**
Magnitude of first peak	M_p	0.14
Time to First Peak	T_p	0.084
Settling Time (2%)	T_s	0.168
Steady-state	$y_s s$	0.1

Table 1: step-response characteristics

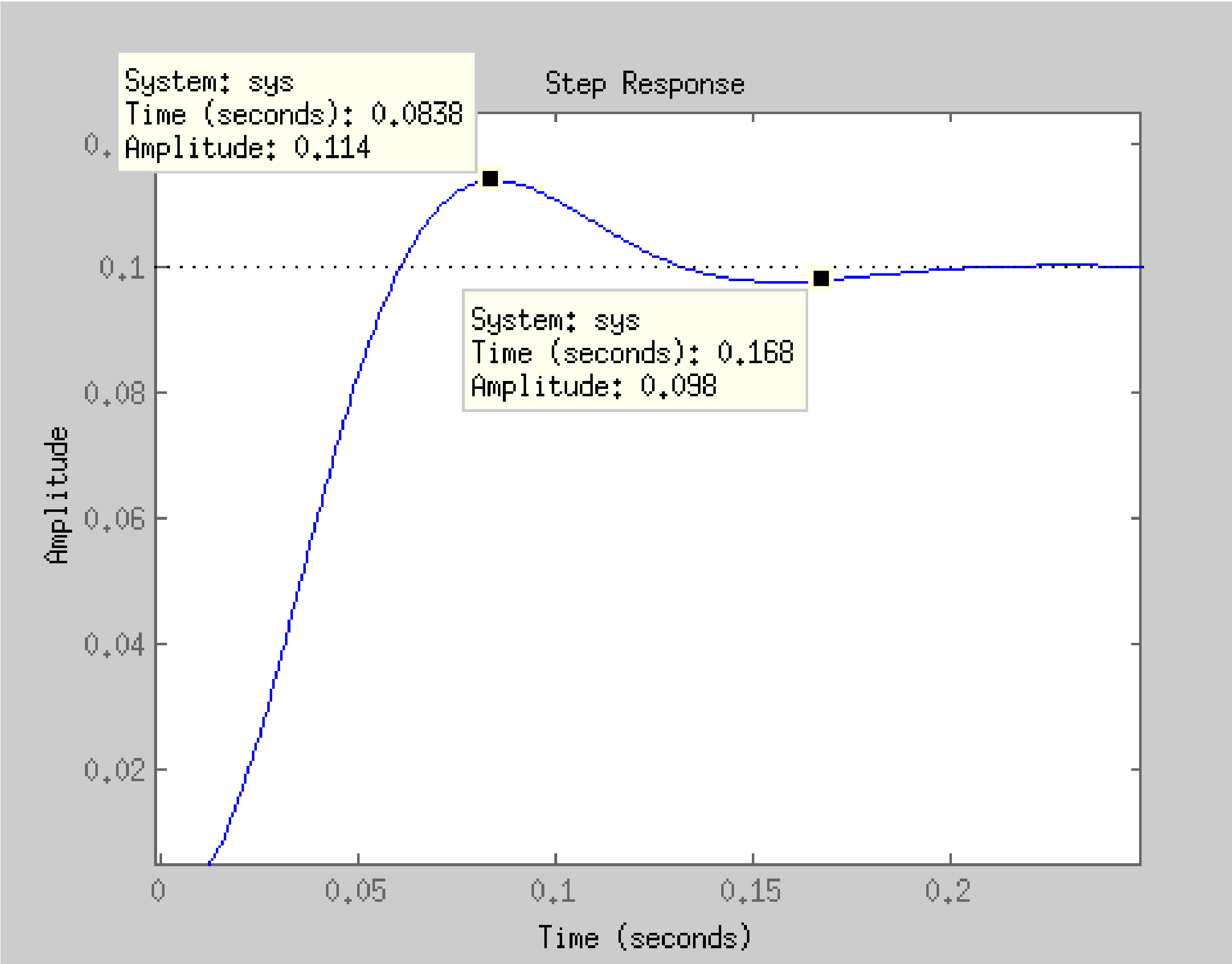

Figure 1: Step response of system

c)

Since the system is strictly proper its stability is determined by the presence of roots in the positive real area.

$$
\begin{aligned}
\text{H(S)} &= \frac{125000}{s^3 + (150)s^2 + (7500)s + 125000(1+K_p)} \\
\text{Q(S)} &= s^3 + (150)s^2 + (7500)s + 250000 \\
a_3 &= 1 \\
a_2 &= 150 \\
a_1 &= 7500 \\
a_0 &= 125000(1+K_p) \\
b_1 &= \frac{-1}{150} \times \begin{vmatrix} 1 & 7500 \\ 150 & 125000(1+K_p) \end{vmatrix} \\
&= 7500 - 833.33(1+K_p) \\
c_0 &= \frac{-1}{7500 - 833.33(1+K_p)} \times \begin{vmatrix} 150 & 125000(1+K_p) \\ 7500 - 833.33(1+K_p) & 0 \end{vmatrix} \\
&= 125000(1+K_p)
\end{aligned}
$$

$$
\begin{array}{c|cc}
3 & 1 & 7500 \\
2 & 150 & 250000 \\
1 & 7500 - 833.33(1+K_p) & 0 \\
0 & 250000 & 0
\end{array}
$$

This means that the system is stable where:

$$
\begin{aligned}
7500 - 833.33(1+K_p) &> 0 \\
K_p &> 8
\end{aligned}
$$