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By filling out the names above, the group members acknowledge that a) they have jointly authored this submission, b) this work represents their original work, c) that they have not been provided with nor examined another person's assignment, either electronically or in hard copy, and d) that this work has not been previously submitted for academic credit.			

LAB 1. INSTRUMENTATION AND MEASUREMENT TECHNIQUES

ASSIGNED DATA

For easily referencing it, the Assigned Data has been placed at the start of this document.

With your pre-lab and post-lab submissions, always include this page at the beginning of your report.

Select your lab session:	☐ morning lab; ☒ afternoon lab; ☒ Tue; ☐ Wed; ☐ Thu
CourseBook Group Number	$GroupNum =$
Assigned bandwidth (open-loop) formula [rad/s]	$GroupNum \times 30 + 350 - rounddown(GroupNum/21.5,0) * 650$ (valid Excel formula syntax; result should be between 300 and 1000)
Assigned bandwidth value (open-loop) [rad/s]	400
Assigned K_p value	3

Prelab

A)

$$
\begin{aligned}
y(t) &= K * (1 - e^{-aTt}) \\
y(\frac{1}{aT}) &= K(1 - e^{-1}) \\
&= 0.63K
\end{aligned}
$$

B)

$$
\begin{aligned}
0.98K &= y(t) \\
&= K(1 - e^{-aTt}) \\
1 &= 0.02e^{aTt} \\
50 &= e^{aTt} \\
\ln 50 &= aTt \\
t &= \frac{\ln 50}{aT}
\end{aligned}
$$

C)

$$
\begin{aligned}
dB &= 20\log_{10}\left(\frac{V_{out}}{v_{in}}\right) \\
dB &= 20\log_{10}\left|\frac{KaT}{jw + aT}\right| \\
dB_{low} &= 20\log_{10}\left|\frac{KaT}{aT}\right| \\
dB_{low} &= 20\log_{10} K
\end{aligned}
$$

$$
\begin{aligned}
\frac{dB_{low}}{\sqrt{2}} &= 20\log_{10}\left|\frac{KaT}{jw_c + aT}\right| \\
\frac{20\log_{10} K}{\sqrt{2}} &= 20\log_{10}\left|\frac{KaT}{400j + aT}\right| \\
\frac{\log_{10} K}{\sqrt{2}} &= \log_{10}\left(\frac{KaT}{\sqrt{1600 + a^2T^2}}\right) \\
10^{\frac{\log_{10} K}{\sqrt{2}}} &= 10^{\log_{10}(\frac{KaT}{\sqrt{1600+a^2T^2}})} \\
K^{\frac{1}{\sqrt{2}}} &= \frac{KaT}{\sqrt{1600 + a^2T^2}} \\
K^{\sqrt{2}} &= \frac{K^2a^2T^2}{1600 + a^2T^2} \\
1600K^{\sqrt{2}} &= (K^2 - K^{\sqrt{2}})a^2T^2 \\
a^2T^2 &= \frac{1600K^{\sqrt{2}}}{K^2 - K^{\sqrt{2}}} \\
aT &= \sqrt{\frac{1600K^{\sqrt{2}}}{K^2 - K^{\sqrt{2}}}}
\end{aligned}
$$

Let K = 3

$$
\begin{aligned}
aT &= \sqrt{\frac{1600 \times 3^{\sqrt{2}}}{3^2 - 3^{\sqrt{2}}}} \\
aT &= 42.09
\end{aligned}
$$

Since T must be $\{1, 10, 100\}$ and a must be (1,10) one possible answer is $a = 4.2$ and $T = 10$.

