# Chapter 7.
# Rate of return analysis

We now examine the third method of a project evaluation. The internal rate of return (IRR) is the most widely used concept, and students are expected to fully understand this approach. Choosing one of mutually exclusive projects with the IRR method is different from the previous approach and it should be carefully considered.

# 1. Rate of return analysis

- *Rate of return* analysis is the most frequently used approach.
- Consider a project whose PW becomes $30,000 after one year. Are you happy with this project?
  - Not necessarily! Why?
  - You might have invested $1 million and received $30,000 of PW.
  - Or, you might have invested $100,000 and got $30,000 of PW.
  - The rate of return of the former is 3% and that of the latter is 30%!
- Why do we use the rate of return analysis?
  - It is expressed as a percentage (%), and is readily understood and compared among several projects.
  - Unlike PW or AW, it doesn't need an interest rate in the calculation.
  - The calculated rate of return is compared with a *minimum attractive rate of return* (MARR).

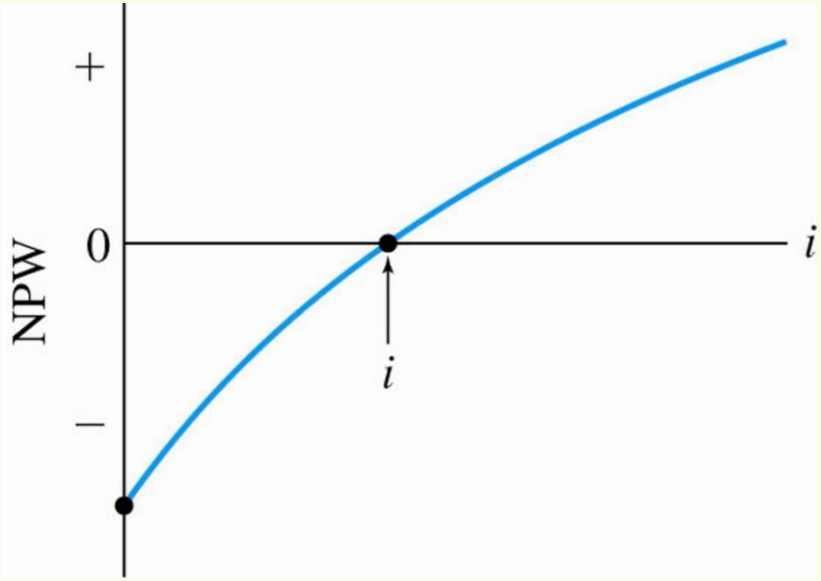
# 1.1. Internal rate of return (IRR)

- Definition of internal rate of return (IRR)
  - the interest rate at which a project *breaks even*, or
  - the interest rate at which PW or EUAW is equal to *zero*, or
  - the interest rate at which the *benefit* equals the *cost*.
- The "*internal*" means that the return depends only on cash flows due to a specific investment.
  - The other case of *external rate of return* considers the concurrent investments in other projects.
- Notes on IRR
  - It is expressed in a rate *per period* (usually one year).
  - If IRR > MARR (PW > 0 or EUAW > 0), the project should be accepted.
  - We can use the *linear interpolation* technique to find the IRR.

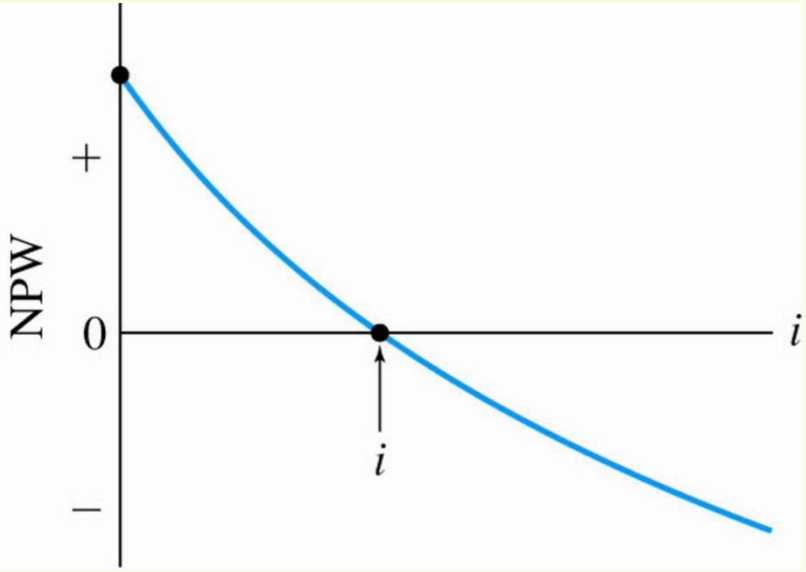
# 1.1. Graphical interpretation of IRR

- Remember that IRR is the interest rate at which *benefit = cost*.
  - *Loan*: the interest rate paid on the unpaid balance of a loan.
  - *Investment*: the interest rate earned on the unrecovered investment

NPW plot for loan




NPW plot for investment



# 1.2. Calculating the Rate of Return

- Formal definition of IRR
  - The interest rate $i^*$ that satisfies the following condition:

$$\sum_{n=0}^{N} \frac{B_n}{(1+i^*)^n} = \sum_{n=0}^{N} \frac{C_n}{(1+i^*)^n}$$

  - A negative IRR means that the project is losing money.
  - We compare IRR with MARR to make an investment decision.
- Steps of calculating IRR
  - Convert various consequences of the investment into a cash flow.
  - Calculate the benefits (B) and costs (C) of a project with unknown $i^*$, either as present worth or equivalent uniform annual worth.
  - Solve for $i^*$ by setting PW(B) = PW(C) or EUAW(B) = EUAW(C).

