MATH 213 ASSIGNMENT 7

1. Find the Laplace transforms of the following functions:

a)
$$f(t) = \begin{cases} e^t, & 0 \le t \le 1 \\ 0, & t > 1 \end{cases}$$

b)
$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$$

c)
$$f(t) = \begin{cases} t^2, & 0 \le t < 3 \\ 0, & t \ge 3 \end{cases}$$

d) $$f(t) = \begin{cases} \sin 4t, & 0 \le t < \frac{\pi}{4} \\ 0, & t > \frac{\pi}{4} \end{cases}$$

2. Sketch the Bode plots of systems with the following transfer functions:

a) $$H(s) = \frac{s^2 + \sqrt{3}\, s + 3}{s^2 + 2s + 4}.$$

b) $$H(s) = \frac{s}{s^2 + 6s + 9}$$

c) $$H(s) = 5\,\frac{s + 1}{s + 4}$$

3. A system has the transfer function

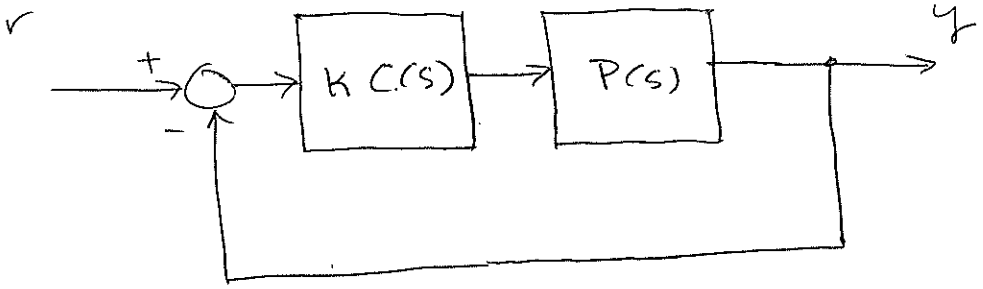
$$G(s) = \frac{s}{s^2 + 2s + 2}$$

Find the (steady-state) response to the input

$$10 \sin(t + 33^\circ)$$

4. (Challenging)

A feedback control system has the following "block diagram":

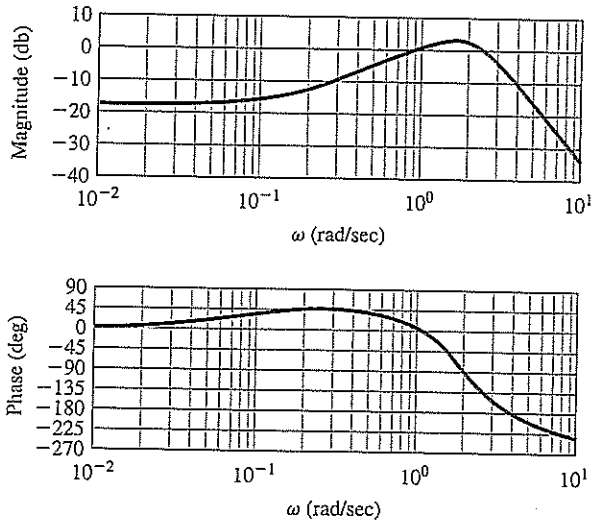


(P(s) is the transfer function of the system under control, and K C(s) is that of the controller; the designer has yet to choose the value of the constant K.)

a) Show that the transfer function $\frac{Y(s)}{R(s)}$ is

$$H(s) = \frac{K\,C(s)\,P(s)}{1 + K\,C(s)\,P(s)}$$

b) The Bode plot corresponding to the transfer function $C(s)P(s)$ is given below:



Comment on the properness of $C(s)P(s)$ and of $H(s)$.

c) For sufficiently small $K > 0$, the transfer function $H(s)$ is stable, but as K is increased, poles of $H(s)$ cross over to the right of the imaginary axis. Use the above Bode plot to find the minimum K for which $H(s)$ is unstable.