D)

$$Y(S) = \frac{KaT}{s + aT}$$
$$Y(jw_c) = \frac{KaT}{jw_c + aT}$$
$$\text{Since } \tau = \frac{1}{aT} \text{ then } aT = \frac{1}{\tau}$$
$$Y(jw_c) = \frac{KaT}{jw_c + \frac{1}{\tau}}$$
$$\frac{dB_{low}}{\sqrt{2}} = \frac{K\frac{1}{\tau}}{jw_c + \frac{1}{\tau}}$$
$$w_c = \frac{(K - \frac{dB_{low}}{\sqrt{2}}) \times \frac{1}{\tau}}{j \times \frac{dB_{low}}{\sqrt{2}}}$$
$$w_c = \frac{(K - \frac{dB_{low}}{\sqrt{2}})}{\tau \times j \times \frac{dB_{low}}{\sqrt{2}}}$$

In this system the relationship between bandwidth and time constant is a coefficient dependent on K.

E)

Let the signal between the ACS-13001 and ACS-13002 be called H(S).

$$
\begin{aligned}
Y(S) &= H(S)G(S) \\
H(S) &= R(S) - Y(S) \\
R(S) &= H(S) + Y(S) \\
&= H(S) + H(S)G(S) \\
&= \frac{G(S)}{1+G(S)} \\
Y(S) &= R(S) \times \frac{G(S)}{1+G(S)} \\
&= \frac{1}{s} \times \frac{\frac{KbT}{s+aT}}{1+\frac{KbT}{s+aT}} \\
&= \frac{KbT}{s(s+aT)(1+\frac{KbT}{s+aT})} \\
&= \frac{KbT}{s^2+(1+K)saT}
\end{aligned}
$$

Now solving for the low frequency gain

$$
\begin{aligned}
dB_{low} &= 20\log\left(\frac{V_{out}}{V_{in}}\right) \\
&= 20\log\left|s \times \frac{KbT}{s^2+(1+K)saT}\right| \\
&= 20\log\left|\frac{KbT}{s+(1+K)aT}\right| \\
&= 20\log\left|\frac{KbT}{jw+(1+K)aT}\right| \\
\text{low frequency gain occurs at} w &= 0 \\
&= 20\log\left|\frac{KbT}{0+(1+K)aT}\right| \\
&= 20\log\left|\frac{KbT}{(1+K)aT}\right| \\
\text{Subbing in values from c) } a &= b \\
&= 20\log\left(\frac{K}{K+1}\right)
\end{aligned}
$$

Now solving for bandwidth

$$
\begin{aligned}
\frac{dB_{low}}{\sqrt{2}} &= 20\log\left|\frac{KaT}{jw_c + (1+K)aT}\right| \\
\frac{20\log(\frac{K}{K+1})}{\sqrt{2}} &= 20\log\left|\frac{KaT}{jw_c + (1+K)aT}\right| \\
\frac{\log(\frac{K}{K+1})}{\sqrt{2}} &= \log\left|\frac{KaT}{jw_c + (1+K)aT}\right| \\
\left(\frac{K}{K+1}\right)^{\frac{1}{\sqrt{2}}} &= \left|\frac{KaT}{jw_c + (1+K)aT}\right| \\
\left(\frac{K}{K+1}\right)^{\frac{1}{\sqrt{2}}} &= \frac{KaT}{\sqrt{w_c^2 + (1+K)^2a^2T^2}} \\
\left(\frac{K}{K+1}\right)^{\sqrt{2}} &= \frac{K^2a^2T^2}{w_c^2 + (1+K)^2a^2T^2} \\
\left(\frac{K}{K+1}\right)^{\sqrt{2}} w_c^2 &= K^2a^2T^2 - \left(\frac{K}{K+1}\right)^{\sqrt{2}} \times (1+K)^2a^2T^2
\end{aligned}
$$

Sub in values from c)

$$
\begin{aligned}
\left(\frac{3}{3+1}\right)^{\sqrt{2}} w_c^2 &= 3^2 \times 4.2^2 \times 10^2 - \left(\frac{3}{3+1}\right)^{\sqrt{2}} \times (1+3)^2 \times 4.2^2 \times 10^2 \\
w_c &= 66.16
\end{aligned}
$$