# Example 1-1. Rate of return

- An engineer invests $5,000 at the end of every year for 40 years. If the engineer wants $1 million in savings at retirement, what interest rate must the investment earn?

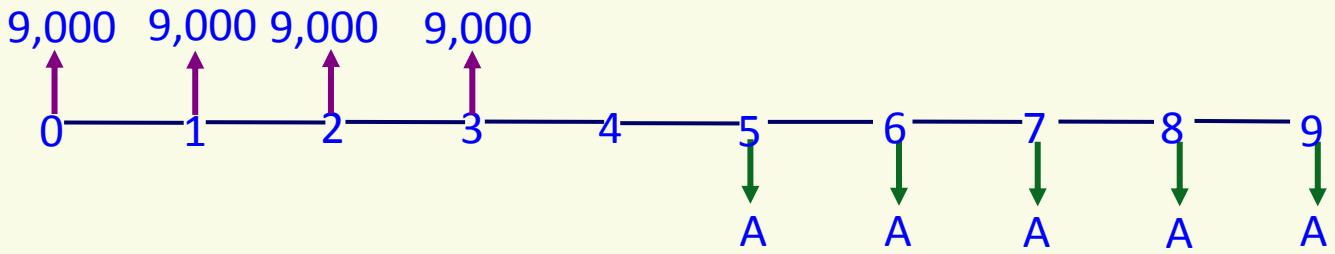
# Example 1-2. Calculation of IRR

- Consider an investment of $1,600 which leads to an annual revenue of $100 in the first year, with an increase of $50 each year in the next 10 years. Calculate the *internal rate of return*.

# EXAMPLE 1-3. IRR FOR STUDENT LOAN (1)

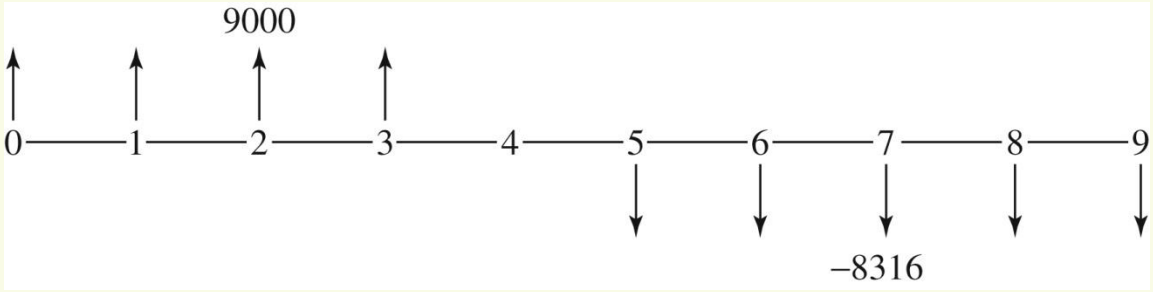
- Maria borrows a student loan of $9,000 each year for 4 years, at the *start* of each year. No interest is charged until graduation, and then the interest rate is charged at 5%.

(a) If Maria makes 5 equal annual payments after graduation (starting from year 5), what is the amount of each payment? First draw a cash flow diagram.



# EXAMPLE 1-3. IRR FOR STUDENT LOAN (2)

(b) What is the internal rate of return (IRR) for Maria's loan? Is this attractive to Maria?



# Example 1-4. Choice of money

- In his will, Frank's grandfather has given Frank the choice between two alternatives.
  - Alternative 1: $20,000 cash
  - Alternative 2: $1,500 cash now plus $1,000 per month for 20 months beginning the first day of next month

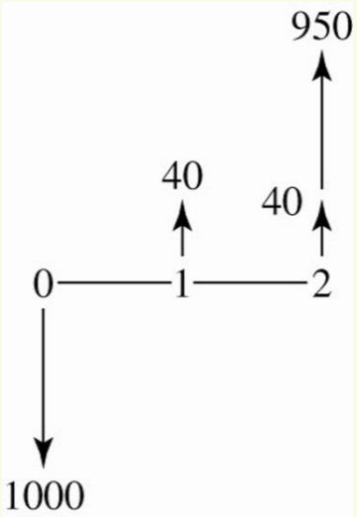
(a) What rate of return are the two alternatives equivalent?

(b) If Frank thinks the rate of return in (a) is too low, which alternative should he select?

# EXAMPLE 1-5. IRR OF BONDS (1)

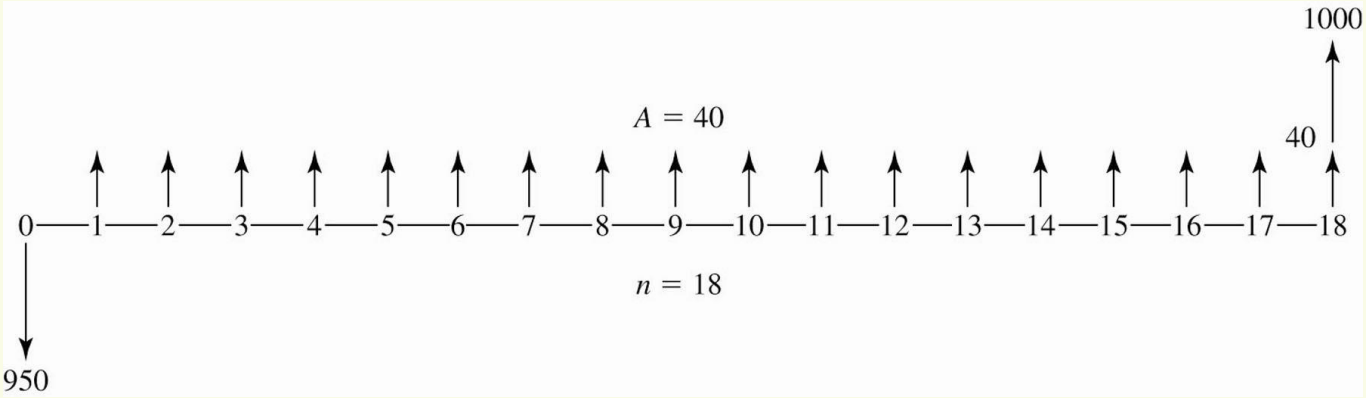
- A new corporate bond was initially sold to an investor for $1,000. The issuing firm promised to pay the coupon value of $40 every 6 months, and the $1,000 face value at the end of 10 years. After 1 year, the buyer of the bond sold it for $950.

