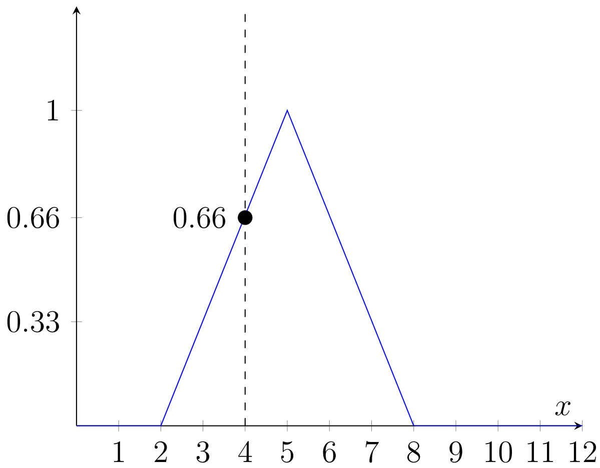
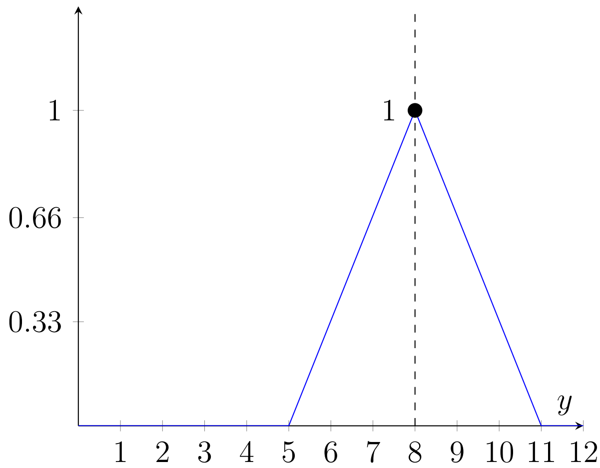
Question 1