(a) What rate of return did the *buyer* receive on this investment?



# EXAMPLE 1-5. IRR OF BONDS (2)

(b) What rate of return can the *new buyer* expect if he/she keeps the bond for its remaining 9-year life?



# EXAMPLE 1-6. IRR OF STOCK

- Jan purchased 100 shares for $18 per share, plus a $45 brokerage commission. Every 6 months, she received a dividend of 50 cents per share. At the end of 2 years, just after receiving the fourth dividend, she sold the stock for $23 per share and paid a $58 brokerage commission. What *annual* rate of return did she receive on her investment?

# 2. Applications of the IRR method

- Fees and cash discounts
  - In many transactions, we often encounter *fees* and *discounts* which are provided by the seller.
  - These practices raise or lower the interest rate on a loan.
  - We need to find the true cost of a loan (or discount), which is its IRR.
  - Examples: Cash rebate vs. a low-interest loan
- Loans and investments
  - Similar approaches of IRR analysis can be applied to loans and investments.
  - Should I buy a parking permit for a year vs. for a term? Choosing a shorter period is a loan.
  - Should I pay insurance payment quarterly or yearly?
  - Should I buy or lease a car? Leasing is a loan.

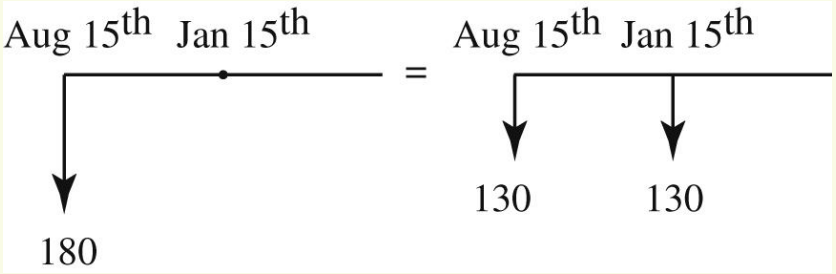
# Example 2-1. Cash discount

- A firm is considering to buy a property. There exist two options to acquire the property whose price is $300,000.
  - Option 1: Finance through the seller with 20% down and the balance due in 5 equal annual payments at 12% interest rate.
  - Option 2: Pay cash with 10% discount. Though the firm does not have $270,000 in cash, it can borrow this amount from a bank.
- What is the IRR for the loan offered by the seller?

# Example 2-2. Parking permit (1)

- There are two options to buy a parking permit on campus.
  - Option 1: Buy an annual parking permit on August 15th at $180.
  - Option 2: Buy semester permits on August 15th and January 15th at $130 each.
- What is the IRR for buying the annual permit?

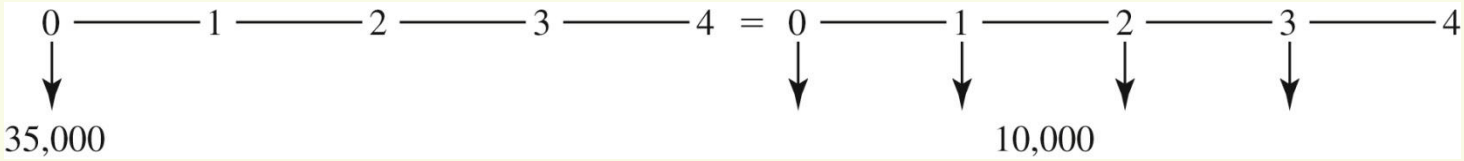
  - In option 2, we are borrowing $50 to delay the payment.
  - We equate the $50 cost difference now with the $130 cost to buy another semester permit in 5 months.



# Example 2-2. Parking permit (2)

# Example 2-3. Insurance payment

- A firm can pay for its liability insurance in one of two options.
  - Annual payment: Pay $35,000 annually
  - Quarterly payment: Pay $10,000 each quarter, starting from now.
- What is the IRR for paying on annual basis?



# Example 2-4. Leasing vs. buying

- There are two options to buy a car which can be used for 2 years.
  - Buying: Buy the car at $40,000, with salvage value of $6,000 at the end of 2 years.
  - Leasing: Lease the car at $2,500 per month, starting from now.
- What is the IRR or the cost of leasing option?

# Example 2-5. College education (1)

- Why do you want to get a college education?
- One of the reasons is to earn higher future earnings in comparison to non-college graduates.
  - Sharon estimates that a college education has a $28,000 equivalent cost at graduation.
  - She believes the benefits of her education will occur throughout 40 years of employment.
  - During the first 10 years, her income will be higher than that of a non-college graduate by $3,000 per year.
  - During the subsequent 10 years, it will be higher by $6,000.
  - During the last 20 years of employment, it will be higher by $12,000.
- What rate of return will she receive from a college education?

# Example 2-5. College education (2)

# 3. Incremental analysis of IRR

- Previous examples of choosing one of two alternatives can be formally examined by the *incremental analysis*.
- Types of projects
  - *Independent* projects: Choose any projects whose IRR > MARR.
  - *Mutually exclusive* projects: A common case in which only one (or no) project should be chosen.
  - Should you choose the project with the highest IRR?
  - No. For mutually exclusive projects, the simple comparison of IRRs of each project is not a correct approach.
- Incremental analysis
  - When there are two or more (mutually exclusive) alternatives, we should perform the IRR analysis by computing the *incremental IRR.*
  - This is the trickiest aspect of the IRR analysis.

# Example 3-1. Why incremental analysis? (1)

- Suppose you must choose one of two mutually exclusive projects.
  - Option 1: Invests $1,000 today and returns $1,500 one year later.
  - Option 2: Invests $2,000 today and returns $2,800 one year later.
- Any money not invested here may be invested elsewhere at the MARR of 6%. Which one should you choose?

# 3.1. Steps of incremental analysis

- For mutually exclusive projects, you should choose option 2. Why?
  - Remember that you only have a *single chance* of investment.
  - For any money not invested here, you should check whether the IRR of the incremental investment earns at least larger than MARR.
- Steps of incremental analysis
  - Order the projects from the lowest to highest *initial cost*.
  - Start with the lowest cost project, and check whether IRR > MARR. If so, it is the *current best*.
  - Check whether IRR of the '*incremental investment*' between the current best and *challenger* exceeds MARR.
  - If so, the challenger becomes the current best. Otherwise, another new challenger is compared with the current best.
  - The process continues until no further challengers exist.

# Example 3-1. Why incremental analysis? (2)

- Let's use the incremental investment approach of IRR in our example.

# Example 3-2. Incremental analysis (1)

- A firm wants to install an equipment whose service life is 5 years with no salvage value. The equipment will save $1,200 per year for 5 years. Two suppliers are considered:
  - Leaseco: It charges three beginning-of-year annual payments of $1,000 each.
  - Saleco: It charges $2,783 now.
- If the MARR is 10%, which supplier should be selected?

| Year | Leaseco | Saleco | Difference between Alternatives: Saleco – Leaseco |
|---|---|---|---|
| 0 | −$1,000 | −$2,783 | −$1,783 |
| 1 | −1,000<br>+1,200 | +1,200 | +1,000 |
| 2 | −1,000<br>+1,200 | +1,200 | +1,000 |
| 3 | +1,200 | +1,200 | 0 |
| 4 | +1,200 | +1,200 | 0 |
| 5 | +1,200 | +1,200 | 0 |

# Example 3-2. Incremental analysis (2)

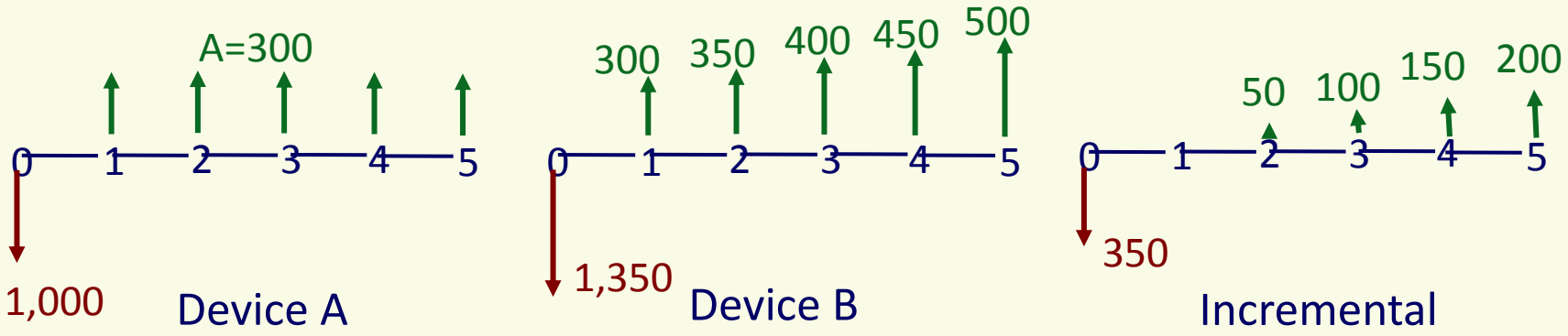
# Example 3-3. Multiple alternatives

- WatAir plans to buy a new metal cutter, out of 3 options, each of which has a life of 10 years with no salvage value. Given a MARR of 15%, which alternative should be chosen?

| | Option 1 | Option 2 | Option 3 |
|---|---|---|---|
| Initial cost | $100,000 | $150,000 | $200,000 |
| Annual savings | $25,000 | $34,000 | $46,000 |

# Example 3-4. Incremental analysis (1)

- A device can be installed to reduce cost. It has no salvage value at the end of 5 years of useful life. Two options exist:
  - Device A: Costs $1,000 and saves $300 annually.
  - Device B: Costs $1,350 and saves $300 the 1st year and increases $50 annually.
- For a 7% MARR, which device should the firm purchase?



# Example 3-4. Incremental analysis (2)

# 4. Problems of IRR analysis

- The main difficulty (or shortcoming) of the IRR analysis is that there may be *multiple IRRs*.