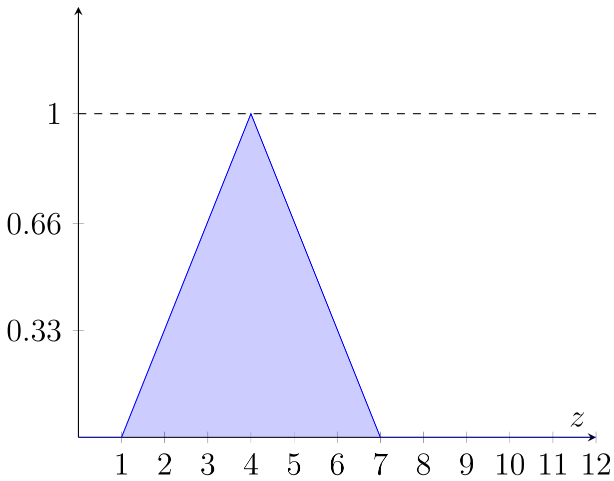


μ_{A_1}

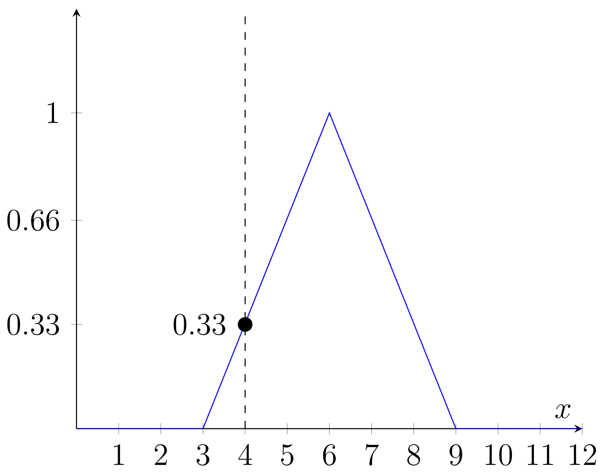


μ_{B_1}

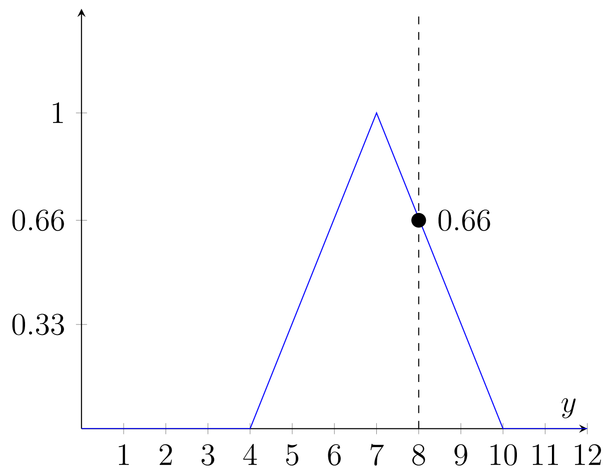
The strength of these two rules is 0.66 and the resulting and the cut on μ_{C_1} will occur at the max, which is 1.



μ_{C_1}

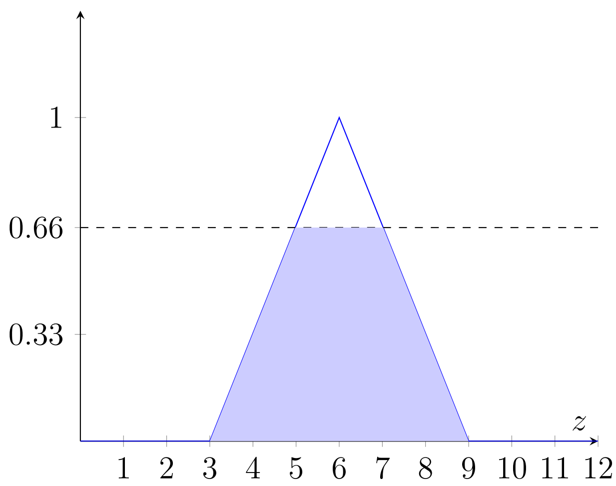


μ_{A_2}

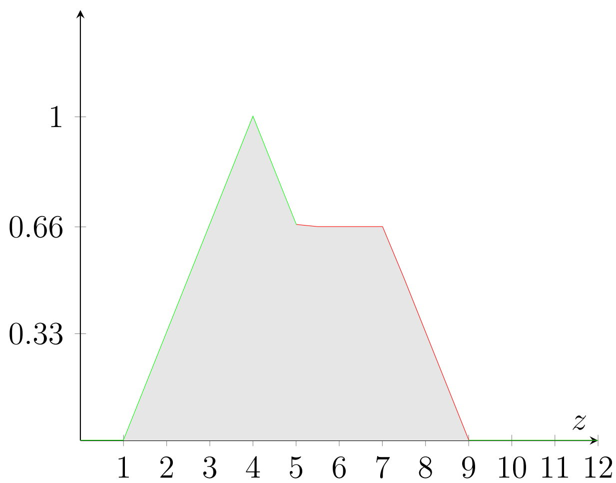


μ_{B_2}

The strength of these two rules is 0.33 and the resulting and the cut on μ_{C_1} will occur at the max, which is 0.66.



μ_{C_2}



output

In this instance we have only one maximum at 4 so the mean of maxima is 4, giving us the defuzzified output at t_1.

When calculating for the centroid of area a slightly different answer occurs.

$$
\begin{aligned}
c &= \frac{\int c\mu_c(c)\delta c}{\int \mu_c(c)\delta c} \\
&= \frac{\int_1^5 \frac{x-1}{3}x\delta x + \int_5^7 0.66x\delta x + \int_7^9 \frac{9-x}{3}x\delta x}{\int_1^5 \frac{x-1}{3}\delta x + \int_5^7 0.66\delta x + \int_7^9 \frac{9-x}{3}\delta x} \\
&= \frac{22.8089}{4.6533} \\
&= 4.9016
\end{aligned}
$$

The centroid of area is slightly higher because the mean of maxima disregards the values outside of its single peak. This is also why the centroid of area returns a more accurate result.