- Suppose a project pays \$1,000 today, costs \$5,000 a year from now, and pays \$6,000 in two years. What is the IRR?

$$1{,}000 + 6{,}000(P/F, i, 2) = 5{,}000(P/F, i, 1)$$

$$1 - 5/(1 + i) + 6/(1 + i)^2 = 0$$

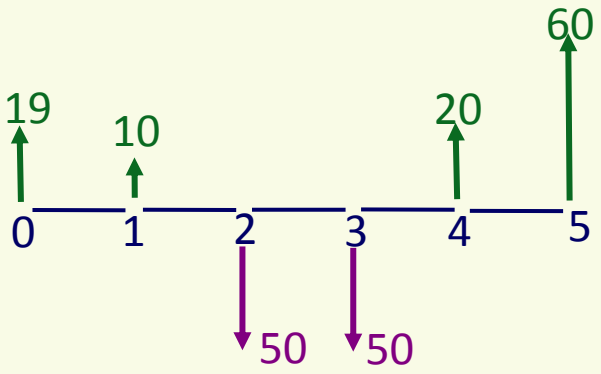
$$(i - 1)(i - 2) = 0 \qquad \Rightarrow i = 100\% \text{ or } 200\%$$

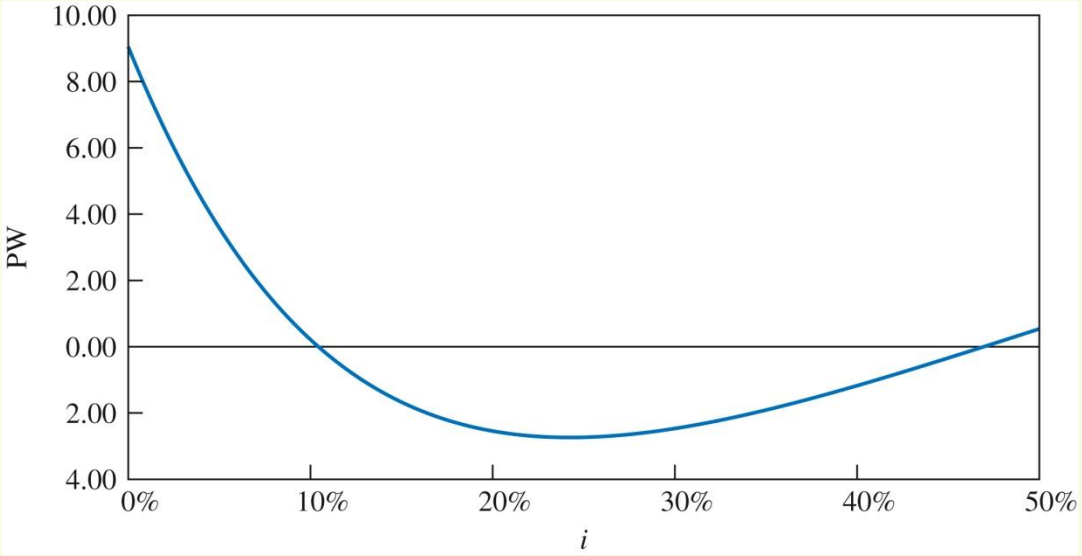
- There are two IRRs for this project. Which one to use?
  - There is no easy criterion of choice.
  - We have to use an alternative IRR method.

# EXAMPLE 4-1. MULTIPLE IRRS

- Consider the following cash flows.

| Year | Cash flow |
|---|---|
| 0 | 19 |
| 1 | 10 |
| 2 | -50 |
| 3 | -50 |
| 4 | 20 |
| 5 | 60 |



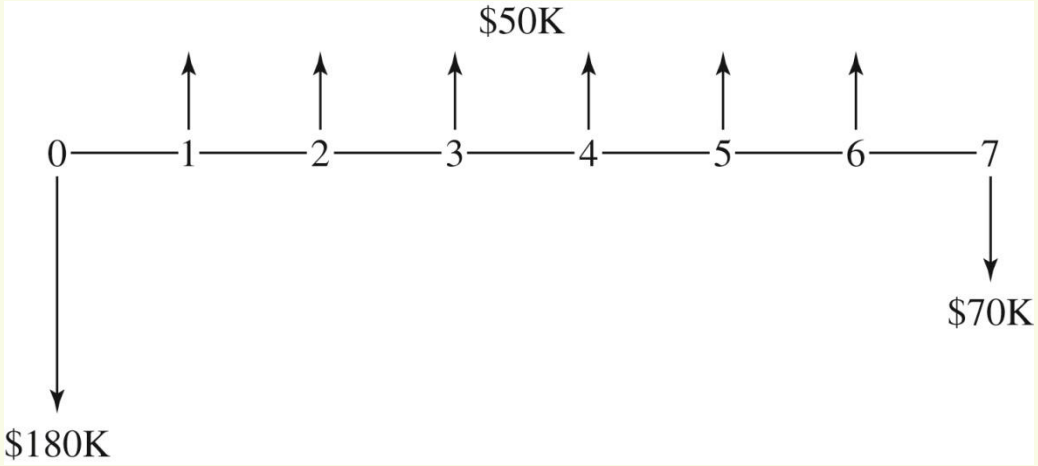



# 4.1. Why multiple solutions?

- Multiple IRRs *may* exist if there are multiple changes in the signs of the cash flows.
  - *Simple investment* (only one change in sign) leads to a single IRR.
  - *Non-simple investment* (multiple changes in sign) *may* produce multiple IRRs.
  - Non-simple investment implies that there are both loans and investments in the given cash flow.
- Dealing with multiple IRRs
  - Multiple IRRs should be suspected if there are multiple sign changes.
  - If a project has one positive IRR and multiple negative IRRs, use the positive IRR.
  - If there are multiple positive IRRs, none of them should be considered a suitable measure of IRR.

# EXAMPLE 4-2. ONE POSITIVE IRR (1)

- Consider the following cash flow. How many IRR for the PW equation exists?




- With two changes in signs, there can be 0, 1 or 2 IRRs for this project.
- Using Excel, we can draw the PW line for different interest rate.

# EXAMPLE 4-2. ONE POSITIVE IRR (2)

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | Year | Cash Flow | | $i$ | PW | | |
| 2 | 0 | −180 | | −40% | −126.39 | =$B$2+NPV(D2,$B$3:$B$9) | |
| 3 | 1 | 50 | | −30% | 219.99 | | |
| 4 | 2 | 50 | | −20% | 189.89 | | |
| 5 | 3 | 50 | | −10% | 114.49 | | |
| 6 | 4 | 50 | | 0% | 50.00 | | |
| 7 | 5 | 50 | | 10% | 1.84 | | |
| 8 | 6 | 50 | | 20% | −33.26 | | |
| 9 | 7 | −70 | | 30% | −59.02 | | |
| 10 | | | | 40% | −78.24 | | |
| 11 | IRR | 10.45% | | 50% | −92.88 | | |
| 12 | root | −38.29% | | | | | |
| 13 | | | | | | | |
| 14 | | | | | | | |
| 15 | | | | | | | |
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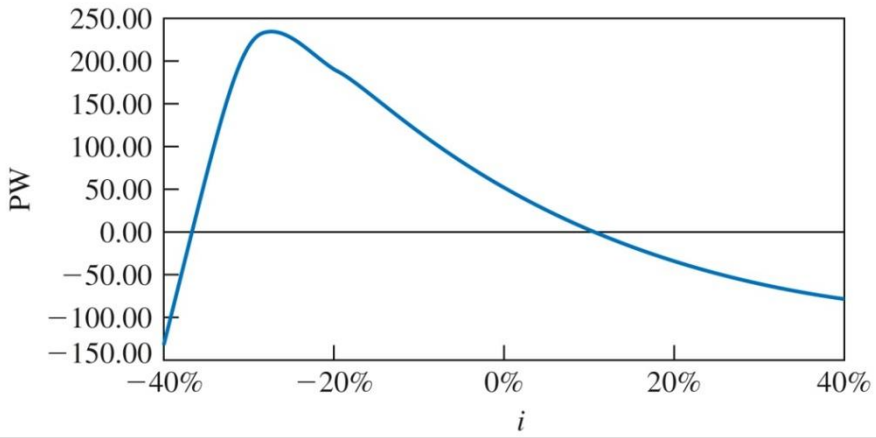
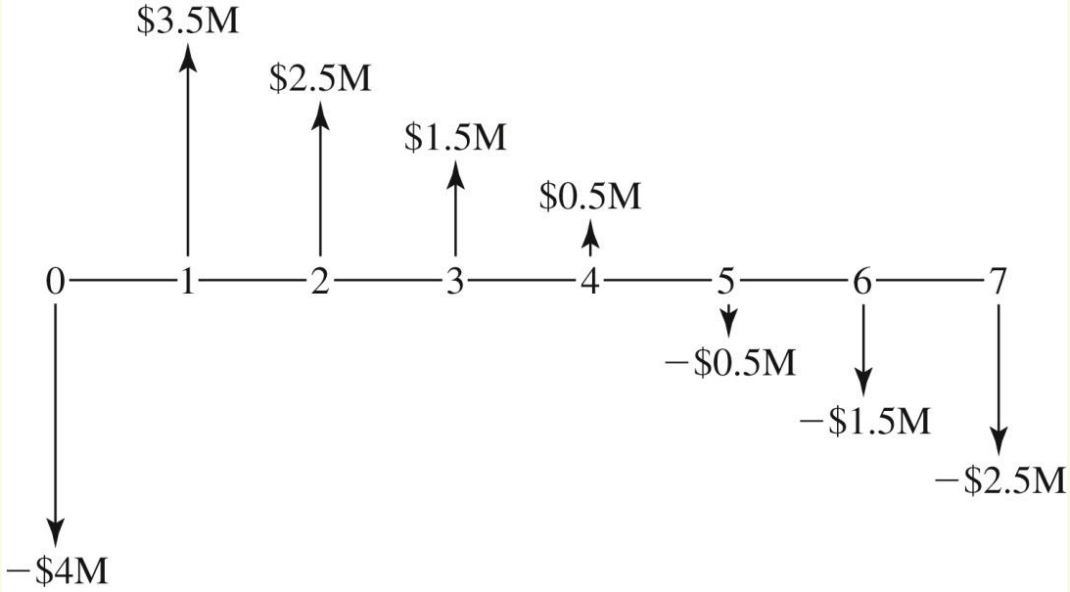


# EXAMPLE 4-3. TWO POSITIVE IRRS (1)

- Consider the following cash flow. How many IRR for the PW equation exists?