Question 2

a)

i)

Classical:

f\K	$1e+3$	$1e+4$	$1e+5$	$5e+5$	$1e+6$	$5e+6$	$1e+7$
100	1	0.8	0.5	0.2	0	0.2	0.8
200	1	0.8	0.5	0.2	0	0.2	0.8
500	1	0.8	0.5	0.2	0.2	0.2	0.8
800	1	0.8	0.5	0.5	0.5	0.5	0.8
1000	1	0.8	0.5	0.8	1	0.8	0.8
2000	1	0.8	0.5	0.8	0.8	0.8	0.8
5000	1	0.8	0.5	0.2	0.2	0.2	0.8

ii)

Mamdani:

f\K	$1e+3$	$1e+4$	$1e+5$	$5e+5$	$1e+6$	$5e+6$	$1e+7$
100	0	0	0	0	0	0	0
200	0	0	0	0	0	0	0
500	0	0.2	0.2	0.2	0.2	0.2	0.2
800	0	0.2	0.5	0.5	0.5	0.5	0.2
1000	0	0.2	0.5	0.8	1	0.8	0.2
2000	0	0.2	0.5	0.8	0.8	0.8	0.2
5000	0	0.2	0.2	0.2	0.2	0.2	0.2

iii)

Product:

f\K	$1e+3$	$1e+4$	$1e+5$	$5e+5$	$1e+6$	$5e+6$	$1e+7$
100	0	0.0	0.0	0.0	0	0.0	0.0
200	0	0.0	0.0	0.0	0	0.0	0.0
500	0.0	0.04	0.1	0.16	0.2	0.16	0.04
800	0.0	0.1	0.25	0.4	0.5	0.4	0.1
1000	0	0.2	0.5	0.8	1	0.8	0.2
2000	0.0	0.16	0.4	0.64	0.8	0.64	0.16
5000	0.0	0.04	0.1	0.16	0.2	0.16	0.04

b)

$$
\begin{aligned}
R &= \begin{bmatrix} 1 & 0.8 & 0.5 & 0.2 & 0 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0.2 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.5 & 0.5 & 0.5 & 0.8 \\ 1 & 0.8 & 0.5 & 0.8 & 1 & 0.8 & 0.8 \\ 1 & 0.8 & 0.5 & 0.8 & 0.8 & 0.8 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0.2 & 0.2 & 0.8 \end{bmatrix} \\
K' &= \begin{bmatrix} 0 \\ 0.8 \\ 0.2 \end{bmatrix} \\
f_1 &= R \circ K' \\
&= \max_{rows}\left(\min_{cols}(K', R)\right) \\
&= \max_{cols}\left(\min_{cols}\left(\begin{bmatrix} 0 \\ 0.8 \\ 0.2 \end{bmatrix}, \begin{bmatrix} 1 & 0.8 & 0.5 & 0.2 & 0 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0.2 & 0.2 & 0.8 \\ 1 & 0.8 & 0.5 & 0.5 & 0.5 & 0.5 & 0.8 \\ 1 & 0.8 & 0.5 & 0.8 & 1 & 0.8 & 0.8 \\ 1 & 0.8 & 0.5 & 0.8 & 0.8 & 0.8 & 0.8 \\ 1 & 0.8 & 0.5 & 0.2 & 0.2 & 0.2 & 0.8 \end{bmatrix}\right)\right) \\
&= \max_{cols}\left(\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.8 & 0.5 & 0.2 & 0 & 0.2 & 0.8 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}\right) \\
&= \begin{bmatrix} 0.8 & 0.8 & 0.5 & 0.2 & 0.2 & 0.2 & 0.8 \end{bmatrix}
\end{aligned}
$$

Question 3

$$
\begin{aligned}
T_{300} &= \left\{\frac{0}{LW} + \frac{1}{HG}\right\} \\
M_{800} &= \left\{\frac{0}{SM} + \frac{1}{LG}\right\} \\
P_{1.3} &= \left\{\frac{0.2}{FR} + \frac{0.8}{NR}\right\}
\end{aligned}
$$