# Example 4-3. Two positive IRRs (2)

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | Year | Cash Flow | | $i$ | PW | | |
| 2 | 0 | −4.00 | | 0% | −0.50 | =$B$2+NPV(D2,$B$3:$B$9) | |
| 3 | 1 | 3.50 | | 5% | 0.02 | | |
| 4 | 2 | 2.50 | | 10% | 0.28 | | |
| 5 | 3 | 1.50 | | 15% | 0.37 | | |
| 6 | 4 | 0.50 | | 20% | 0.36 | | |
| 7 | 5 | −0.50 | | 25% | 0.29 | | |
| 8 | 6 | −1.50 | | 30% | 0.19 | | |
| 9 | 7 | −2.50 | | 35% | 0.06 | | |
| 10 | | | | 40% | −0.08 | | |
| 11 | root | 4.73% | | 45% | −0.22 | | |
| 12 | root | 37.20% | | 50% | −0.36 | | |
| 13 | | | | | | | |
| 14 | | | | | | | |
| 15 | | | | | | | |
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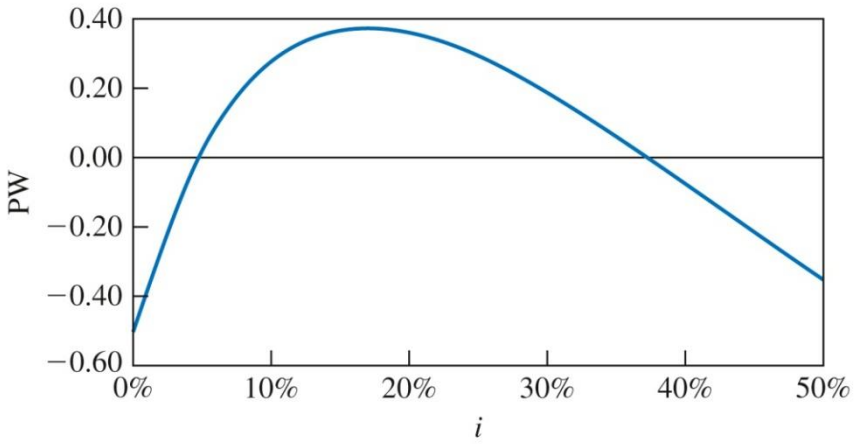
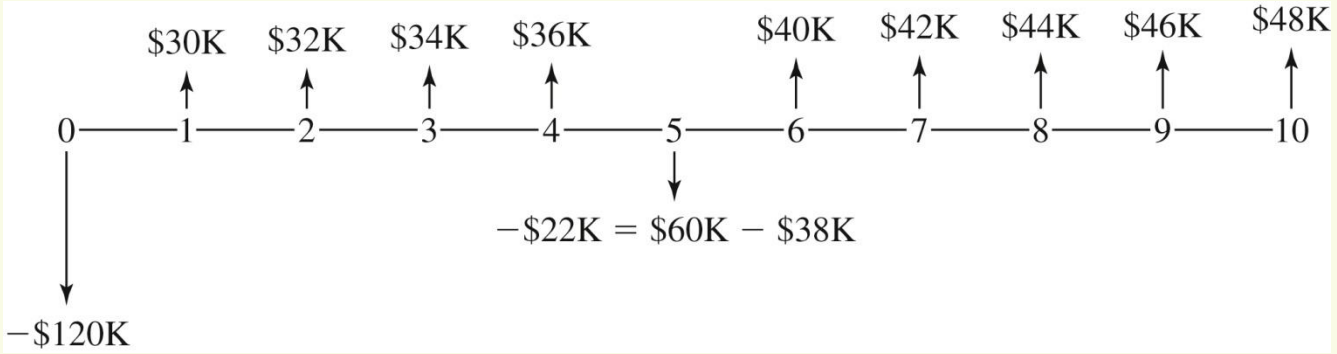


# EXAMPLE 4-4. NON-SIMPLE INVESTMENT (1)

- Consider the following cash flow. How many IRR for the PW equation exists?




- The cash flow is non-simple because there are multiple sign changes.
- However, there is a single IRR.

# Example 4-4. Non-simple investment (2)

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | Year | Cash Flow | | $i$ | PW | | |
| 2 | 0 | −120 | | −80% | 579646330 | =$B$2+NPV(D2,$B$3:$B$12) | |
| 3 | 1 | 30 | | −60% | 735723 | | |
| 4 | 2 | 32 | | −40% | 17651 | | |
| 5 | 3 | 34 | | −30% | 1460 | | |
| 6 | 4 | 36 | | 0% | 210 | | |
| 7 | 5 | −22 | | 10% | 73 | | |
| 8 | 6 | 40 | | 20% | 7 | | |
| 9 | 7 | 42 | | 30% | −28 | | |
| 10 | 8 | 44 | | 40% | −48 | | |
| 11 | 9 | 46 | | 50% | −62 | | |
| 12 | 10 | 48 | | 60% | −71 | | |
| 13 | | | | 70% | −78 | | |
| 14 | IRR | 21.69% | | 80% | −83 | | |
| 15 | | | | 90% | −87 | | |
| 16 | | | | | | | |
| 17 | | | | | | | |
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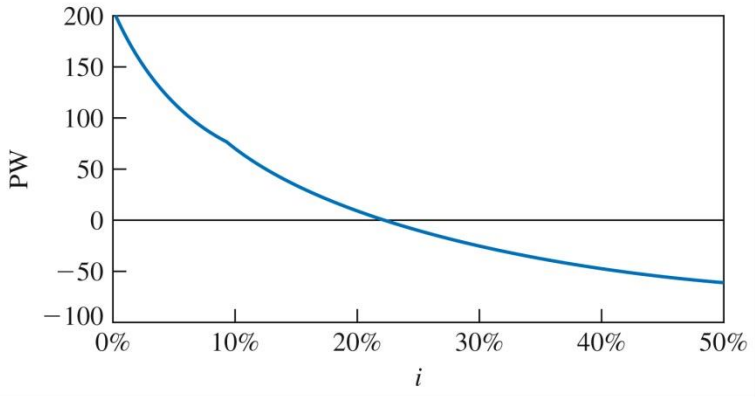


# 4.2. What to do with multiple IRRs?

- Modified IRR (MIRR)
  - For the cash flow with multiple IRRs, we use the *modified IRR*.
  - There exists only one MIRR for any project.
- How to calculate MIRR?
  - We consider two interest rates, one for investment and one for financing (which is often lower than the investment rate).
  - For revenue, calculate the *future worth* with the investment rate.

    $$\text{FW} = R(F/P, i(\text{investment}))$$

  - For expense, calculate the *present worth* with the financing rate, and the convert it to future worth with MIRR.

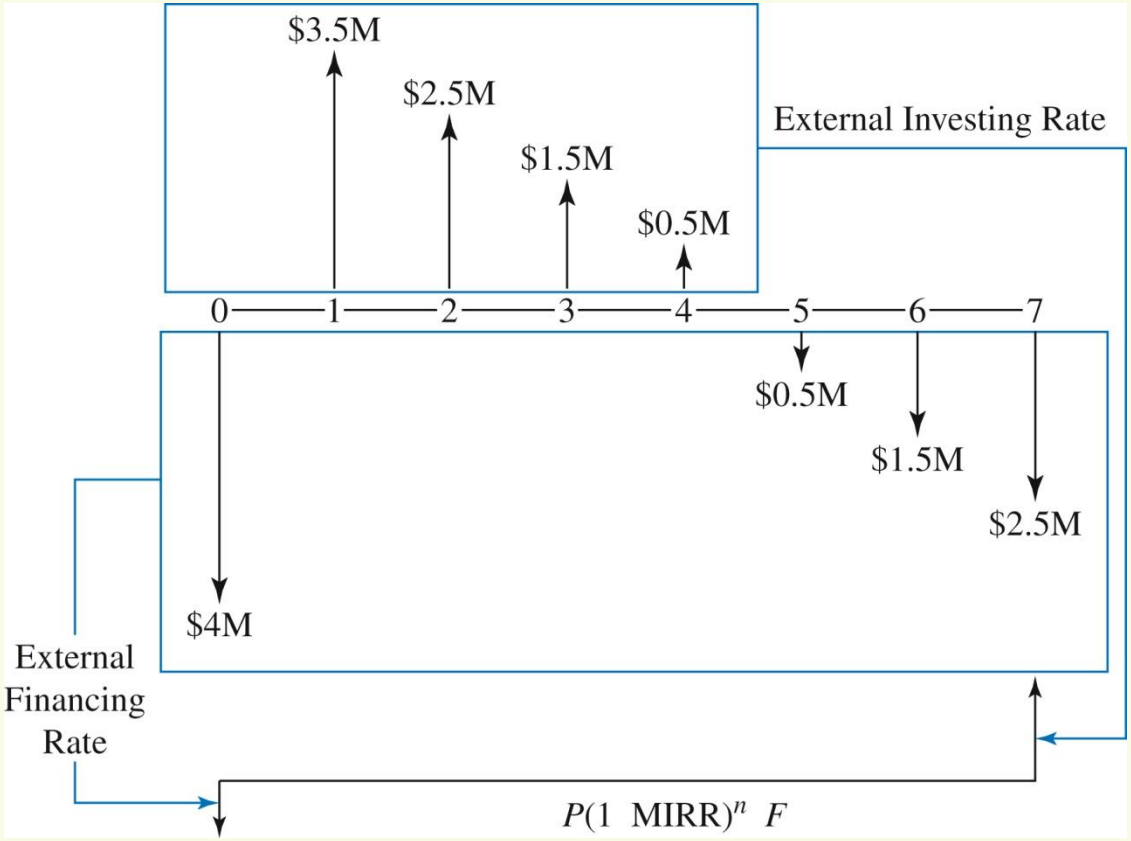
    $$\text{PW} = E(P/F, i(\text{financing})) \quad \Rightarrow \quad \text{FW} = (F/P, \text{MIRR})\text{PW}$$

  - Find the MIRR which makes the PW and FW equivalent

    $$(F/P, \text{MIRR})E(P/F, i(\text{financing})) = R(F/P, i(\text{investment}))$$

# Example 4-5. Modified IRR (1)

- Consider the cash flow in Example 4-3. If the firm normally borrows money at 8% and invests at 15%, find MIRR.



# Example 4-5. Modified IRR (2)

- Find the present worth of the *expenses* with the financing rate.

  $PW = 4 + 0.5(P/F, 8\%, 5) + 1.5(P/F, 8\%, 6) + 2.5(P/F, 8\%, 7)$

  $= 4 + 0.5(0.6806) + 1.5(0.6302) + 2.5(0.5835) = 6.744$

- Find the future worth of the *receipts* with the investing rate.

  $FW = 3.5(F/P, 15\%, 6) + 2.5(F/P, 15\%, 5) + 1.5(F/P, 15\%, 4)$

  $+ 0.5(F/P, 15\%, 3) = 16.507$

- Find the MIRR that makes both worths equivalent.

  $(1 + MIRR)^n PW = FW$

  $\Rightarrow (1 + MIRR) = (16.507/6.744)^{1/7}$

  $\Rightarrow$ MIRR = **13.64%**

- In practice, you better use Excel to find MIRR.

  "= MIRR(values, finance_rate, invest_rate)"

# Summary of chapter 7

- Definition and calculation of IRR
  - Discounts and fees
  - Investments and loans
- Incremental analysis
- Multiple IRRs and Modified IRR (MIRR)

- Selective end-of-chapter problems
  - 5, 9, 13, 15, 26, 28, 32, 36, 46(a), 54, 56, 73